ECSE 304-305B Assignment 3 Winter 2007

Return by noon, Thursday, 1 February

Question 3.1

The diagram on the following page gives the abstracted Markov chain representation of a collection of hurricane random walk (HRW) events in the Atlantic and the Gulf of Mexico in 2004.

Assume that with probability 1 on Day 1 a hurricane is generated off the African coast at location $A,$ and that (as shown in the diagram) on Day 2 with probability $\frac{1}{2}$ it arrives at the North Atlantic (NA) location and with probability $\frac{1}{2}$ at the Mid-Atlantic (MA) location.

Further assume that on Days 3, 4 and 5 the hurricane makes the successive Markov random transitions with the transition (i.e. conditional) properties indicated on the diagram. Assume that the $\frac{1}{3}$ probability path from the Central States (CS) location leads to a location North Carolina (NC) which is not displayed. Further assume that with probability 1 a hurricane at NC, SEC (South East Coast) or NA goes to the location A on the next day.

(i) Given that $P(A) = 1$, find the conditional probabilities:

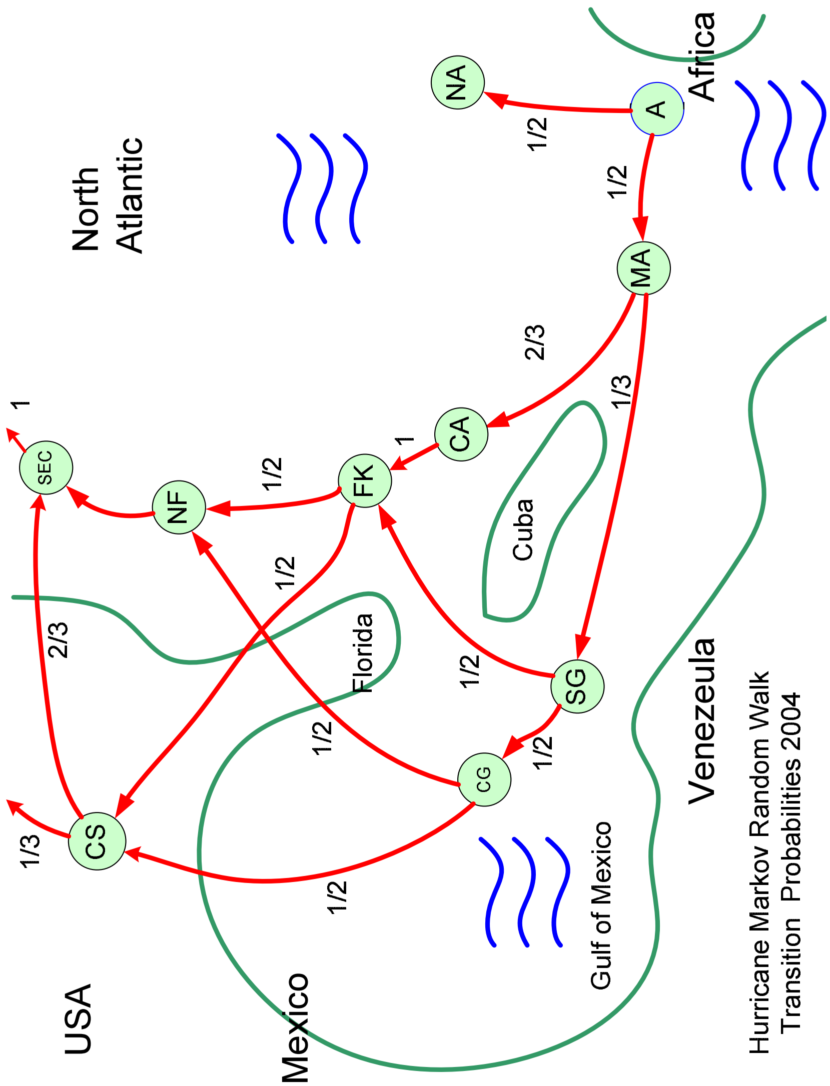
(a) $P(CS|MA),$ (c) $P(CS|A)$, where NF denotes North Florida.

Do this by using the Markov property together with the conditional version of the Total Probability Theorem, namely, $P(A|C) = P(A|B,C)P(B|C) \quad + \quad P(A|B^c,C)P(B^c|C),$ (two element partition case: $B \cup B^c$). Do this at each stage working forwards or backwards in time.

(ii) Given that $P(A) = 1$, find the conditional probabilities:

(a) $P(MA|CS)$, (b) $P(CA|SEC).$

(iii) Take the 11 component vector $[NC, SEC, CS, NF, CG, FK, SG, CA, MA, NA, A]^T$ to denote the state vector of the system. Find the transition matrix of the HRW model. Assume that on Day 1: $P(SG) = \frac{3}{4}, P(CA) = \frac{1}{4}$; then use the transition matrix to find the hurricane probability distribution on Day 3.


Hurricane Markov Random Walk
Transition Probabilities 2004
North
Atlantic
Africa
Venezeula
Cuba
Florida
Gulf of Mexico
USA
Mexico
NA
A
MA
CA
FK
NF
SEC
SG
CG
CS
1/2
1/2
2/3
1/3
1
1/2
1
1/2
2/3
1/2
1/2
1/2
1/2
1/2
1/3

Question 3.1

Question 3.2

The communication channel in Figure 1 corresponds to a Markov chain on $S = \{\alpha, \beta\} \times \{\gamma, \delta\} \times$

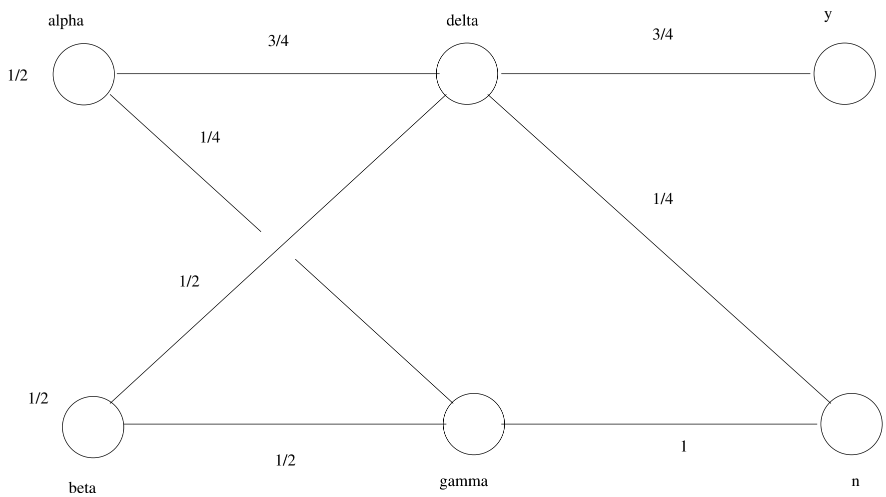


Figure 1: Markovian Communication Channel

$\{y, n\}$, with the indicated prior and conditional probabilities.

Find: (i) $P(n|\alpha)$; (ii) $P(n|\beta)$; (iii)$P(\alpha|n)$

Question 3.3

A web page search algorithm has two independent stages: at Stage 1, the algorithm makes repeated independent attempts at steps $n = 1,\ 2, ..$, to connect with page W_1. The probability of success is $p > 0$ and of failure $1 - p$. Once a success occurs, the algorithm passes to Stage 2, where it makes repeated independent attempts at steps $n = 1,\ 2, ..$, to connect with page W_2. The probability of success is $q > 0$ and of failure $1 - q$. The algorithm halts when it connects to page W_2.

(a) Find the probability the algorithm halts after making exactly n steps of Stage 1 and then making exactly m steps of Stage 2.

(b) Find the probability that the algorithm has a success in Stage 1 at any instant over the infinite range $1, 2, ...$ and then halts after making exactly m steps in Stage 2

(c) Take $p = 1/2$ and $q = 1/3$. Find the probability that the algorithm makes an odd number $2n + 1, n = 0, 1, ...,$ of steps at Stage 1 and then halts at any unspecified iteration in the infinite range $m = 1, 2, ...$ at Stage 2

Question 3.4

In a physical experiment a particle starts in state S_1 with probability 1 and then jumps to either state S_L or state S_R with probability p and $1 - p$ respectively. From state S_L, it jumps to S_1, S_L, S_R, with probabilities $1 - \alpha - \beta$, α and β respectively; and from state S_R, it jumps to S_1, S_L, S_R, with probabilities $1 - \alpha^{'} - \beta^{'}$, $\alpha^{'}$ and $\beta^{'}$ respectively.

(a) Give the transition matrix T for this Markov chain with the probability p_{LL} corresponding to the S_L to S_L transition in the top left position, and with p_{11} corresponding to the S_1 to S_1 transition in the bottom right position.

(b) (i) Assume $\alpha = \frac{1}{2}$, $\beta = 0$, $\alpha^{'} = 0$, $\beta^{'} = \frac{1}{2}$, and $p = (1 - p) = \frac{1}{2}$. Find the probability that a particle will be found in state S_L after two jumps if it starts in state S_1 with probability 1. Do this by evaluating $(T^n)p_0$ or, equivalently, $T^{n-1}(Tp_0)$, for appropriate n and initial probability vector p_0.

(ii) Show which of the occupancy probability vectors $(p_L, p_R, p_1)^T = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})^T$ or $(\frac{1}{4}, \frac{1}{4}, \frac{1}{2})^T$ is a steady state probability for the Markov chain.