Class Test 2, 2006

Solutions

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1. [4] In general, the electric field inside a conductor with steady current flowing in it is:

A zero
B non-zero, but uniform throughout the volume of the conductor
C non-uniform, but not varying in time
D non-uniform and varying in time

2. [4] Which quantity has the unit with standard abbreviation Wb?

A magnetic flux density → T
B magnetic flux
C magnetic field intensity → Am^{-1}
D magnetic flux linkage → Wb-turns

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3. $\nabla \cdot \underline{D} = \rho \Rightarrow \rho = \nabla \cdot \varepsilon \underline{E} = \varepsilon \nabla \cdot \underline{E}$

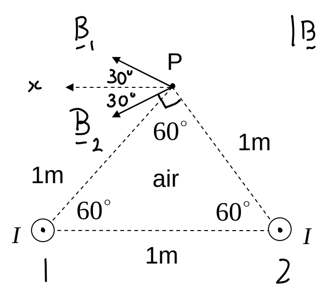
To get $\nabla \cdot \underline{E}$, we use:

$\nabla \cdot \underline{J} = 0 \Rightarrow \nabla \cdot \sigma (\underline{E} + \underline{E}_i) = 0 \Rightarrow \sigma \nabla \cdot (\underline{E} + \underline{E}_i) = 0$

$$\Rightarrow \nabla \cdot \underline{E} = -\nabla \cdot \underline{E}_i$$
$$= -\nabla \cdot x^2 \underline{a}_x = -\frac{\partial}{\partial x} x^2 = -2x$$

So $\rho = \varepsilon(-2x) = -2x\varepsilon$ D

4.



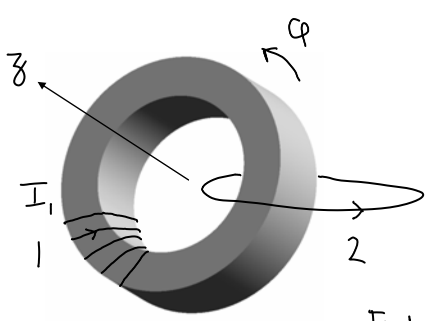
$$|\underline{B}| = B_x = (\underline{B}_1 + \underline{B}_2) \cdot \underline{a}_x$$
$$= B_1 \cos 30^\circ + B_2 \cos 30^\circ$$
$$= \frac{\mu_0 I}{2\pi r} \cos 30^\circ + \frac{\mu_0 I}{2\pi r} \cos 30^\circ$$
$$= \frac{\mu_0 I}{\pi r} \cos 30^\circ$$
$$= \frac{(4\pi \times 10^{-7}) \times 1 \times (\sqrt{3}/2)}{\pi \times 1}$$
$$= 0.3464\ \mu T$$

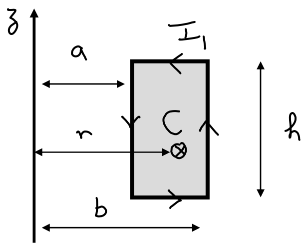
C

For ∞ straight wire:

$$\underline{B}_1 = \frac{\mu_0 I}{2\pi r} \underline{a}_\varphi$$

5.





Put current I_1 in 1 and find L_{12}.

r, φ, z coords.

Find $\underline{B}_1$, due to I_1:

$$\oint_C \underline{H}\cdot d\underline{l} = I_c$$

$$\Rightarrow \quad 2\pi r H_\varphi = -N I_1 \quad \Rightarrow \quad B_\varphi = -\frac{\mu N I_1}{2\pi r}$$

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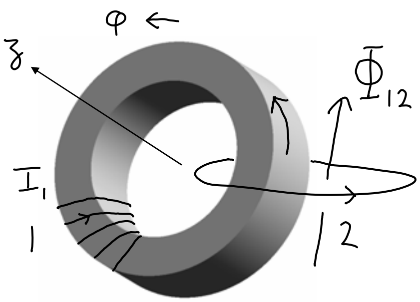

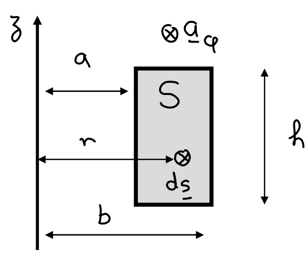

$\underline{B}_1$ is zero outside the toroid, so the flux Φ_{12} passing through coil 2 is just the flux flowing around inside the toroid.

i.e. $$\Phi_{12} = \int_S \underline{B}_1 \cdot d\underline{s}$$

$$= \int_{z=0}^{h} \int_{r=a}^{b} \left(-\frac{\mu N I_1}{2\pi r} \underline{a}_\varphi \right) \cdot \left(dr\,dz\, \underline{a}_\varphi \right)$$

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i.e. $\Phi_{12} = -\frac{\mu N I_1 h}{2\pi} \int_{r=a}^{b} \frac{1}{r}\, dr = -\frac{\mu N I_1 h}{2\pi} \ln\left(\frac{b}{a}\right)$

Then $\Lambda_{12} = N_2\, \Phi_{12} = 1.\, \Phi_{12} = -\frac{\mu N I_1 h}{2\pi} \ln\left(\frac{b}{a}\right)$

and $L_{12} = \frac{\Lambda_{12}}{I_1} = -\frac{\mu N h}{2\pi} \ln\left(\frac{b}{a}\right)$

$\parallel$

L_{21}

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