ECSE 353 ELECTROMAGNETIC FIELDS AND WAVES FORMULAS

$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B}$ $\qquad$ $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

$\nabla\cdot(\psi\mathbf{A}) = \psi\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla\psi$ $\qquad$ $\nabla\times(\psi\mathbf{A}) = \psi\nabla\times\mathbf{A} + \nabla\psi\times\mathbf{A}$

$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$ $\qquad$ $\nabla(\psi V) = \psi\nabla V + V\nabla\psi$

$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$ $\qquad$ $\nabla\cdot\nabla V = \nabla^2 V$ $\qquad$ $\nabla\cdot(\nabla\times\mathbf{A}) = 0$ $\qquad$ $\nabla\times\nabla V = 0$

$$\int_V \nabla\cdot\mathbf{A}\,dv = \oint_S \mathbf{A}\cdot d\mathbf{s} \quad \text{(Divergence thm.)} \qquad \int_S \nabla\times\mathbf{A}\cdot d\mathbf{s} = \oint_C \mathbf{A}\cdot d\mathbf{l} \quad \text{(Stokes's thm.)}$$

Cartesian Coordinates (x,y,z) $\qquad$ $\mathbf{dl} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z$

$$\nabla V = \mathbf{a}_x\frac{\partial V}{\partial x} + \mathbf{a}_y\frac{\partial V}{\partial y} + \mathbf{a}_z\frac{\partial V}{\partial z} \qquad \nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathrm{A} = \mathbf{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (r,ϕ,z) $\qquad$ $\mathbf{dl} = dr\,\mathbf{a}_r + r d\phi\,\mathbf{a}_\phi + dz\,\mathbf{a}_z$

$$\nabla V = \mathbf{a}_r\frac{\partial V}{\partial r} + \mathbf{a}_\phi\frac{\partial V}{r\partial\phi} + \mathbf{a}_z\frac{\partial V}{\partial z} \qquad \nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \mathbf{a}_r\left(\frac{\partial A_z}{r\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{a}_z\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{r\partial\phi}\right)$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R,θ,ϕ) $\qquad$ $\mathbf{dl} = dR\,\mathbf{a}_R + R d\theta\,\mathbf{a}_\theta + R\sin\theta\,d\phi\,\mathbf{a}_\phi$

$$\nabla V = \mathbf{a}_R\frac{\partial V}{\partial R} + \mathbf{a}_\theta\frac{\partial V}{R\partial\theta} + \mathbf{a}_\phi\frac{1}{R\sin\theta}\frac{\partial V}{\partial\phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathrm{A} = \mathbf{a}_R\frac{1}{R\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right) + \mathbf{a}_\theta\frac{1}{R}\left(\frac{1}{\sin\theta}\frac{\partial A_R}{\partial\phi} - \frac{\partial}{\partial R}(RA_\phi)\right)$$
$$+\,\mathbf{a}_\phi\frac{1}{R}\left(\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right)$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

Electrostatics

$\nabla \cdot \mathbf{D} = \rho \qquad \oint_S \mathbf{D}\cdot d\mathbf{s} = Q \qquad \mathbf{a}_{n2}\cdot(\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$

$\nabla \times \mathbf{E} = 0 \qquad \oint_C \mathbf{E}\cdot d\mathbf{l} = 0 \qquad \mathbf{E}_{1t} = \mathbf{E}_{2t}$

Potential : $\mathbf{E} = -\nabla V \quad V_{12} = -\int_2^1 \mathbf{E}\cdot d\mathbf{l} \quad \nabla\cdot\varepsilon\nabla V = -\rho$

Sources in Free Space : $\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\mathbf{a}_R\frac{q}{R^2} \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

Infinite Line Charge : $\mathbf{E} = \mathbf{a}_r\frac{\rho_l}{2\pi\varepsilon_0 r} \qquad V = \frac{\rho_l}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r}\right)$

Electric Dipole : $\mathbf{p} = q\mathbf{d} \qquad V = \frac{\mathbf{p}\cdot\mathbf{a}_R}{4\pi\varepsilon_0 R^2}$

Polarization Charge : $\rho_p = -\nabla\cdot\mathbf{P} \qquad \rho_{ps} = \mathbf{P}\cdot\mathbf{a}_n$

Flux Density : $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} = \varepsilon_0\varepsilon_r\mathbf{E}$

$C = \frac{Q}{V}$ Parallel Plate: $C = \frac{\varepsilon S}{d}$ Coax: $C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)}$ (Fm^{-1})

Wires: $C = \frac{\pi\varepsilon}{\cosh^{-1}\left(\frac{D}{2a}\right)}$ (Fm^{-1})

Energy : $W_e = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k$ or $\frac{1}{2}\int_{V'}\rho\, V\, dv'$ or $\frac{1}{2}\int_{V'}\mathbf{E}\cdot\mathbf{D}\, dv'$ or $\frac{1}{2}CV^2$

Physical Constants

$\mu_0 = 4\pi\times 10^{-7}$ $(\mathrm{Hm^{-1}})$ $\qquad \varepsilon_0 \approx \frac{1}{36\pi}\times 10^{-9}$ $(\mathrm{Fm^{-1}})$ $\qquad 1\ \mathrm{Npm^{-1}} = 8.686\ \mathrm{dBm^{-1}}$

$\eta_0 \approx 377\,\Omega$ or $120\pi\,\Omega \qquad c \approx 3\times 10^8$ $(\mathrm{ms^{-1}})$

Steady Electric Currents

$\mathbf{J} = Nq\mathbf{u}$ $\quad \int_S \mathbf{J}\cdot d\mathbf{s} = I$ $\quad \mathbf{J}_s = N_s q\mathbf{u}$ $\quad \int_C \mathbf{J}_s \cdot \mathbf{a}_n dl = I$ $\quad \mathbf{J} = \sigma(\mathbf{E}+\mathbf{E}_i)$

$\text{EMF} = \oint_C (\mathbf{E}+\mathbf{E}_i)\cdot d\mathbf{l}$ $\quad \text{EMF} = I\sum_k R_k$ $\quad \text{EMF}_k = \int_{\text{Cond } k} \mathbf{E}_i \cdot d\mathbf{l}$

$\nabla\cdot\mathbf{J} = 0$ $\quad \oint_S \mathbf{J}\cdot d\mathbf{s} = 0$ $\quad J_{1n} = J_{2n}$

Joules' Law: $P = \int_V \mathbf{E}\cdot\mathbf{J}\,dv$ or VI

$R = -\int_2^1 (\mathbf{E}+\mathbf{E}_i)\cdot dl \big/ I_{12}$ $\quad R_{cylinder} = d/(\sigma S)$ $\quad C/G = \varepsilon/\sigma$ $\quad R_s = 1/(\delta\sigma)$

Magnetostatics and Faraday's Law

$\nabla\cdot\mathbf{B} = 0$ $\quad \oint_S \mathbf{B}\cdot d\mathbf{s} = 0$ $\quad B_{1n} = B_{2n}$

$\nabla\times\mathbf{H} = \mathbf{J}$ $\quad \oint_C \mathbf{H}\cdot d\mathbf{l} = I_C$ $\quad \mathbf{H}_{1t} - \mathbf{H}_{2t} = \mathbf{J}_s \times \mathbf{a}_{n2}$

$\mathbf{m} = \mathbf{a}_z IS$ $\quad \mathbf{B} = \frac{\mu_0 m}{4\pi R^3}(\mathbf{a}_R 2\cos\theta + \mathbf{a}_\theta \sin\theta)$ $\quad \mathbf{J}_m = \nabla\times\mathbf{M}$ $\quad \mathbf{J}_{ms} = \mathbf{M}\times\mathbf{a}_n$

Field Intensity: $\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} = \frac{1}{\mu_0\mu_r}\mathbf{B}$

Infinite Line Current: $\mathbf{B} = \mathbf{a}_\phi \frac{\mu I}{2\pi r}$ $\quad$ Infinite Solenoid: $\mathbf{B} = \mathbf{a}_z \mu nI$

$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ $\quad \oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s}$

$\Lambda_{12} = N_2\Phi_{12} = N_2\int_{S_2} \mathbf{B}_1\cdot d\mathbf{s}_2$ $\quad L_{12} = \frac{\Lambda_{12}}{I_1}$ $\quad L = L_{11} = \frac{\Lambda_{11}}{I_1}$

Coax: $L_{ext} = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$ (Hm^{-1}) $\quad$ Wires: $L_{ext} = \frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$ (Hm^{-1})

Energy: $W_m = \frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{N} L_{jk} I_j I_k$ or $\frac{1}{2}\int_{V'} \mathbf{B}\cdot\mathbf{H}\,dv'$ $\quad$ Wire: $L_{int} = \frac{\mu}{8\pi}$ (Hm^{-1})