Class Test 1, 2006

Solutions

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1. [4] Which of the following quantities is a vector?
   - A permittivity
   - B electric flux
   - (C) polarization
   - D polarization volume charge density

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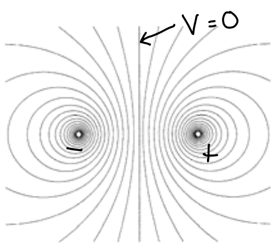
2. [4] Which of the following does this figure represent?
   - A the equipotentials of a pair of equal charges
   - (B) the equipotentials of a pair of opposite charges
   - C the electric field lines of a pair of equal charges
   - D the electric field lines of a pair of opposite charges



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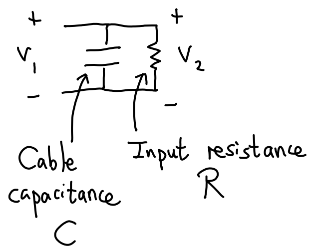
3.



Rise time of $v_2 = \tau$

$= CR$

$C = C'\ell$, so $\tau = C'R\ell$

↑ Capacitance/m

$$C' = \frac{2\pi\varepsilon}{\ln b/a} \qquad C'_{new} = \frac{2\pi\varepsilon}{\ln b_{new}/a}$$

Need $C'_{new} = \frac{1}{2}C'$ to reduce τ by $1/2$

ie $\ln \frac{b_{new}}{a} = 2\ln\frac{b}{a} \Rightarrow \frac{b_{new}}{a} = \left(\frac{b}{a}\right)^2 = 9$ (D)

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4. $\underline{E} = \underline{a}_R / R$

$\underline{P} = \varepsilon_0 \underline{a}_R$

$\underline{D} = \varepsilon_0 \underline{E} + \underline{P} = \varepsilon_0 \left(\frac{1}{R} + 1\right) \underline{a}_R$

$\rho = \nabla \cdot \underline{D} = \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 D_R\right)$ (spherical coords.)

$= \varepsilon_0 \frac{1}{R^2} \frac{\partial}{\partial R} \left(R + R^2\right)$

$= \frac{(1 + 2R)}{R^2} \varepsilon_0$ (D)

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5. (a) $\underline{D} = \varepsilon_0 \underline{E} + \underline{P} \Rightarrow \underline{P} = \underline{D} - \varepsilon_0 \underline{E}$

$$= \varepsilon \underline{E} - \varepsilon_0 \underline{E}$$

$$= -(\varepsilon - \varepsilon_0) E_2 \underline{a}_x$$

(b) $\rho_p = -\nabla \cdot \underline{P} = +\nabla \cdot \underbrace{(\varepsilon - \varepsilon_0) E_2 \underline{a}_x}_{\text{uniform}} = 0$

At surface $x = 0$: $\rho_{ps} = \underline{P} \cdot \underline{a}_n = \left(-(\varepsilon - \varepsilon_0) E_2 \underline{a}_x\right) \cdot \underline{a}_x$

$$= -(\varepsilon - \varepsilon_0) E_2$$

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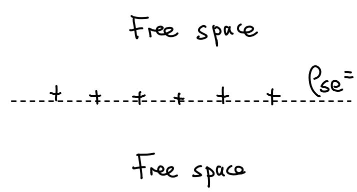
(c) Equivalent system:



$$\rho_{se} = \rho_s + \rho_{ps}$$

$$= \rho_s - (\varepsilon - \varepsilon_0) E_2$$

(d) From equivalent system: $E_1 = E_2 = \dfrac{\rho_{se}}{2\varepsilon_0} = \dfrac{\rho_s - (\varepsilon - \varepsilon_0) E_2}{2\varepsilon_0}$

$\Rightarrow 2\varepsilon_0 E_2 = \rho_s - (\varepsilon - \varepsilon_0) E_2$

$\Rightarrow (\varepsilon + \varepsilon_0) E_2 = \rho_s \Rightarrow E_1 = E_2 = \dfrac{\rho_s}{\varepsilon + \varepsilon_0}$

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