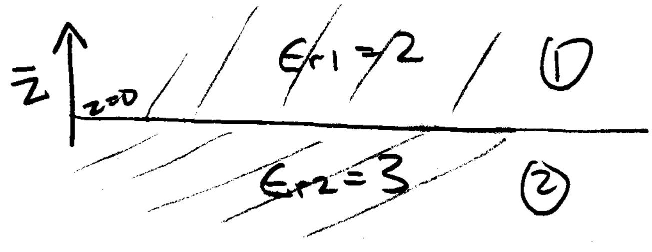
Problem Set 4

3-25 ✓, 3-26 ✓, 3-27 ✓, 3-28, 3-29

3-25



$\bar{E}_1 = \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z(5+z)$

$\bar{E}_2 = ?$

$\bar{D}_2 = ?$

Boundary Conds

$\bar{E}_{1t} = \bar{E}_{2t}$

$a_{n2} \cdot (\bar{D}_1 - \bar{D}_2) = \underbrace{\rho_s}_{=0 \text{ for dielectrics}}$

$\hookrightarrow \bar{D}_{1n} = \bar{D}_{2n}$

At $z=0$ (plane),

$\bar{E}_1 = \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z 5$

$\bar{E}_{1t} = \bar{a}_x 2y - \bar{a}_y 3x$

$\bar{E}_{1n} = \bar{a}_z 5$

$\bar{E}_{1t}(z=0) = \bar{E}_{2t}(z=0) = \bar{a}_x 2y - \bar{a}_y 3x$

why no z component? Cuz we are looking for tangential component to interface, which is parallel to z = 0 plane and thus must have no z component!

$\bar{D}_{1n}(z=0) = \bar{D}_{2n}(z=0)$

$$\bar{D}_{1n}\Big|_{z=0} = \bar{D}_{2n}\Big|_{z=0} \rightarrow \epsilon_{r1}\cancel{\epsilon_0}\,\bar{E}_{1n}(z=0) = \epsilon_{r2}\cancel{\epsilon_0}\,\bar{E}_{2n}(z=0))$$

$$2\,\bar{E}_{1n}(z=0) = 3\,\bar{E}_{2n}(z=0))$$

$$\bar{E}_{2n}(z=0) = \frac{2}{3}\,(\bar{E}_{1n}(z=0)) = \frac{2}{3}\,\bar{a}_z 5$$

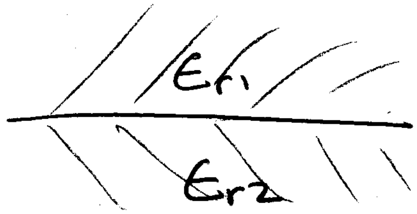
$$\bar{E}_{2n}(z=0) = \bar{a}_z\,\frac{10}{3}$$

$$\therefore \boxed{\begin{aligned} \bar{E}_2\,(z=0) &= \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z \tfrac{10}{3} \\ \bar{D}_2\,(z=0) &= (\bar{a}_x 2y - a_y 3x + \bar{a}_z \tfrac{10}{3})\,3\epsilon_0 \end{aligned}}$$

All we know and can find about region (2) from boundary conditions!

Can't find $\bar{E}_2$ & $\bar{D}_2$ in any region because B.C.'s give us only enough information to solve for fields at the boundary.

3-26



$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}$$

$$\bar{P} = \bar{D} - \epsilon_0 \bar{E}$$

$$\bar{P} = \epsilon_r \epsilon_0 \bar{E} - \epsilon_0 \bar{E}$$

$$\underline{\bar{P} = \epsilon_0 (\epsilon_r - 1)\, \bar{E}}$$

① $\bar{P}_t = \epsilon_0 (\epsilon_r - 1)\, \bar{E}_t$ tang. component

② $\bar{P}_n = \epsilon_0 (\epsilon_r - 1)\, \bar{E}_n$ normal component.

From ①, $\bar{E}_t = \dfrac{\bar{P}_t}{\epsilon_0(\epsilon_r - 1)}$

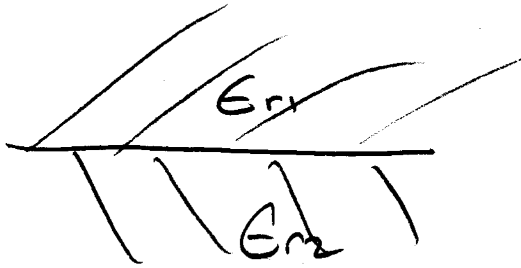
$$\bar{E}_{t1} = \bar{E}_{t2} \longrightarrow \frac{\bar{P}_{t1}}{\cancel{\epsilon_0}(\epsilon_{r1} - 1)} = \frac{\bar{P}_{t2}}{\cancel{\epsilon_0}(\epsilon_{r2} - 1)}$$

$$\boxed{\frac{1}{\epsilon_{r1} - 1}\, \bar{P}_{t1} = \frac{1}{\epsilon_{r2} - 1}\, \bar{P}_{t2}}$$

$$\bar{D}_{1n} = \bar{D}_{2n} \rightarrow \epsilon_{r1} \bar{E}_{n1} = \epsilon_{r2} \bar{E}_{n2}$$

$$\boxed{\frac{\epsilon_{r1}}{\epsilon_{r1} - 1}\, \bar{P}_{n1} = \frac{\epsilon_{r2}}{\epsilon_{r2} - 1}\, \bar{P}_{n2}}$$

3-27



What are BC's for V?

What do we know? $\longrightarrow$ (1) $\bar{E}_{1t} = \bar{E}_{2t}$

(2) $\bar{D}_{1n} = \bar{D}_{2n}$ (for dielectric interface w/ no ρ_s)

(3) $\bar{E} = -\nabla V$

From (3), $\bar{E} = -\left(\underbrace{\frac{\partial V}{\partial n} \bar{a}_n}_{\text{normal component}} + \underbrace{\frac{\partial V}{\partial t} \bar{a}_t}_{\text{tangential component.}} \right)$

So $\bar{E}_n = -\frac{\partial V}{\partial n}$ & $\bar{E}_t = -\frac{\partial V}{\partial t}$

From (1), $\bar{E}_{1t} = \bar{E}_{2t}$

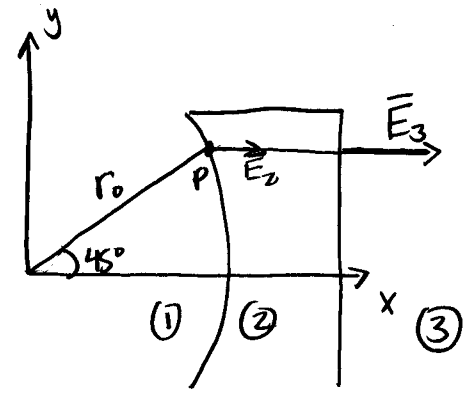
$$\cancel{-}\frac{\partial V_1}{\cancel{\partial t}} = \cancel{-}\frac{\partial V_2}{\cancel{\partial t}} \longrightarrow \boxed{V_1 = V_2}$$

From (2) $\bar{D}_{1n} = \bar{D}_{2n}$

$\epsilon_{r1} \bar{E}_{1n} = \epsilon_{r2} \bar{E}_{2n}$

$$\epsilon_{r1} \left(\cancel{-} \frac{\partial V_1}{\partial n} \right) = \epsilon_{r2} \left(\cancel{-} \frac{\partial V_2}{\partial n} \right) \longrightarrow \boxed{\epsilon_{r1} \frac{\partial V_1}{\partial n} = \epsilon_{r2} \frac{\partial V_2}{\partial n}}$$

3-28



$$\bar{E}_1 = \bar{a}_r 5 - \bar{a}_\phi 3$$

Find ϵ_{r2} so that $\bar{E}_3$ is parallel to x-axis

Assume $\bar{E}_2 = \bar{a}_r E_{2r} + \bar{a}_\phi E_{2\phi}$

Boundary condition: tangential components of E are continuous across interface 1-2

$$\bar{a}_n \times \bar{E}_1 = \bar{a}_n \times \bar{E}_2$$

$$E_{1\phi} = E_{2\phi}$$

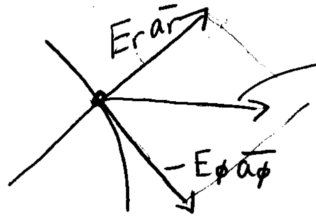
$$\underline{E_{2\phi} = -3}$$

For $\bar{E}_3$ to be parallel to x-axis, $\bar{E}_2$ must also be parallel to x-axis.

For $\bar{E}_2$ to be parallel to x-axis,

$$E_{2\phi} = -E_{2r}$$

why? ⟶



parallel to x-axis

if E_r is not equal to $-E_\phi$, resultant vector will not be parallel to x-axis. Add vectors graphically to prove this!

So $E_{2\phi} = -E_{2r} \longrightarrow E_{2r} = 3$

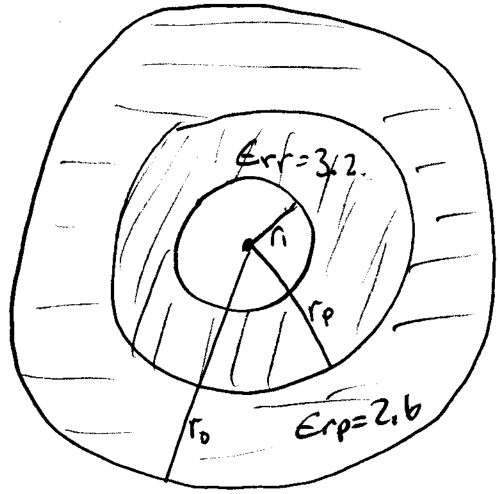
Boundary condition 2: $\underbrace{\bar{a}_n \cdot \bar{D}_1 = \bar{a}_n \cdot \bar{D}_2}_{\text{normal components of } \bar{D} \text{ are cts..}}$

$\longrightarrow \epsilon_1 E_{r1} = \epsilon_2 E_{r2} \rightarrow \epsilon_{r1} \cancel{\epsilon_0} 5 = \epsilon_{r2} \cancel{\epsilon_0} 3$

$\longrightarrow \epsilon_{r1} = 1$ (air)

so $\boxed{\epsilon_{r2} = \frac{5}{3} = 1.667}$

3-29



$r_i = 0.4\,cm = 0.004\ m$

$r_o = 0.832\,cm = 2.08\, r_i$

25% of dielectric strength of rubber ($\epsilon_{rr}=3.2$)

$\hookrightarrow\ 0.25 \times \underbrace{25\times10^6}_{\text{in table 3.1}} = 6.25\times10^6\ (V/m)$

25% of dielectric strength of polystyrene ($\epsilon_{rp}=2.6$)

$\hookrightarrow\ 0.25 \times 20\times10^6 = 5\times10^6\ (V/m)$

a) $r_p = 1.75\, r_i$ so $r_o = 1.189\, r_p$

$$\text{Max } E_r = \frac{\rho_\ell}{2\pi\epsilon_0}\left(\frac{1}{3.2 r_i}\right) \longrightarrow \frac{\rho_\ell}{2\pi\epsilon_0} = 6.25\times10^6 \times 3.2 r_i = \underline{20\times10^6 r_i}$$

$$\text{Max } E_p = \frac{\rho_\ell}{2\pi\epsilon_0}\left(\frac{1}{2.6 r_p}\right) = \frac{\rho_\ell}{2\pi\epsilon_0}\left(\frac{1}{4.55 r_i}\right) \longrightarrow \frac{\rho_\ell}{2\pi\epsilon_0} = 5\times10^6 \times 4.55 r_i$$

$$= \underline{22.75\times10^6\, r_i}$$

$$\frac{\text{Max } E_r}{\text{Max } E_p} = \frac{4.55}{3.2} > \frac{6.25}{5} \longrightarrow \text{Max } E_r \text{ determines the allowable } \rho_e/2\pi\epsilon_0$$

35

$$\therefore \quad \frac{\rho_e}{2\pi\epsilon_0} = 20\times10^6 \times 0.004 = 8\times10^4$$

Voltage rating $V_{max} = 8\times10^4\left(\frac{1}{2.6}\ln\frac{r_o}{r_p} + \frac{1}{3.2}\ln\frac{r_p}{r_i}\right)$

$$= 8\times10^4\left(\frac{1}{2.6}\ln 1.189 + \frac{1}{3.2}\ln 1.75\right)$$

$$\underline{V_{max} = 19.3 \text{ kV}}$$

b) $r_p = 1.35\, r_i$, $r_o = 1.54\, r_p$

Max $r_p = \frac{\rho_e}{2\pi\epsilon_0}\left(\frac{1}{2.6 r_p}\right) = \frac{\rho}{2\pi\epsilon_0}\left(\frac{1}{3.51 r_i}\right)$, which

determines the allowable $\frac{\rho_e}{2\pi\epsilon_0} = 5\times10^6 \times 3.51\, r_i = 1.76\times10^6 r_i$

$$= 7\times103$$

$$V_{max} = 7\times10^3\left(\frac{1}{2.6}\ln 1.54 + \frac{1}{3.2}\ln 1.35\right)$$

$$\underline{V_{max} = 1.82 \text{ kV}}$$