PROBLEM SET #11

8-6, 8-7, 8-16, 8-21, 8-26

8-6

$$E(t,z) = \bar{a}_x\, 2\cos(10^8 t - z/\sqrt{3}) - \bar{a}_y \sin(10^8 t - z/\sqrt{3}) \quad (V/m)$$

Phasor: $\bar{E} = \bar{a}_x 2e^{-jz/\sqrt{3}} + a_y j e^{-jz/\sqrt{3}} \quad (V/m)$

a) $w = 10^8$ rad/s $\longrightarrow$ $f = \frac{10^8}{2\pi} = \underline{1.59 \times 10^7 (Hz)}$

$\beta = \frac{1}{\sqrt{3}}$ rad/s $\longrightarrow$ $\lambda = \frac{2\pi}{\beta} = \underline{2\sqrt{3}\,\pi\ (m)}$

b) $u = \frac{c}{\sqrt{\epsilon_r}} = \frac{w}{\beta} \longrightarrow \epsilon_r = \left(\frac{\beta c}{w}\right)^2 = 3$

c) Left hand elliptically polarized.

d) $\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{3}} \quad (\Omega)$

$$\bar{H} = \frac{1}{\eta}\, \bar{a}_z \times \bar{E} = \frac{\sqrt{3}}{120\pi}\left(\bar{a}_y 2e^{-jz/\sqrt{3}} - \bar{a}_x j e^{-jz/\sqrt{3}}\right)$$

so $\bar{H}(z,t) = \frac{\sqrt{3}}{120\pi}\left[\bar{a}_x \sin(10^8 t - z/\sqrt{3}) + \bar{a}_y \cos(10^8 t - z/\sqrt{3})\right] (A/m)$

8-7.

Let $\alpha = \omega t - kz$

$$\bar{E} = \bar{a}_x E_{10} \sin\alpha + \bar{a}_y E_{20} \sin(\alpha+\psi) = \bar{a}_x E_x + \bar{a}_y E_y$$

$$\frac{E_x}{E_{10}} = \sin\alpha \qquad \frac{E_y}{E_{20}} = \sin(\alpha+\psi) = \sin\alpha\cos\psi + \cos\alpha\sin\psi$$

$$= \frac{E_x}{E_{10}}\cos\psi + \sqrt{1-\left(\frac{E_x}{E_{10}}\right)^2}\,\sin\psi$$

$$\left(\frac{E_y}{E_{20}} - \frac{E_x}{E_{10}}\cos\psi\right)^2 = \left(1 - \frac{E_x}{E_{10}}\right)\sin^2\psi$$

$$\left(\frac{E_y}{E_{20}\sin\psi}\right)^2 + \left(\frac{E_x}{E_{10}\sin\psi}\right)^2 - 2\frac{E_x}{E_{10}}\frac{E_y}{E_{20}}\frac{\cos\psi}{\sin^2\psi} = 1 \quad (1)$$

which is the equation of an ellipse.

In order to find the parameters of the polarization ellipse, rotate the coordinate axes x-y counterclockwise by an angle θ to x'-y'. Assume the equation of the ellipse in terms of the new coordinates to be

$$\left(\frac{E_x'}{a}\right)^2 + \left(\frac{E_y'}{b}\right)^2 = 1 \quad (2)$$

8-16

$$\mathcal{P}_{av} = |E|^2/2\eta_0 = 10^{-2} \ (W/cm^2)$$

a) $|E| = \sqrt{0.02\eta_0} = 2.75 \ (V/cm) = 275 \ (V/m)$

$|H| = \dfrac{|E|}{\eta_0} = 7.28\times10^{-3} \ (A/cm) = 0.728 \ (A/m)$

b) $\mathcal{P}_{av} = \dfrac{|E|^2}{2\eta_0} = 1300 \ (W/m^2)$

$|E| = 990 \ (V/m)$

$|H| = 2.63 \ (A/m)$

where $E_x' = E_x \cos\theta + E_y \sin\theta$ (3)

$E_y' = -E_x \sin\theta + E_y \cos\theta$ (4)

Substituting (3) & (4) in (2) and rearranging:

$$E_x^2\left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right) + E_y^2\left(\frac{\sin^2\theta}{a^2} + \frac{\cos^2\theta}{b^2}\right) - 2E_xE_y \sin\theta\cos\theta\left(\frac{1}{b^2} - \frac{1}{a^2}\right) = 1 \quad (5)$$

Comparing (1) & (5), we obtain

$$\begin{cases} \dfrac{\cos^2\theta}{a^2} + \dfrac{\sin^2\theta}{b^2} = \dfrac{1}{E_{10}^2 \sin^2\psi} & (6) \\ \dfrac{\sin^2\theta}{a^2} + \dfrac{\cos^2\theta}{b^2} = \dfrac{1}{E_{20}^2 \sin^2\psi} & (7) \\ \sin\theta\cos\theta\left(\dfrac{1}{b^2} - \dfrac{1}{a^2}\right) = \dfrac{\cos\psi}{E_{10}E_{20}\sin^2\psi} & (8) \end{cases}$$

Eqns (6), (7) and (8) can be solved for three unknowns:

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2E_{10}E_{20}\cos\psi}{E_{10}^2 - E_{20}^2}\right)$$

$$a = \sqrt{2\Big/\left(\frac{1}{E_{10}^2}(1+\sec 2\theta) + \frac{1}{E_{20}^2}(1-\sec 2\theta)\right)}\,\sin\psi$$

$$b = \sqrt{2\Big/\left(\frac{1}{E_{10}^2}(1-\sec 2\theta) + \frac{1}{E_{20}^2}(1+\sec 2\theta)\right)}\,\sin\psi$$

In particular, if $E_{10} = E_{20} = E_0$: $\theta = 45°$, $a = \sqrt{2}E_0\cos\frac{\psi}{2}$

$b = \sqrt{2}E_0\sin\frac{\psi}{2}$

8-21 Given $\bar{E}_i = E_0(\bar{a_x} - j\bar{a_y})e^{-j\beta z}$

a) Assume reflected

$$\bar{E}_r(z) = (\bar{a}_x E_{rx} + \bar{a_y} E_{ry})e^{j\beta z}$$

Boundary condition at $z=0$: $\bar{E}_i(0) + \bar{E}_r(0) = 0$

$$\longrightarrow \bar{E}_r(z) = E_0(-\bar{a_x} + j\bar{a_y})e^{j\beta z}$$

which is a left-hand circularly polarized wave in the $-z$ direction.

b) $\bar{a}_{n2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s \longrightarrow -\bar{a_z} \times [\bar{H}_i(0) + \bar{H}_r(0)] = \bar{J}_s$

$$\bar{H}_i(0) = \frac{1}{\eta_0}\bar{a_z} \times \bar{E}_i(0) = \frac{E_0}{\eta_0}(j\bar{a_x} + \bar{a_y})$$

($\bar{H}_2 = 0$ in a perfect conductor)

$$\bar{H}_r(0) = \frac{1}{\eta_0}(-\bar{a_z}) \times \bar{E}_r(0) = \frac{E_0}{\eta_0}(j\bar{a_x} + \bar{a_y})$$

$$\bar{H}_1(0) = \bar{H}_i(0) + \bar{H}_r(0) = \frac{2E_0}{\eta_0}(\bar{a_x} - j\bar{a_y})$$

$$\bar{J}_s = -\bar{a_z} \times \bar{H}_1(0) = \frac{2E_0}{\eta_0}(\bar{a_x} - j\bar{a_y})$$

c) $\bar{E}_1(z,t) = Re[\bar{E}_i(z) + \bar{E}_r(z)]e^{j\omega t}$

$$= Re\, E_0[(\bar{a_x} - j\bar{a_y})e^{-j\beta z} + (-\bar{a_x} + j\bar{a_y})e^{j\beta z}]e^{j\omega t}$$

$$= Re\, E_0[-2j(\bar{a_x} - j\bar{a_y})\sin\beta z]e^{j\omega t}$$

$$= 2E_0 \sin\beta z(\bar{a_x}\sin\omega t - \bar{a_y}\cos\omega t)$$

8-26

For normal incidence: $1+\Gamma=\tau$,

where $|\Gamma| \le 1$

If $|\tau|=|\Gamma|$; $\Gamma < 0$ and $\eta_1-\eta_2=2\eta_2$,

$\rightarrow \eta_1 = 3\eta_2 \rightarrow |\Gamma| = \frac{1}{2}$

$\therefore \quad S = \dfrac{1+|\Gamma|}{1-|\Gamma|} = 3$

$S_{dB} = 20\log_{10}3 = \underline{9.54\,(dB)}$