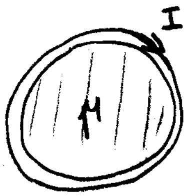
Problem Set #7

6-22, 6-32, 7-2, 7-5

6-22

a)



$r < a$: $\bar{H} = \bar{a}_z \, nI$

$\bar{B} = \bar{a}_z \, \mu n I$

$\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \bar{a}_z \left(\frac{\mu}{\mu_0} - 1\right) nI$

$a < r < b$: $\bar{H} = \bar{a}_z \, nI$

$\bar{B} = \bar{a}_z \, \mu_0 n I$

$\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \bar{a}_z \left(\overset{1}{\cancel{\frac{\mu_0}{\mu_0}}} - 1\right) nI = 0$

b) $\bar{J}_m, \bar{J}_{ms} = ?$

$\bar{J}_m$ = volume current density $= \bar{\nabla} \times \bar{M} = ?$

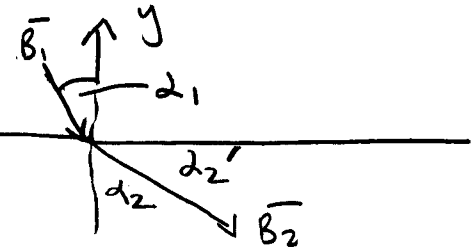
$$\bar{J}_m = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & \left(\frac{\mu}{\mu_0} - 1\right) nI \end{vmatrix} = \underline{\underline{0}}$$

$\bar{J}_{ms}$ = surface current density $= \bar{M} \times \bar{a}_n$ outward unit normal

$\bar{a}_n = \bar{a}_r$

$$\bar{J}_{ms} = \bar{M} \times \bar{a}_n = (\bar{a}_z \times \bar{a}_r)\left(\frac{\mu}{\mu_0} - 1\right) nI = \underline{\underline{\bar{a}_\phi \left(\frac{\mu}{\mu_0} - 1\right) nI}}$$

6-32



a) $\bar{B}_1 = \bar{a}_x\, 0.5 - \bar{a}_y\, 10 \quad \text{mT}$

find $\bar{B}_2$

$\bar{B}_2 = \bar{a}_x\, B_{2x} - \bar{a}_y B_{2y}$

remember boundary conditions

$B_{1n} = B_{2n}$

$\bar{a}_{n2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s$

$$\underbrace{H_{2x} = \frac{B_{2x}}{5000\mu_0} = H_{1x}}_{\text{tangential components}} = \frac{0.5}{\mu_0}$$

so $\rightarrow$ $B_{2x} = 2500 \text{ mT}$

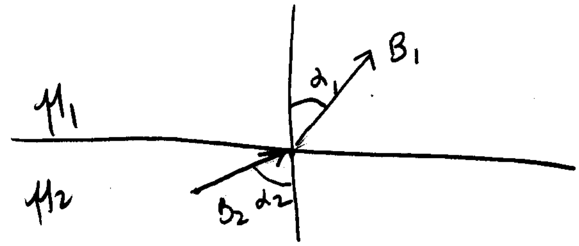
$B_{2y} = B_{1y} = -10 \text{ mT}$

so $\boxed{\bar{B}_2 = \bar{a}_x\, 2500 - \bar{a}_y\, 10 \quad \text{mT}}$

$$\tan\alpha_2 = \frac{\mu_2}{\mu_1}\tan\alpha_1 = 5000\left(\frac{B_{1x}}{B_{1y}}\right) = 250$$

$\alpha_2 = 89.8°$

so $\boxed{\alpha_2' = 0.2°}$ $\rightarrow$ angle $\bar{B}_2$ makes with interface b)



$\bar{B}_2 = a_x 10 + a_y\, 0.5 \text{ mT}$

$H_{1x} = \dfrac{B_{1x}}{\mu_1} = H_{2x} = \dfrac{B_{2x}}{\mu_2} \rightarrow B_{1x} = \dfrac{B_{2x}}{\mu_{r2}} = \dfrac{10}{5000} = 0.002 \text{ mT}$

$B_{1y} = B_{2y} = 0.5 \text{ mT}$

So $\boxed{\bar{B}_1 = \bar{a}_x\, 0.002 + \bar{a}_y\, 0.5 \quad \text{mT}}$

$\tan \alpha_1 = \dfrac{B_{1x}}{B_{1y}} = \dfrac{0.002}{0.5} = 0.004$

$\boxed{\alpha_1 = 0.23^\circ}$

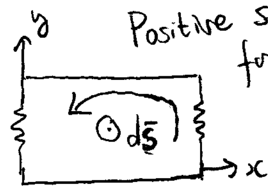
angle $\bar{B}_1$ makes w/ normal to interface.

7-2

$$\bar{B} = \bar{a}_z\, 3\cos\left(5\pi 10^7 t - \tfrac{2}{3}\pi x\right)\cdot 10^{-6}\ \mathrm{T}$$

$R = 15\,\Omega$, find i

Find V, then find i.



Positive sense for loop

$$V = -\frac{d}{dt}\int_S \bar{B}\cdot d\bar{s}$$

doesn't vary so $\int dy = 0.2$

$$d\bar{s} = dx\,dy\,\bar{a}_z$$

$$\int_S \bar{B}\cdot d\bar{s} = \int_0^{0.6} \left(\bar{a}_z 3\cos\left(5\pi 10^7 t - \tfrac{2}{3}\pi x\right)\times 10^{-6}\right) \cdot \left(\bar{a}_z\, 0.2\,dx\right)$$

$$= -\frac{0.18}{2\pi}\left[\sin(5\pi 10^7 t - 0.4\pi) - \sin 5\pi 10^7 t\right]\cdot 10^{-6}\ (\mathrm{Wb})$$

$$V = -\frac{d}{dt}\int \bar{B}\cdot d\bar{s} = 45\left[\cos(5\pi 10^7 t - 0.4\pi) - \cos 5\pi 10^7 t\right]\ \mathrm{V}$$

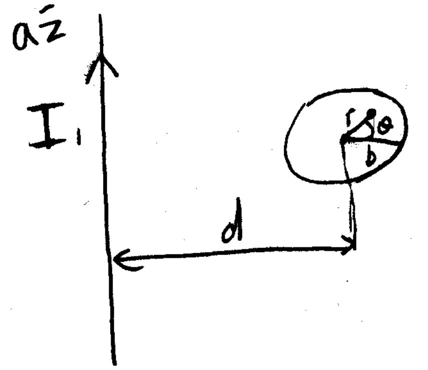
$$i = -\frac{V}{2R} = -1.5\left[\cos(5\pi 10^7 t - 0.4\pi) - \cos 5\pi 10^7 t\right]$$

Because + dir^n for i is opposite to + sense chosen for loop

$$\boxed{i = -1.76\sin(5\pi 10^7 t - 0.2\pi)\ \ (\mathrm{A})}$$

7-5

a)



$$\Lambda_{12} = \int_{S_2} B_1 \cdot dS_2$$

B at $P(r,\theta)$ is $\bar{a_\phi} \dfrac{\mu_0 I}{2\pi(d + r\cos\theta)}$

flux linkage

$$\Lambda_{12} = \frac{\mu_0 I_1}{2\pi} \int_0^b \int_0^{2\pi} \frac{r\,dr\,d\theta}{d + r\cos\theta}$$

Difficult integral: $\displaystyle\int_0^{2\pi} \frac{dx}{a + b\cos x} = \frac{2\pi}{\sqrt{a^2 - b^2}}$

$$= \frac{\mu_0 I_1}{2\pi} \int_0^b \frac{2\pi r\,dr}{\sqrt{d^2 - r^2}} = \mu_0 I \left(d - \sqrt{d^2 - b^2}\right)$$

mutual inductance

$$L_{12} = \mu_0 \left(d - \sqrt{d^2 - b^2}\right)$$

mutual flux

$$\Phi_{12} = L_{12} I_1 = L_{12} \underbrace{I \sin(wt)}_{I_1 \rightarrow \text{ac current}}$$

$$\Phi_{12} = \mu_0 I \sin(wt) \left(d - \sqrt{d^2 - b^2}\right)$$

$$V = -\frac{d\Phi}{dt} = -\mu_0 I w (\cos wt) \left(d - \sqrt{d^2 - b^2}\right)$$

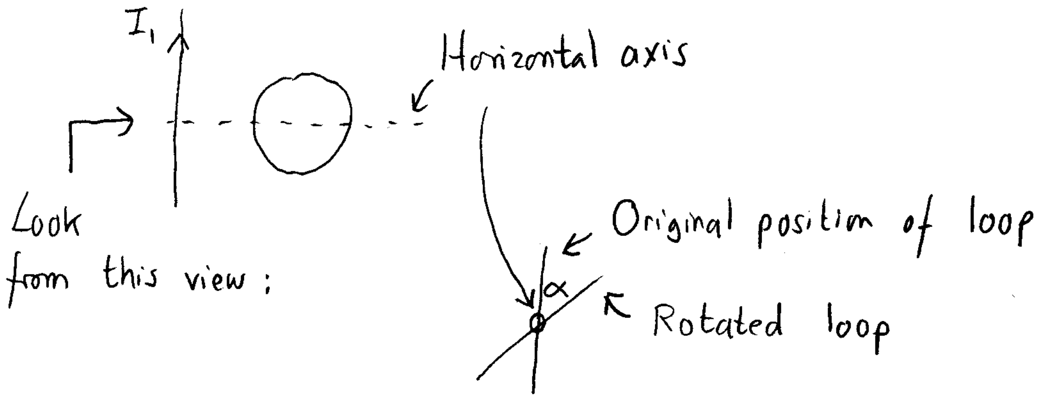
$$V = V_m \cos(wt)$$

$$I_{\text{millammeter}} = 0.3\,\text{mA} = \frac{|V_m|}{\sqrt{2}\,R} = \frac{\mu_0 I w \left(d - \sqrt{d^2 - b^2}\right)}{\sqrt{2}\,R}$$

$$\rightarrow I = \frac{0.3 \times 10^{-3} \times \sqrt{2} \times 0.01}{(4\pi \times 10^{-7}) \times (2\pi \times 60)\left(0.15 - \sqrt{0.15^2 - 0.1^2}\right)} = 0.234\ \text{A}$$

$$\boxed{I = 0.234\,\text{A}}$$

b) What angle about the horizontal axis should the circular loop be rotated by to reduce reading to 0.2 mA? ie $\frac{2}{3}$ of original value.

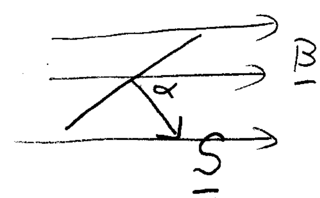


If the flux density were <u>uniform in space</u> with value $\underline{B}_0$:

$$\Phi_{12}^{new} = \int_{\substack{\text{Rotated}\\ \text{loop}}} \underline{B} \cdot d\underline{s} = \underline{B}_0 \cdot \underline{S}^{new}$$

$$= B_0 S \cos\alpha$$

$\uparrow$ area of loop



But $\Phi_{12}^{orig} = B_0 S$

So to reduce flux by $\frac{2}{3}$: $\quad B_0 S \cos\alpha = \frac{2}{3} B_0 S \Rightarrow \underline{\underline{\alpha = 48.2^\circ}}$

This also reduces the EMF, and hence the current, by $2/3$

<u>HOWEVER</u> : $\underline{B}$ is <u>NOT</u> uniform and so the above argument is not strictly correct.

(The true answer involves another (more difficult) integral over the rotated loop.)