Class Test 2, 2005

Solutions

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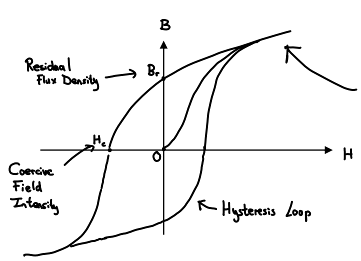
1. [4] The unit of electromotive force is:
A V B Vm^{-1} C N D Nm^{-1}

2. [4] In general, in a ferromagnetic material:
A The magnetization is uniform.
B The magnitude of the magnetization is directly proportional to the magnitude of the magnetic field, **H**.
C The magnitude of the magnetization is directly proportional to the magnitude of the magnetic flux density, **B**.
D The magnitude of the magnetization is never greater than a certain value, no matter how large **B** or **H** is.

$B = \mu_0 H + M$



$$\left.\begin{array}{l} M = M_{sat} \\ \dfrac{dB}{dH} = \mu_0 \end{array}\right\} \text{Saturation}$$

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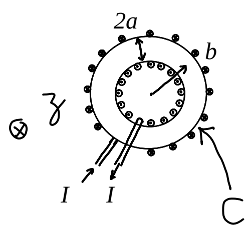
3.



$b = 1\text{ cm}$, $a = 0.1\text{ cm}$ } So $a \ll b$

$N = 1000$

$I = 1\text{ mA}$

$$\boxed{\int_C \underline{J}_s \cdot \underline{a}_n \, dl = I}$$

Current crossing $C = IN = \int_C \underline{J}_s \cdot \underline{a}_n \, dl = \int_{\varphi=0}^{2\pi} J_s \, \underline{a}_z \cdot \underline{a}_z \, (a+b) \, d\varphi$

$$= 2\pi J_s (a+b)$$

So $J_s = \dfrac{IN}{2\pi(a+b)} \simeq \dfrac{IN}{2\pi b} = \dfrac{10^{-3}\,10^{3}}{2\pi \cdot 10^{-2}} = 15.9 \text{ Am}^{-1}$ A

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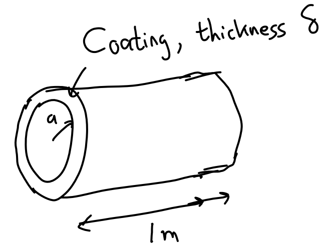
4.

$\delta \ll a$



$R = \dfrac{l}{\sigma S}$

In this case: $l = 1\text{ m}$

$S = \pi(a+\delta)^2 - \pi a^2 = \pi(a^2 + 2a\delta + \delta^2 - a^2)$

$= \pi(2a\delta + \delta^2)$

So $R' = \dfrac{1}{\sigma\pi(2a\delta+\delta^2)} \simeq \dfrac{1}{\sigma\pi\, 2a\delta}$ $(\Omega \text{ m}^{-1})$ C

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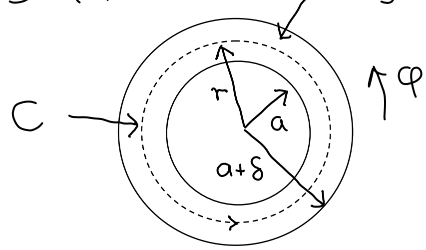
5 (a)

Coating (thickness exaggerated)



Ampère's Law:

$$\oint_C \underline{H}\cdot d\underline{l} = I\left(\frac{\pi r^2 - \pi a^2}{\pi(a+\delta)^2 - \pi a^2}\right)$$

$$\Rightarrow \quad 2\pi r H_\varphi = I\left(\frac{r^2 - a^2}{(a+\delta)^2 - a^2}\right)$$

So $$\underline{B} = \mu_0 \underline{H} = \mu_0\, \underline{a}_\varphi \frac{I}{2\pi r}\left(\frac{r^2-a^2}{2a\delta+\delta^2}\right) \quad //$$

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(b) Energy/m in coating $= W_m' = \frac{1}{2}\int_{\text{Coating}} \underline{H}\cdot\underline{B}\, dv = \frac{1}{2\mu_0}\int_{\text{Coating}} B^2\, dv$

$$= \frac{1}{2\mu_0}\int_{r=a}^{a+\delta}\int_{z=0}^{1}\int_{\varphi=0}^{2\pi}\left[\frac{\mu_0 I}{2\pi r}\left(\frac{r^2-a^2}{2a\delta+\delta^2}\right)\right]^2 r\,d\varphi\, dz\, dr$$

$$= \frac{\mu_0 I^2}{8\pi^2}\frac{2\pi}{(2a\delta+\delta^2)^2}\int_{r=a}^{a+\delta}\frac{r^4 - 2a^2r^2 + a^4}{r}\, dr$$

$$= \frac{\mu_0 I^2}{4\pi\delta^2}\frac{1}{(2a+\delta)^2}\left[\frac{r^4}{4} - a^2r^2 + a^4\ln r\right]_a^{a+\delta}$$

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$$= \frac{\mu_0 I^2}{4\pi\delta^2} \frac{1}{(2a+\delta)^2} \left[\frac{(a+\delta)^4 - a^4}{4} - a^2\left((a+\delta)^2 - a^2\right) + a^4 \ln\frac{a+\delta}{a} \right]$$

But also $W_m' = \frac{1}{2} L_{int}' I^2$ where L_{int}' = internal inductance /m of coating

So

$$L_{int}' = \frac{\mu_0}{2\pi\delta^2} \frac{1}{(2a+\delta)^2} \left[\frac{(a+\delta)^4 - a^4}{4} - a^2\left((a+\delta)^2 - a^2\right) + a^4 \ln\frac{a+\delta}{a} \right]$$

(Hm^{-1})

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(c) (Supplementary)

Let $\alpha = \delta/a$. Derive an approximate inductance for $\alpha \ll 1$.

$$L_{int}' = \frac{\mu_0}{2\pi\alpha^2 a^4} \frac{a^4}{(2+\alpha)^2} \left[\frac{(1+\alpha)^4 - 1}{4} - \left((1+\alpha)^2 - 1\right) + \ln(1+\alpha) \right]$$

$$= \frac{\mu_0}{2\pi} \frac{1}{\alpha^2} \frac{1}{(2+\alpha)^2} \left[\frac{(1 + 4\alpha + 6\alpha^2 + 4\alpha^3 + \alpha^4 - 1)}{4} - (1 + 2\alpha + \alpha^2 - 1) + \ln(1+\alpha) \right]$$

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$$= \frac{\mu_0}{2\pi} \frac{1}{\alpha^2} \frac{1}{(2+\alpha)^2} \left[-\alpha + \frac{\alpha^2}{2} + \alpha^3 + \frac{\alpha^4}{4} + \ln(1+\alpha) \right]$$

But $\ln(1+\alpha) \approx \alpha - \frac{\alpha^2}{2} + \frac{\alpha^3}{3}$

So $L'_{int} \approx \frac{\mu_0}{2\pi} \frac{1}{4\alpha^2} \frac{4\alpha^3}{3} = \frac{\mu_0}{6\pi} \left(\frac{\delta}{a}\right) \quad (\mathrm{Hm}^{-1})$

Compare this with wire: $L'_{int} = \frac{\mu_0}{8\pi} \quad (\mathrm{Hm}^{-1})$

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