ECSE 304-305B Assignment 6 Winter 2007

Return by 12.00 pm, 2nd March

Question 6.1

A positive scalar random variable X with a density is such that $EX = \mu < \infty$, $EX^2 = \infty$.

(a) Using the Markov or Chebyshev inequalities, estimate $P(X^2 \geq \alpha^2)$, $\alpha > 0$.

(b) Explain why the distribution function $F_X(x) = P(X \leq x)$, $x \in \mathbb{R}$ is a continuous function of x.

(c) Show that there exist a real number $\gamma > 0$ such that $e^{-\gamma} = P(X \leq \gamma)$

(d) Split an integral representing Ee^{-X} at $\gamma > 0$, where γ is given in (c), and hence (justifying each step) give a lower bound for Ee^{-X} of the form $ae^{-2\gamma}$, $a > 0$. Determine a value for the positive number a.

Question 6.2

Find (a) the mean value μ, and (b) the variance σ^2 of an RV X with the *Laplace density*

$$f_X(x) = \frac{1}{2b} e^{-2|x-m|/2b},$$

where b and m are real constants, $b > 0$ and $-\infty < m < \infty$.

Find the corresponding characteristic function $\Phi_X(\omega)$ and verify the values found above for μ, σ^2 by use of the Moment Theorem.

Question 6.3

(a) The exponential random variable Z has the density $f_Z(\cdot) = \{5\mu e^{-5\mu z}, z \in R_+, \mu > 0\}$. Find the characteristic function $\Phi_Z(\omega), \omega \in R$.

(b) Let the random variable W be defined by $W = 3(Z_1) - 3(Z_2)$, where Z_1 and Z_2 are independent identically distributed exponential random variables with parameter λ. Find $\Phi_W(\omega), \omega \in R$.

(c) Using the one-to-one relation of characteristic functions and densities and pa rt (b), find the density of W. (Hint: check Question 2.)

Question 6.4

Many people believe that the daily change of price of a company's stock on the stock market is a random variable with mean 0 and variance σ^2. That is, if Y_n represents the price of the stock on the nth day, then

$$Y_n = Y_{n-1} + X_n \qquad n \geq 1$$

where X_1, X_2,... are independent and identically distributed random variables with mean 0 and variance σ^2. Suppose that the stock's price today is 100. If $\sigma^2 = 1$, what can you say about the probability that the stock's price will exceed 105 after 10 days?