# Problem Set 10

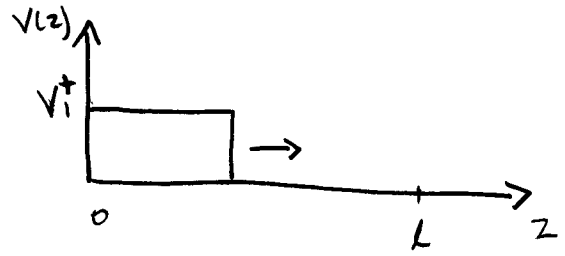
9-33, 9-34, 9-35, 9-36

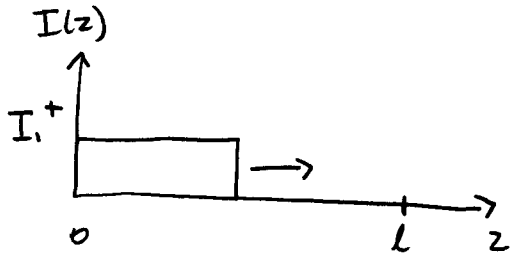
## 9-33

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - \frac{Z_0}{Z_L}}{1 + \frac{Z_0}{Z_L}} = +1$$

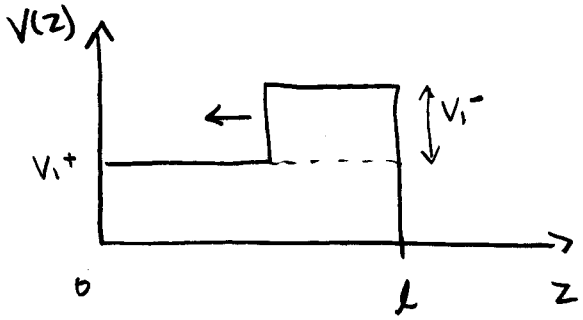
$$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{-Z_0}{Z_0} = -1$$

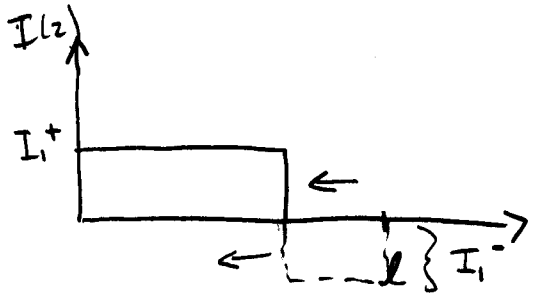
a) $0 < t < T (= \ell/u)$



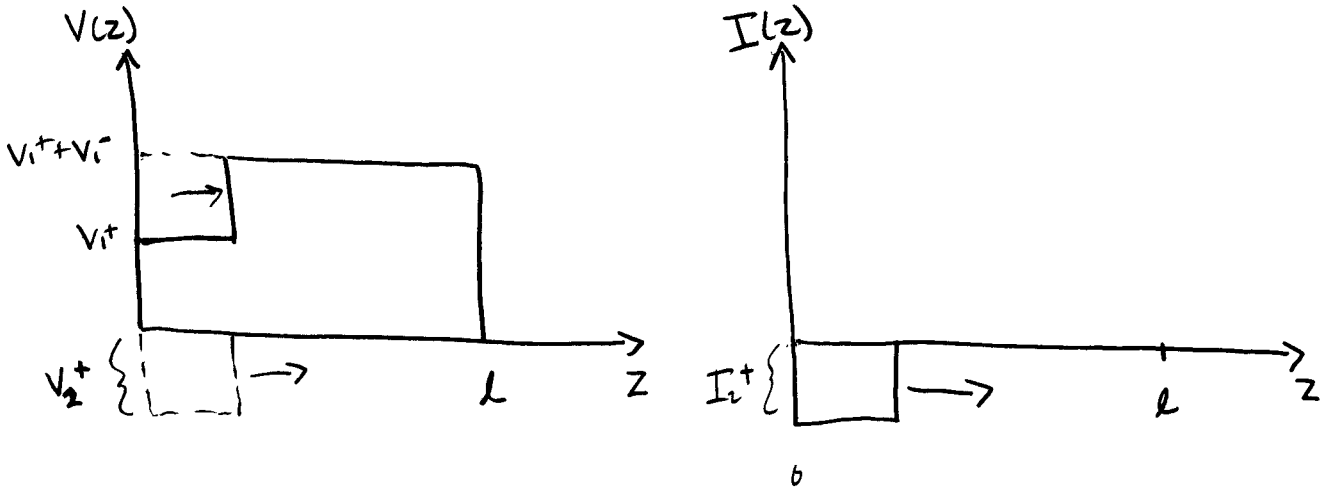




b) $T < t < 2T$



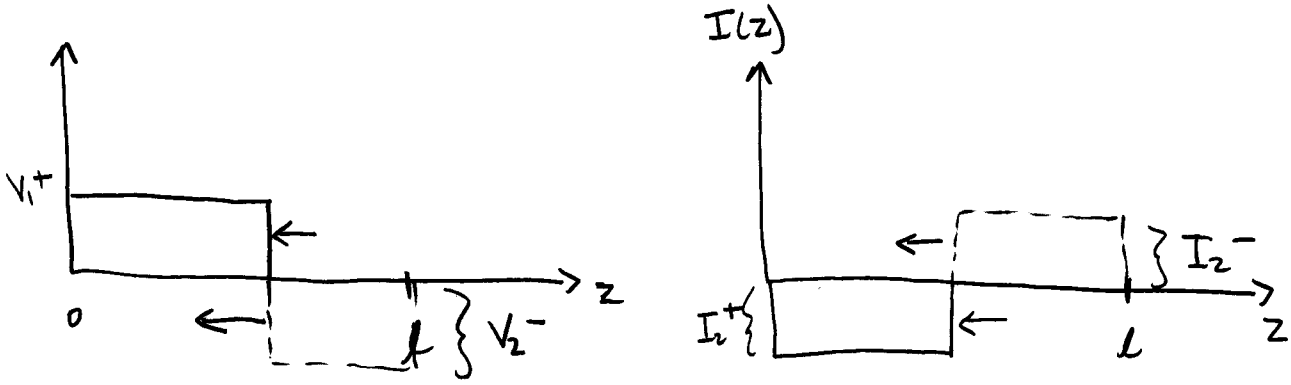



c) $2T < t < 3T$




b

d) $3T < t < 4T$




$V_1^+ = V_0$ , $V_1^- = V_1^+ = V_0$

$V_2^+ = -V_1^- = -V_0$

$V_2^- = V_2^+ = -V_0$

$I_1^+ = \frac{V_0}{R_0}$ , $I_1^- = -I_1^+ = -V_0/R_0$

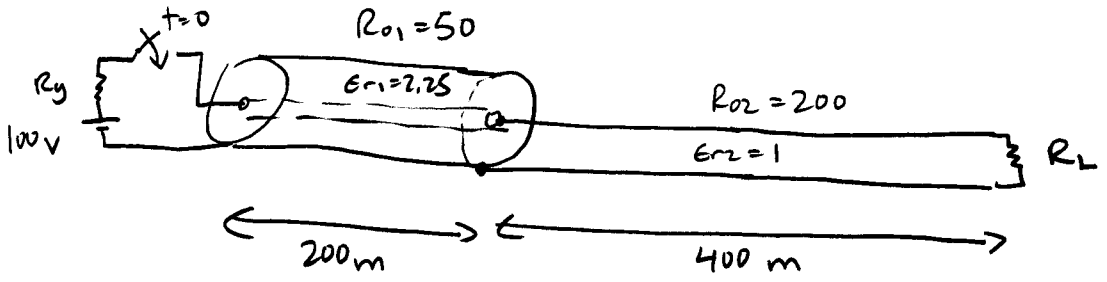
$I_2^+ = I_1^- = -V_0/R_0$

$I_2^- = -I_2^+ = \frac{V_0}{R_0}$

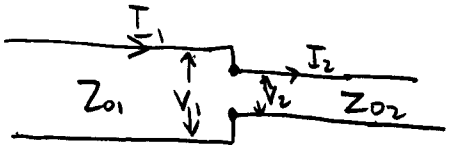
At $t = 4T$, both $V(z)$ & $I(z)$ revert back to the conditions at $t = 0$. and the cycle repeats itself with a period $\underline{4T}$.

9-34






At junction of 2 lines w/ $Z_{01}$ & $Z_{02}$




$V_1 = V_2 \rightarrow V_1^+ + V_1^- = V_2^+$

$I_1 = I_2 \rightarrow I_1^+ + I_1^- = \frac{(V_1^+ - V_1^-)}{Z_{01}} = \frac{V_2^+}{Z_{02}}$

$$V_1^- = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} V_1^+ \quad , \quad V_2^+ = \frac{2Z_{02}}{Z_{02} + Z_{01}} V_1^+$$

$$I_1^- = -\frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} I_1^+ \quad , \quad I_2^+ = \frac{2Z_{01}}{Z_{02} + Z_{01}} I_1^+$$

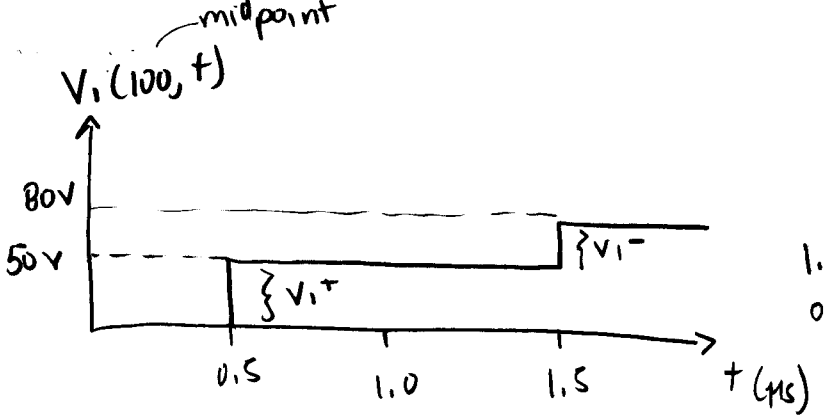
a) $V_1^+ = \frac{V_g}{2} = 50\ V$ $\quad I_1^+ = \frac{V_g}{50+50} = 1\ A$

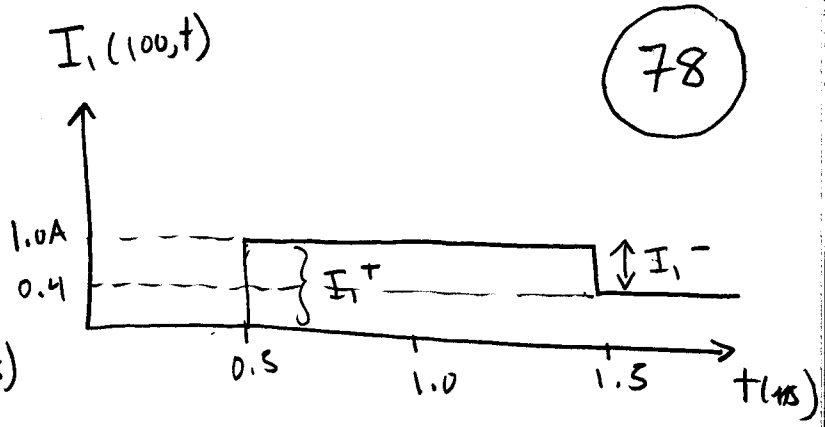
$V_1^- = \frac{200-50}{200+50} V_1^+ = 30\ V$ $\quad I_1^- = \frac{200-50}{200+50} I_1^+ = -0.6\ A$

$V_2^+ = \frac{2\times 200}{200+50} V_1^+ = 80\ V$ $\quad I_2^+ = \frac{2\times 50}{200+50} I_1^+ = 0.4\ A$

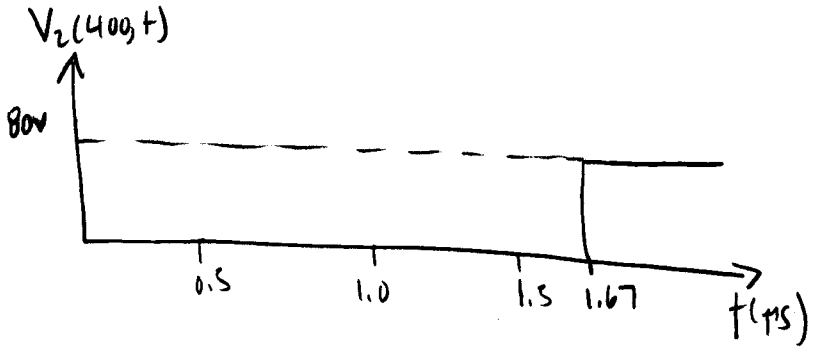
No transient waves on coaxial cable after $V_1^-$ and $I_1^-$ reach input terminals (at $2t_1 = 2\ l_1/u_1 = 400/2\times10^8 = 2\ \mu s$) and no transient waves on two-wire line after $V_2^+$ and $I_2^+$ reach the load $R_L$ (at $t_2 = t_1 + l_2/u_2 = 1\times10^{-6} + 400/3\times10^8 = 2.33\ \mu s$).

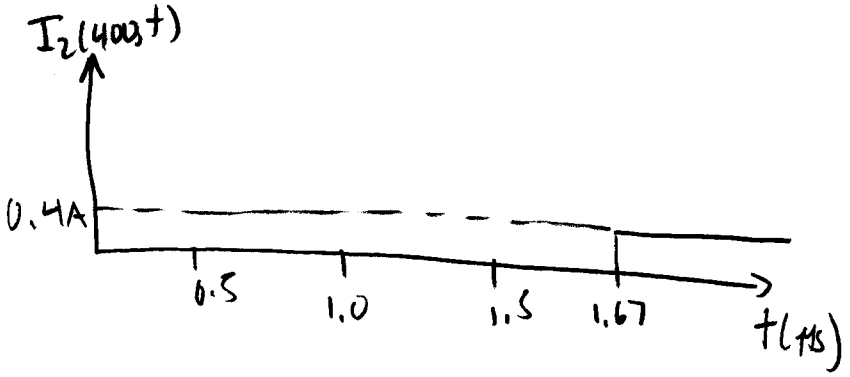
b) On the coaxial cable, it takes $t_{1/2} = 0.5\ \mu s$ for $V_1^+$ & $I_1^+$ to reach midpoint ($x = 100m$). The reflected waves $V_1^-$ and $I_1^-$ arrive at the midpoint at $t = 3t_{1/2} = 1.5\ \mu s$). There are no changes after that.




c) On two wire system, steady state is reached at $t = t_1 + \frac{200}{3\times10^8} = 1.67\,\mu s$ (distance to midpoint from junction!)



## 9-35

$$R_g = \frac{R_0}{2} \rightarrow \Gamma_g = \frac{\frac{R_0}{2} - R_0}{\frac{R_0}{2} + R_0} = \frac{-\frac{1}{2}}{\frac{3}{2}} = -\frac{1}{3}$$

$$R_L = \infty \rightarrow \Gamma_L = 1$$

$$T = \ell/u$$

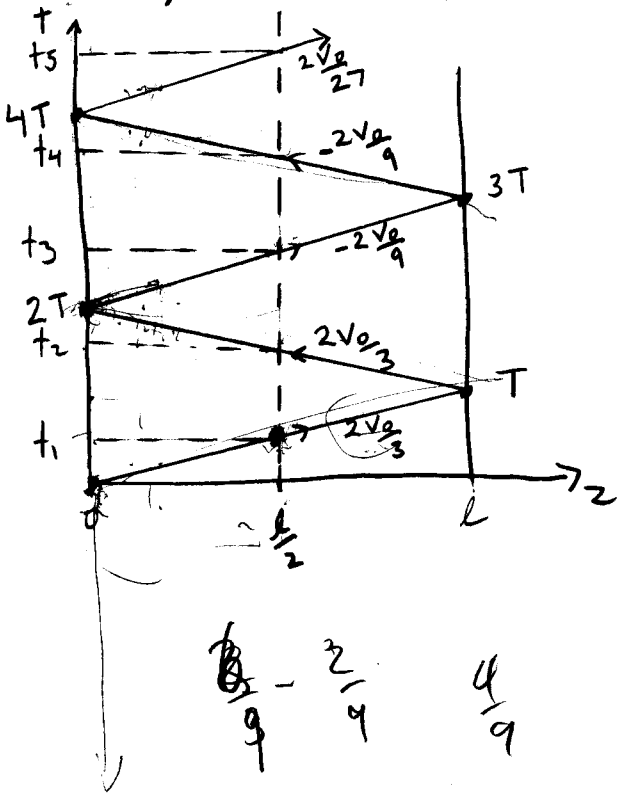
$$V_1^+ = \frac{R_0}{R_0 + \frac{R_0}{2}} V_0 = \frac{2}{3} V_0$$

$$I_1^+ = \frac{V_1^+}{R_0} = \frac{2}{3}\frac{V_0}{R_0}$$

$$t_1 = \frac{1}{2}\frac{\ell}{2u} = \frac{T}{2}$$

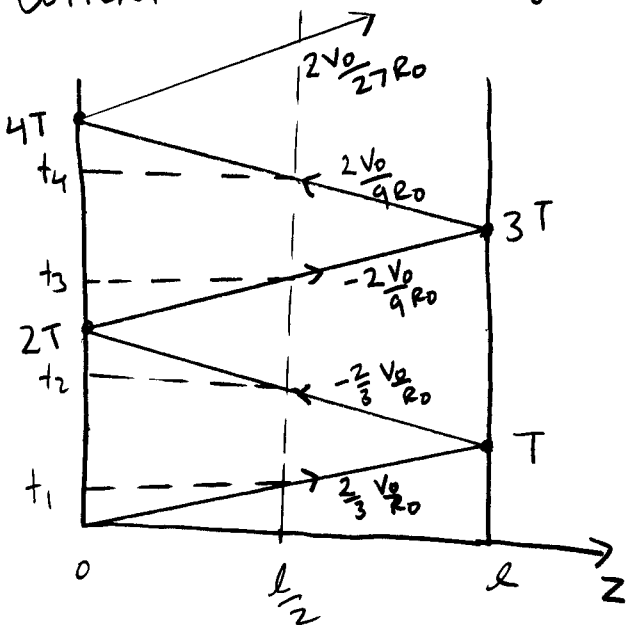
a)

Voltage Reflection Diagram

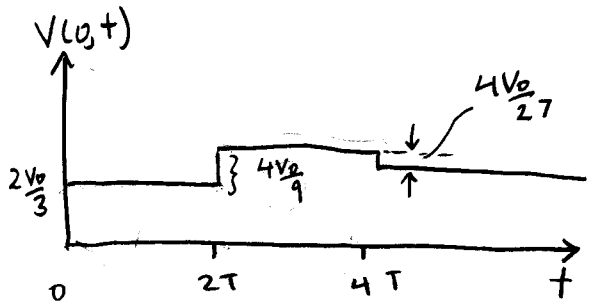


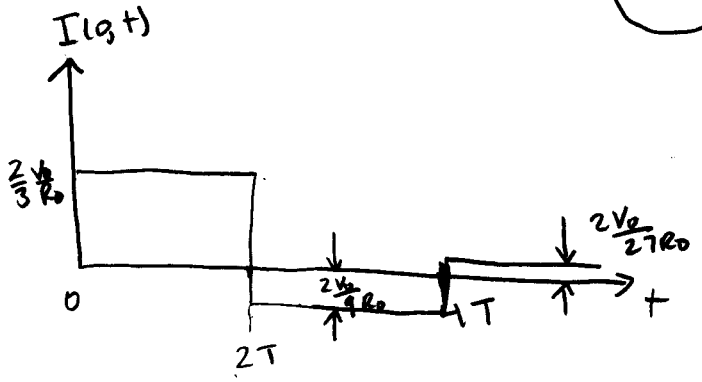

$$\frac{8}{9} - \frac{2}{9} \quad \frac{4}{9}$$

Current Reflection Diagram

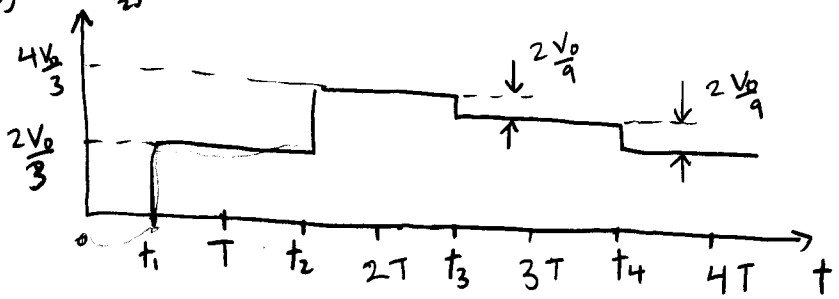


b)

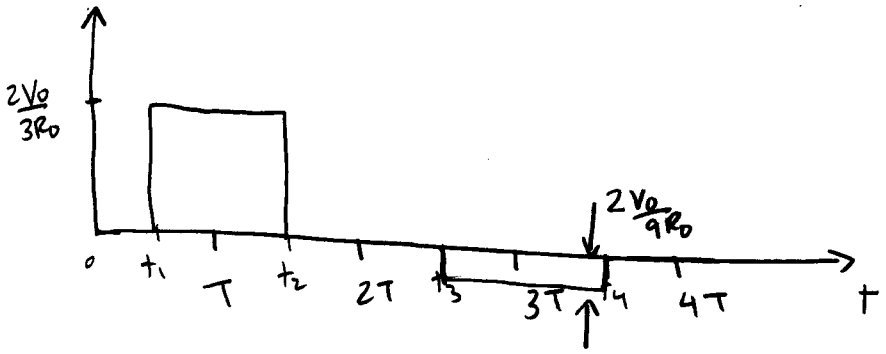

V(0,t)
$\frac{4V_0}{27}$
$\frac{2V_0}{3}$
$\frac{4V_0}{9}$
0
2T
4T
t


I(0,t)
$\frac{2}{3}\frac{V_0}{R_0}$
$\frac{2V_0}{27R_0}$
$\frac{2V_0}{9R_0}$
0
2T
4T
t

c) $V(l_2,t)$


$\frac{4V_0}{3}$
$\frac{2V_0}{3}$
$\frac{2V_0}{9}$
$\frac{2V_0}{9}$
0
$t_1$
T
$t_2$
2T
$t_3$
3T
$t_4$
4T
t

$I(l_2,t)$


$\frac{2V_0}{3R_0}$
$\frac{2V_0}{9R_0}$
0
$t_1$
T
$t_2$
2T
$t_3$
3T
$t_4$
4T
t

9-36

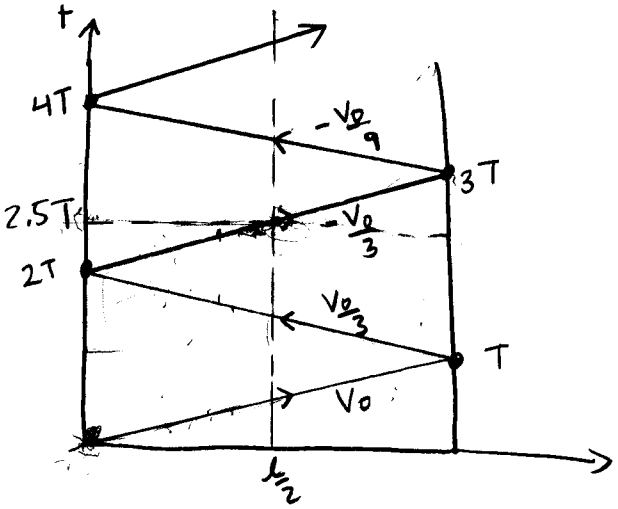
$R_L = 2R_0$ $\quad R_g = 0 \rightarrow \Gamma_g = -1$

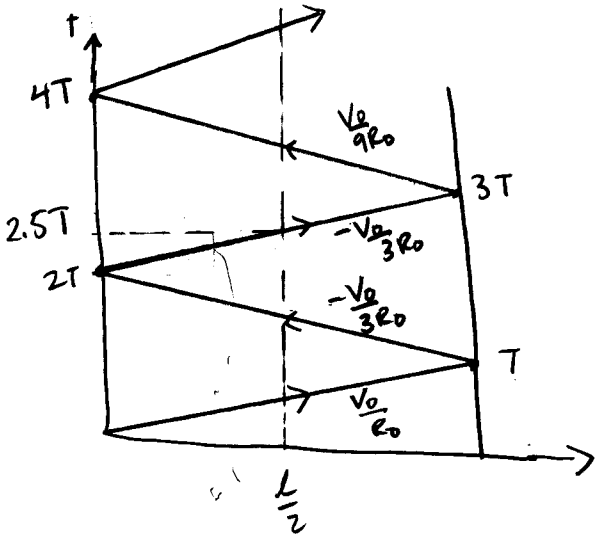
$\hookrightarrow \Gamma_L = \frac{1}{3}$ $\quad T = \frac{\ell}{u}$

$V_1^+ = V_0$

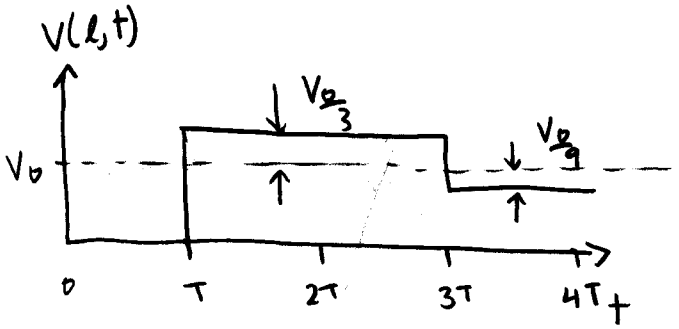
$I_1^+ = \frac{V_0}{R_0}$

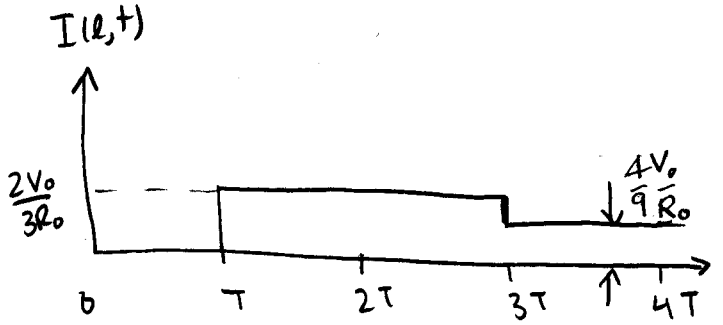
a) Voltage Reflection Diagram



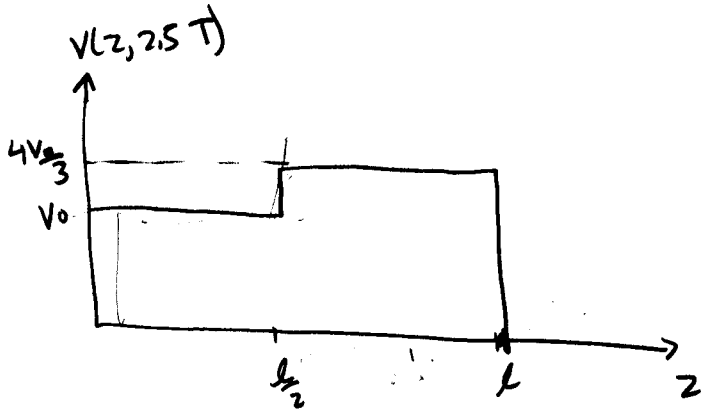



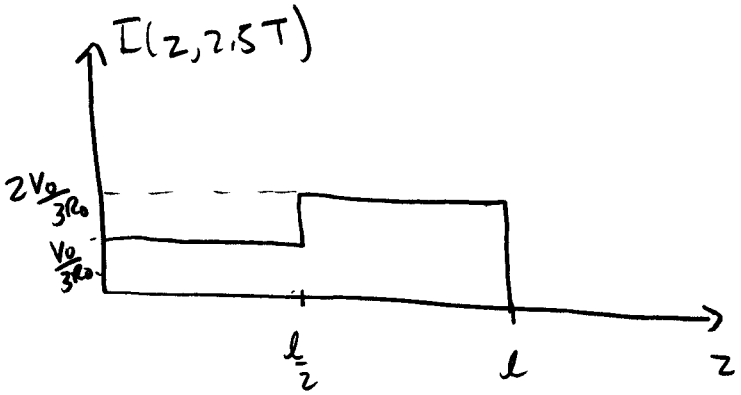
b)


V(l,t)
$V_0$
$\frac{V_0}{3}$
$\frac{V_0}{9}$
0
T
2T
3T
4T
t


I(l,t)
$\frac{2V_0}{3R_0}$
$\frac{4V_0}{9R_0}$
0
T
2T
3T
4T

c)


V(z, 2.5 T)
$\frac{4V_0}{3}$
$V_0$
$\frac{l}{2}$
l
z


I(z, 2.5 T)
$\frac{2V_0}{3R_0}$
$\frac{V_0}{3R_0}$
$\frac{l}{2}$
l
z