ECSE 304-305B   Assignment 4   Winter 2007

Return by noon, Thursday, 8 February

Question 4.1 (SG p. 152)

Let X be a random point be selected from the interval $0, 3)$ (i.e. by the EPP, or, equivalently, such that X has a linearly increasing distribution function). What is the probability that $X^2 - 5X + 6 > 0$?

Question 4.2

For constants a and b, a random variable X has PDF

$$f_X(x) = \begin{cases} ax^2 + bx & 0 \leq x \leq 5, \\ 0 & otherwise \end{cases}$$

What conditions on a and b are necessary and sufficient to guarantee that $f_X(x)$ is a valid PDF?

Question 4.3

A student is allowed to take a certain test three times and the student's final score will be the maximum of the test score. Thus

$$\mathbf{X} = max\{\mathbf{X_1}, \mathbf{X_2}, \mathbf{X_3}\}, \tag{1}$$

where $\mathbf{X_1}, \mathbf{X_2}, \mathbf{X_3}$ are the three test scores and $\mathbf{X}$ is the final score. Assume the score takes values $x, 0 \leq x \leq 100$, and the results of the tests form an independent set of random variables, each with distribution function $F_{X_i} = F(x) = P(\mathbf{X_i} \leq x), \;\; 1 \leq i \leq 3$. Let the distribution function of $\mathbf{X}$ be written as

$$F_{\mathbf{X}}(x) = \{P(\mathbf{X} \leq x), \quad 0 \leq x \leq 160\}. \tag{2}$$

(a) Find which of the following gives $F_{\mathbf{X}}(\cdot)$ and give the derivation:

(i) $F^{1/3}(x^3)$,   (ii) $F^3(x)$,   (iii) $F(x^3)$

(b) If $F(x) = \{0, x < 0; \; \frac{x}{160}, \; 0 \leq x \leq 160; \; 1, 160 \leq x\}$, and given that any score $\mathbf{X}_i$ is uniformly distributed over $[0, 160]$, find $F_{\mathbf{X}}(120) = Pr\{\mathbf{X} \leq 120\}$.