Class Test 2, 2007

Solutions

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1. [4] Kirchhoff's Current Law is a consequence of which of the following equations?

A $\mathbf{J} = \sigma(\mathbf{E}+\mathbf{E}_i)$

B $\text{EMF} = \oint_C (\mathbf{E}+\mathbf{E}_i)\cdot d\mathbf{l}$

C $\nabla \cdot \mathbf{J} = 0$

D $P = \int_V \mathbf{E}\cdot\mathbf{J}dv$

2. [4] Which of the following is a true statement about the change of magnetic field intensity, **H**, and magnetic flux density, **B**, across an interface between two materials?

A The normal part of **H** must be continuous.

B The normal part of **H** must be discontinuous if the materials have different permeabilities.

C The tangential part of **B** must be continuous if the materials have the same permeability.

D The tangential part of **B** must be discontinuous if there is surface current on the interface.

$B_{1n} = B_{2n} \Rightarrow$

$\mu_1 H_{1n} = \mu_2 H_{2n}$

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3. $\underline{B} = -\underline{a}_x\, y\, t^2 \quad \Rightarrow \quad -\dfrac{\partial \underline{B}}{\partial t} = 2\underline{a}_x\, y\, t$

$$\nabla \times \underline{E} = \underline{a}_x\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \underline{a}_y\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \underline{a}_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

A $\quad \nabla \times \underline{E} = \nabla \times (\underline{a}_x y^2 t) = -\underline{a}_z\, 2yt \neq -\dfrac{\partial B}{\partial t}$

B $\quad \nabla \times \underline{E} = \nabla \times (\underline{a}_z y^2 t) = \underline{a}_x\, 2yt = -\dfrac{\partial \underline{B}}{\partial t}$

C $\quad \nabla \times \underline{E} = \nabla \times (2\underline{a}_x y^2 t) = -\underline{a}_z\, 4yt \neq -\dfrac{\partial B}{\partial t}$

D $\quad \nabla \times \underline{E} = \nabla \times (2\underline{a}_z y^2 t) = \underline{a}_x\, 4yt \neq -\dfrac{\partial B}{\partial t}$

Only B satisfies Faraday's Law.

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4. Assume "long" = "infinite"

From Formula Sheet, $\underline{B} = \underline{a}_z \mu n I = \underline{a}_z \mu_0 \mu_r n I$

Also $\quad \dfrac{1}{\mu_0}\underline{B} - \underline{M} = \dfrac{1}{\mu_0 \mu_r}\underline{B}$

$$\Rightarrow \quad \underline{M} = \frac{1}{\mu_0}\left(1 - \frac{1}{\mu_r}\right)\underline{B} = \frac{1}{\mu_0}\left(1 - \frac{1}{\mu_r}\right)\underline{a}_z \mu_0 \mu_r n I$$

$$= \underline{a}_z\,(\mu_r - 1)\, n I$$

So $\quad I = \dfrac{M}{n(\mu_r - 1)} = \dfrac{10^5}{(100 \times 10^2)(10-1)} = \dfrac{10}{9}$ A $\qquad$ B

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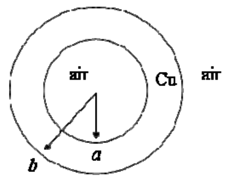
5.



(a) $I_a = \int_{\text{Sphere } R=a} \underline{J}\cdot d\underline{s}$

$= \int_{\substack{\text{Sphere}\\ R=a}} J_0\, \underline{a}_R \cdot \underline{a}_R\, ds$

$= 4\pi a^2 J_0$

(b) $\oint_S \underline{J}\cdot d\underline{s} = 0 \Rightarrow I_b = I_a = 4\pi a^2 J_0$

(c) $\oint_S \underline{J}\cdot d\underline{s} = 0 \Rightarrow I_R = I_a = 4\pi a^2 J_0$

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(d) $I_R = 4\pi a^2 J_0 = \int_{\substack{\text{Sphere}\\ R}} \underline{J}\cdot d\underline{s} = \int_{\substack{\text{Sphere}\\ R}} J_R(R)\, \underline{a}_R \cdot \underline{a}_R\, ds$

$= 4\pi R^2 J_R$

So $J_R = \dfrac{a^2}{R^2} J_0$ and $\underline{J} = \dfrac{a^2 J_0}{R^2}\, \underline{a}_R$

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(e) $\underline{E} = \frac{1}{\sigma}\underline{J} = \frac{a^2}{\sigma R^2} J_0\, \underline{a}_R$

$$V_{ab} = -\int_b^a \underline{E}\cdot d\underline{l} = -\int_{R=b}^{a} \frac{a^2}{\sigma R^2} J_0\, \underline{a}_R \cdot \underline{a}_R\, dR$$

$$= \frac{a^2 J_0}{\sigma} \Big[\frac{1}{R}\Big]_b^a = \frac{a^2 J_0}{\sigma}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{J_0}{\sigma}\frac{a}{b}(b-a)$$

(f) $$\text{Resistance} = \frac{V_{ab}}{I_a} = \frac{J_0}{\sigma}\frac{a}{b}(b-a) \Big/ 4\pi a^2 J_0$$

$$= \frac{(b-a)}{4\pi ab\,\sigma} \qquad \left(\text{compare } \frac{d}{S\sigma}\right)$$

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