ECSE 304-305B   Assignment 1   Winter 2007

Return by noon, Thursday, 18th January

Question 1.1 Assume the failure probabilities of a set of three components are such that: $P[(F_2 \cap F_3) \cap F_1] = 0.003$; $P[(F_2 \cap F_3) \cap \overline{F_1}] = 0.019$ and $P[(\overline{F_2} \cup \overline{F_3}) \cap \overline{F_1}] = 0.009$, where $\overline{F_1}$, etc, denotes complement. Find $P[F_2 \cap F_3]$ and $P[F_1]$. Furthermore, show why one cannot find separately $P[F_2]$ and $P[F_3]$.
Hint: use the Inclusion - Exclusion relation for probabilities.

Question 1.2 Let $A \subset S$, $B \subset S$, where A and B are not necessarily disjoint. Let $\{A_i \subset A, 1 \leq i < \infty\}$ be pairwise disjoint, and let $\{B_i \subset B, 1 \leq i < \infty\}$ be pairwise disjoint.

(i) Let $X \subset A$ and assume $P(A_i) > 0, 1 \leq i < \infty$, and let N be any fixed finite integer. Set $\alpha = P(X \cap (\cup_{i=1}^{N} A_i))$, and set $\beta = P(X) \cdot (\sum_{i=1}^{N} P(A_i))$ and $\gamma = \sum_{i=1}^{N} P(X \cap A_i)$. Prove, which of β or γ, or both, is equal to α for all possible $X \subset A$.

(ii) Give a formula for $P(\cup_{i=1}^{\infty}(A_i \cap B_i))$ in terms of $P(A_i \cap B_i), 1 \leq i < \infty$; do this by use of one of the probability axioms and justify its application.

(iii) In case $A \cap B = \phi$, show what the value of $P(\cup_{i=1}^{\infty}(A_i \cap B_i))$ must be.

Question 1.3 In a branching experiment BE, a coin is such that $Prob(H) = \frac{1}{3}$, $Prob(T) = \frac{2}{3}, (H = head, T = tail)$. The coin is repeatedly tossed until a head is obtained and at that instant BE stops. Distinct tosses are independent, i.e. the probability of a specified head or tail pair at distinct instants is the product of the individual probabilities.

(a) A sample point (i.e. outcome) for BE consists of a sequence (possibly empty) of tails T and a terminating head H. Are the outcomes $\{H, TH, T^2H, ..., T^{N-1}H, ...\}$ distinct? If so, why?

(b) Use Axiom III to derive a simple expression for the probability that BE stops before or at the N^{th} toss, i.e. at an outcome at n, $1 \leq n \leq N, N < \infty$. (Recall that for BE a sample point is an (elementary) event.)

(c) Show whether or not the sample points $\{H, TH, ..., T^{N-1}H, ...\}$ are pairwise independent, i.e. $P(E \cap F) = P(E).P(F)$ for sample points E, F.

Question 1.4 In a specified 24-hour period, a student wakes up at a time T_1 and goes to sleep at a (not necessarily strictly) later time $T_2 \geq T_1$.

(a) Find the sample space for this experiment where the outcome is taken to be the pair (T_1, T_2).

(b) Specify the region A of the plane corresponding to the event "student is awake at 9am."

(c) Specify and sketch the set B in the plane corresponding to the event "student is asleep more time than the student is awake."

(d) Sketch the region corresponding to the event $D = A^c \cap B$ and describe the corresponding event in words. Assuming the Equiprobability Principle holds for the outcomes of the experiment, find the probability of the event D.

Question 1.5 All of the following parts are for Loto 6-49; see Pages 3 and 4, Lecture Notes 3: Counting Probabilities Continued.

(a) What is the probability of getting exactly four numbers correct on one ticket?

(b) What is the probability of getting exactly one or two numbers correct on one ticket? Give the answer correct to three decimal places.

(c) What is the probability of getting exactly one number correct on at least one of two tickets in a single draw? Give the answer correct to three decimal places. (Hint: use the probabilities of complementary events.)

Question 1.6 (From Chapter 1, Question 26,page 25, SG.) For a Democratic candidate to win an election, she must win districts I, II and III. Polls have shown that the probability of wining I and III is 0.55, losing II but not I is 0.34, and losing II and III but not I is 0.15. Find the probability that this candidate will win all three districts. (Draw a Venn diagram.)