Class Test 1, 2007

Solutions

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1\. [4] Which of these statements about the electric potential difference between two points is *not* true?

A It is the path integral of electric field from one point to the other.
B It is the work done per unit charge in moving a charge….
C Its units are volts per metre.
D It is zero if both points are in the same conductor, under static conditions.

Units = VOLTS.

2\. [4] "Polarization" means

A volume density of electric dipole moment
B volume density of electric charge
C surface density of electric dipole moment
D surface density of electric charge

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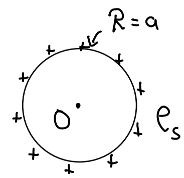
3.



Gaussian surface $R > a$:

$$\oint \underline{D}\cdot d\underline{s} = Q \Rightarrow 4\pi R^2 D_R = 4\pi a^2 \rho_s$$

$$\Rightarrow E_R = \frac{a^2}{R^2}\frac{\rho_s}{\varepsilon_0}$$

Gaussian surface $R<a$:

$$\oint \underline{D}\cdot d\underline{s} = Q \Rightarrow 4\pi R^2 D_R = 0 \Rightarrow E_R = 0$$

$$V(R=a) = -\int_{R=\infty}^{R=a} \underline{E}\cdot d\underline{l} = -\int_{R=\infty}^{a} \frac{a^2}{R^2}\frac{\rho_s}{\varepsilon_0}\,dR = \frac{\rho_s}{\varepsilon_0}a^2\left[\frac{1}{a}\right] = \frac{\rho_s}{\varepsilon_0}a$$

Inside, $\underline{E}=0$ so $V(R=0) = V(R=a) = \frac{\rho_s}{\varepsilon_0}a$ (A)

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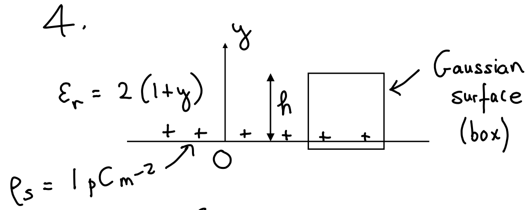
4.



Assume $\underline{E} = \underline{a}_y E_y(y)$

Gauss: $$\oint_S \underline{D}\cdot d\underline{s} = Q \Rightarrow \int_{Top} D_y\,\underline{a}_y\cdot\underline{a}_y\,ds = \rho_s S$$

Area of top of box

$$\Rightarrow D_y(h)\,S = \rho_s S \Rightarrow D_y = \rho_s$$

$$\Rightarrow E_y(h) = \frac{D_y}{\varepsilon(h)} = \frac{\rho_s}{\varepsilon_0\,2(1+h)}$$

So

$$E_y(0.02) = \frac{10^{-12}}{\frac{1}{36\pi}\times 10^{-9}\times 2(1+0.2)} = 47.1\ \mathrm{mV\,m^{-1}}$$ (B)

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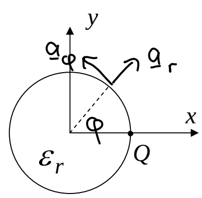
5.



(a) $\underline{E} = E_0\, \underline{a}_x$

$= E_r\, \underline{a}_r + E_\varphi\, \underline{a}_\varphi$

$\Rightarrow \quad E_0\, \underline{a}_x \cdot \underline{a}_r = E_r$

$E_0\, \underline{a}_x \cdot \underline{a}_\varphi = E_\varphi$

$\Rightarrow \quad E_r = E_0 \cos\varphi$

$E_\varphi = -E_0 \sin\varphi$

(b) $$V_P - V_Q = -\int_Q^P \underline{E}\cdot d\underline{l} = -\int_{\varphi=0}^{\alpha} \left(E_r \underline{a}_r + E_\varphi \underline{a}_\varphi\right) \cdot b\, \underline{a}_\varphi\, d\varphi$$

$$= +bE_0 \int_0^{\alpha} \sin\varphi\, d\varphi = -bE_0(\cos\alpha - 1) = \underline{bE_0(1-\cos\alpha)}$$

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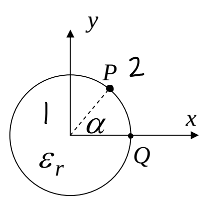
(c)



Interface conditions:

$\underline{E}_{1t} = \underline{E}_{2t}$

$\Rightarrow \quad E_{1\varphi} = E_{2\varphi}$

$\underline{a}_{n2} \cdot (\underline{D}_1 - \underline{D}_2) = \rho_s$

$\Rightarrow \quad D_{1r} - D_{2r} = 0$

$\Rightarrow \quad \varepsilon_r E_{1r} = E_{2r}$

So at P:

$E_{2\varphi} = -E_0 \sin\alpha$

$E_{2r} = \varepsilon_r E_0 \cos\alpha$

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$$E_{2x} = \underline{E}_2 \cdot \underline{a}_x = E_{2r}\, \underline{a}_r \cdot \underline{a}_x + E_{2\varphi}\, \underline{a}_\varphi \cdot \underline{a}_x$$

$$= E_{2r} \cos\alpha - E_{2\varphi} \sin\alpha$$

$$= + E_0 \varepsilon_r \cos^2\alpha + E_0 \sin^2\alpha$$

$$E_{2y} = \underline{E}_2 \cdot \underline{a}_y = E_{2r}\, \underline{a}_r \cdot \underline{a}_y + E_{2\varphi}\, \underline{a}_\varphi \cdot \underline{a}_y$$

$$= E_{2r} \sin\alpha + E_{2\varphi} \cos\alpha$$

$$= E_0 \varepsilon_r \cos\alpha \sin\alpha - E_0 \cos\alpha \sin\alpha$$

[Check, $\varepsilon_r = 1$: $E_{2x} = E_0$

$E_{2y} = 0$]

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