PROBLEM SET #6

5-3, 5-8, 5-9, 6-3, 6-6

5-3

R_1 = resistence per unit length of core $= \frac{1}{\sigma S_1} = \frac{1}{\sigma \pi a^2}$

R_2 = resistence per unit length of coating $= \frac{1}{(0.1\sigma) S_2}$

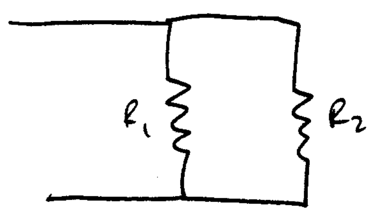
let b = thickness of coating $\rightarrow S_2 = \pi(a+b)^2 - \pi a^2$

$$\underline{S_2 = \pi(2ab + b^2)}$$

a) 2 resistances are in parallel.

To make the resistence per unit length of the wire reduce by 50%, put a resistence in parallel so that $R_1 = R_2$.

Why



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{eq} = \frac{1}{2}\cancel{R_1} = \frac{\cancel{R_1} R_2}{R_1 + R_2}$$

$$\frac{1}{2}R_1 + \frac{1}{2}R_2 = R_2$$

$$\frac{1}{2}R_2 = \frac{1}{2}R_1 \rightarrow \underline{\underline{R_2 = R_1}}$$

$R_1 = R_2$

$$\frac{1}{\cancel{\sigma}\cancel{\pi}a^2} = \frac{1}{(0.1\cancel{\sigma})\cancel{\pi}(2ab+b^2)}$$

$$b^2 + 2ab - 10a^2 = 0$$

$$b = \frac{-2a \pm \sqrt{4a^2 - (4\times(-10a^2))}}{2}$$

$$b = -a \pm \sqrt{11a^2} = a(\sqrt{11} - 1)$$

$$\boxed{b = a(\sqrt{11} - 1) = 2.32\,a}$$

b) $I_1 = I_2 = \frac{I}{2}$ → why, current divides equally into 2 branches of equal resistence.

$$\boxed{J_1 = \frac{I_1}{\pi a^2} = \frac{I}{2\pi a^2}}$$

$$J_2 = \frac{I_2}{S_2}$$

$S_2 = ?$

$$S_2 = \pi(a+b)^2 - \pi a^2$$

$$S_2 = \pi(\cancel{a} + \sqrt{11}a - \cancel{a})^2 - \pi a^2$$

$$S_2 = 10a^2\pi$$

so $\boxed{J_2 = \frac{I_2}{10\pi a^2} = \frac{I}{20\pi a^2}}$

$$\boxed{E_1 = \frac{J_1}{\sigma} = \frac{I}{2\pi a^2\sigma}} \qquad \boxed{E_2 = \frac{J_2}{0.1\sigma} = \frac{I}{2\pi a^2\sigma}}$$

so $J_1 = 10J_2$ & $E_1 = E_2$

5-8

a) $R = \dfrac{l}{\sigma S} = \dfrac{V}{I} \longrightarrow \sigma = \dfrac{l I}{S V}$

$$\sigma = \frac{(1000)\cdot(1/6)}{(\pi(0.5\times10^{-3})^2)(6)} = 3.54\times10^{7}\,(S/m)$$

b) $E = \dfrac{V}{l} = \dfrac{6\ V}{1000\ m} = 6\times10^{-3}\,(V/m)$

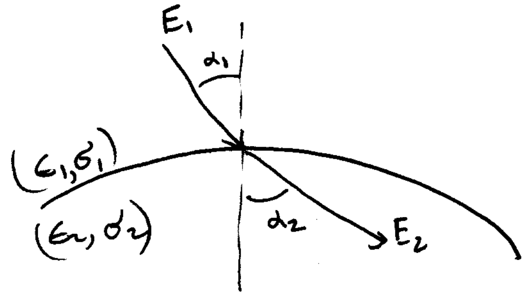
c) $P = VI = 6V \times \frac{1}{6}A = 1\,(W)$

d) $\rho_e = -\dfrac{\sigma}{\mu_e}$ $\qquad \mu_e = 1.4\times10^{-3}\ m^2 V/s)$

$$u = \left|\frac{J}{\rho_e}\right| = \left|\frac{\mu_e}{\sigma} J\right| = |\mu_e E| = 1.4\times10^{-3}\times 6\times10^{-3}$$

$$u = 8.4\times10^{-6}\,(m/s)$$

5-9



a) $E_2 = ?$

$$E_{1t} = E_{2t} \rightarrow E_1 \sin\alpha_1 = E_2 \sin\alpha_2$$

$$J_{1n} = J_{2n} \rightarrow \sigma_1 E_{1n} = \sigma_2 E_{2n} \rightarrow \sigma_1 E_1 \cos\alpha_1 = \sigma_2 E_2 \cos\alpha_2$$

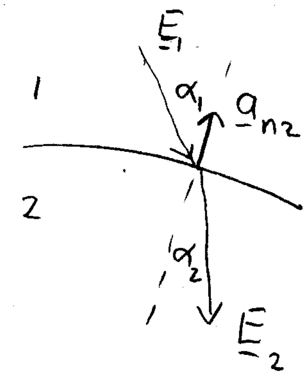
$$E_2^2 \sin^2\alpha_2 = E_1^2 \sin^2\alpha_1$$

$$+ \quad E_2^2 \cos^2\alpha_2 = \frac{\sigma_1^2}{\sigma_2^2} E_1^2 \cos^2\alpha_1$$

$$E_2^2 = E_1^2 \sin^2\alpha_1 + \frac{\sigma_1^2}{\sigma_2^2} E_1^2 \cos^2\alpha_1$$

$$\therefore \boxed{\begin{array}{l} E_2 = E_1 \sqrt{\sin^2\alpha_1 + \left(\frac{\sigma_1}{\sigma_2}\cos\alpha_1\right)^2} \\ \tan\alpha_2 = \frac{\sigma_2}{\sigma_1}\tan\alpha_1 \longrightarrow \alpha_2 = \tan^{-1}\left(\frac{\sigma_2}{\sigma_1}\tan\alpha_1\right) \end{array}}$$

b) $$\left(\underline{D}_1 - \underline{D}_2\right) \cdot \underline{a}_{n2} = \rho_s$$



$$\Rightarrow \quad \varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$$

where $E_{1n} = \underline{E}_1 \cdot \underline{a}_{n2}$

& $E_{2n} = \underline{E}_2 \cdot \underline{a}_{n2}$

But $\sigma_2 E_{2n} = \sigma_1 E_{1n}$, so $E_{2n} = \dfrac{\sigma_1}{\sigma_2} E_{1n}$

$$\Rightarrow \quad \rho_s = \left(\varepsilon_1 - \varepsilon_2 \frac{\sigma_1}{\sigma_2}\right) E_{1n}$$

Now $E_{1n} = \underline{E}_1 \cdot \underline{a}_{n2} = -E_1 \cos\alpha_1$ (see figure above)

So $$\boxed{\rho_s = \left(\frac{\sigma_1}{\sigma_2}\varepsilon_2 - \varepsilon_1\right) E_1 \cos\alpha_1}$$

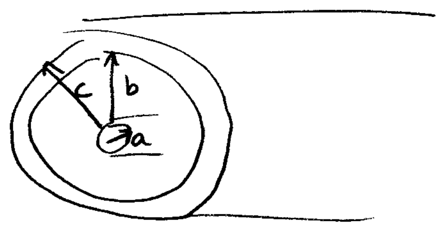
c) If both media are perfect dielectrics, $\sigma_1 = \sigma_2 = 0$

& $$\frac{\tan\alpha_2}{\tan\alpha_1} = \frac{\varepsilon_2}{\varepsilon_1}$$

So $$E_2 = E_1 \left[\sin^2\alpha + \left(\frac{\varepsilon_1}{\varepsilon_2}\cos\alpha_1\right)^2\right]^{\frac{1}{2}}$$

– eqn (3-130) in Chapter 3

6-3



Find $\overline{B}$ for all regions.

$$\oint_c B \cdot dl = \mu I$$

Region 1:
$0 \le r_1 \le a$

$$\overline{B}_1 = \bar{a}_\phi B_{\phi 1} \qquad dl = a_\phi r_1 d\phi$$

$$\oint_{c_1} B_1 \cdot dl = \int_0^{2\pi} B_{\phi 1} a_1 d\phi = 2\pi r_1 B_{\phi 1}$$

$$I_1 = \frac{\cancel{\pi} r_1^2}{\cancel{\pi} a^2} I$$

$$B_{\phi 1} = \frac{1}{2\cancel{\pi}\cancel{r_1}} \cdot \frac{\cancel{\pi} r_1^{\cancel{2}}}{\pi a^2} I \mu = \frac{\mu r_1 I}{2\pi a^2}$$

so $$\boxed{\overline{B} = \bar{a_\phi} \frac{\mu r_1 I}{2\pi a^2} \qquad 0 \le r_1 \le a}$$

Region 2:

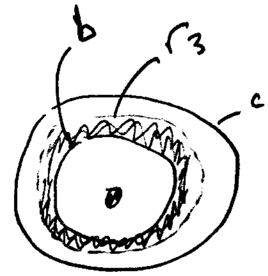
$a \le r_2 \le b$

$$\oint_{C_2} B_2 \cdot dl = 2\pi r_2 B_{\phi 2}$$

$$I_2 = I.$$

$$B_{\phi 2} = \frac{1}{2\pi r_2} \cdot \mu I$$

so $\boxed{\bar{B} = \bar{a}_\phi \frac{\mu I}{2\pi r_2} \qquad a \le r_2 \le b}$



Region 3:
$b \le r_3 \le c$

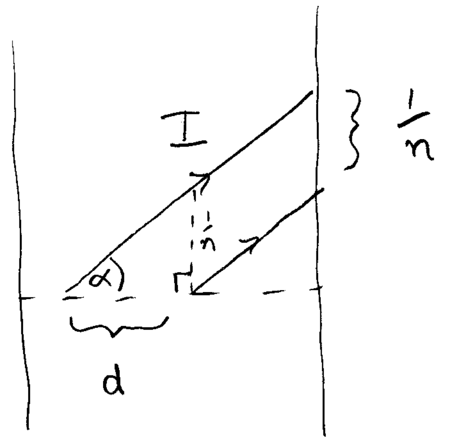
$$\oint_{C_3} B_3 \cdot dl = 2\pi r_3 B_{\phi 3}$$

$$I_3 = \overset{\text{from inside}}{I} - \frac{\pi r_3^2 - \pi b^2}{\pi c^2 - \pi b^2} I \quad \text{current of shaded part}$$

$$I_3 = \frac{\pi c^2 - \cancel{\pi b^2} - \pi r_3^2 + \cancel{\pi b^2}}{\pi c^2 - \pi b^2} I = \frac{\cancel{\pi} c^2 - \cancel{\pi} r_3^2}{\cancel{\pi} c^2 - \cancel{\pi} b^2}$$

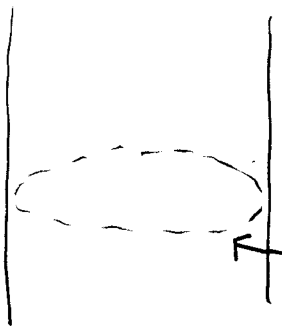
so $\boxed{\bar{B} = \bar{a}_\phi \left(\frac{c^2 - r_3^2}{c^2 - b^2}\right) \frac{\mu I}{2\pi r_3} \qquad b \le r_3 \le c}$

6-6



$\tan\alpha = \frac{1}{n} / d$

$\Rightarrow d = \frac{1}{n \tan\alpha}$



So number of turns crossing this circle is $\frac{2\pi b}{d}$

or $2\pi b\, n \tan\alpha$

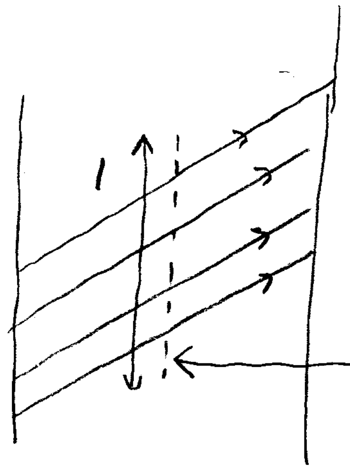
(NOT just $2\pi b\, n$)

ie axial current, I_1, is $I \times 2\pi b\, n \tan\alpha$

From Ampère's Law, the field $\underline{B}_1$ created by this is

$$2\pi r\, B_{1\varphi} = \mu_0\, 2\pi b n I \tan\alpha$$

$$\Rightarrow B_{1\varphi} = \begin{cases} \dfrac{b\mu_0 n I \tan\alpha}{r} & r > b \\ 0 & 0 < r < b \end{cases}$$



Number of turns crossing this unit-length line is n

ie circumferential (φ) current, I_2, per unit length is nI

But this is the same as a regular solenoid with n turns/m carrying current I.

From p. 231, Ex 6-3:

$$B_{2z} = \begin{cases} \mu_0 n I & 0 < r < b \\ 0 & r > b \end{cases}$$

Total field $\underline{B}$ is $\underline{B}_1 + \underline{B}_2$

$$= \begin{cases} \underline{a}_z \, \mu_0 n I & 0 < r < b \\ \dfrac{b \mu_0 n I \tan\alpha}{r} & r > b \end{cases}$$