**ECSE 304-305B        Assignment 2        Winter 2007**

Return by noon, Thursday, 25th January

**Question 2.1** Let a coin have a probability $\frac{2}{3}$ of coming down Heads in a toss and $\frac{1}{3}$ Tails. Generalize the method in the example below to this biased coin case so as to find the probability of getting three or more heads in five tosses. (This is an example of the use of a *multinomial distribution with parameter* $r$ *=2*.)

Express your answer in terms of fractions and then evaluate it to three decimal places.

**Example:** Find the probability of getting four or more heads in six tosses of an unbiased coin.

**Solution:** $P(4 \text{ or more heads in 6 tosses}) = P(4) + P(5) + P(6)$ where,

$$P(k)=P(k \text{ heads in 6 tosses})=\begin{pmatrix} 6 \\ k \end{pmatrix}/2^6$$

so $P(4$ or more heads in 6 tosses$)=(15+6+1)/2^6$=11/32.

**Question 2.2** Assume that the probability of any particular child being a girl in a given group of families is $\frac{6}{10}$ and the probability the child is a boy is $\frac{4}{10}$. By a double application of the method used to solve Question 2.1, find the probability that among five families, each with six children, at least three of the families have five or more girls? Express your answer in terms of fractions and then evaluate it to three decimal places

**Question 2.3**

A graph $G$ has $N$ nodes $\{n_i; 1 \leq i \leq N\}$ and a set $U$ of undirected edges. $G$ is called a *clique* if each of its $N$ nodes is connected by exactly one edge to every other node. For a clique graph $G$ with $N > 2$ give expressions for the number of:

(a) Distinct unordered edges $(n, n')$ in $G$, where $n, n'$ are distinct nodes in $G$.

(b) Distinct unordered edge pairs $((n, n'), (m, m'))$ in $G$ , where $(n, n')$,$(m, m')$ are distinct edges.

(c) Distinct unordered edges pairs in $G$ which connect at a node, i.e. are of the form $((n', n), (n, n''))$ where $n, n', n''$ are distinct nodes.

(d) Verify your answers to (b) and (c) for a triangle and for a clique graph on four nodes.

**Question 2.4**

In a specific application of the Partition Function of statistical thermodynamics, a substance in a vessel has $n = 10^{10}$ particles at a temperature $\mu = 1/100$, in appropriate units. There are four energy levels $e_i = i^2$, for $i = 1, 2, 3, 4$.

(a) Using the approximate version of Sterling's formula in the lecture notes, find an expression for the ratio of the probability of the most likely (corresponding to the most equal) distribution of particles to the probability of the most unlikely distribution of particles. Do this respecting the Conservation of Particles Constraint (1), but neglecting the Energy Conservation Constraint (2).

(b) Find an expression for the number of particles in each energy level in the most likely configuration (subject to (1) and (2)) by applying the formulas for each $n_i$ in the notes.

(c) Give a formula for the energy per particle in the most likely configuration (subject to (1) and (2)) by using the formula for $\frac{E}{N}$ in the notes.

NB Check web vista for the statistical thermodynamics formulas (typos now corrected).

**Question 2.5**

The elevator of a four-floor building leaves the first floor with six passengers and stops at all of the remaining three floors. If it is equally likely that a passenger gets of at any of these three floors, what is the probability that, at each stop, of the elevator at least one passenger departs?

Hint: This is Problem 28, Chapter 2, Section 2.2 of GS. Use the counting of permutations method which is used to solve Problem 27 (answers in the back of the book). No explicit probabilities are used until you obtain the solution.