Class Test 1, 2005

Solutions

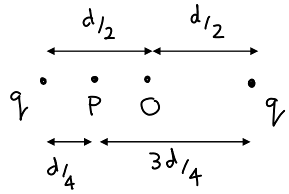
1. [4] In general, charge on a conductor under static conditions is distributed:
   - A uniformly through the volume of the conductor.
   - B non-uniformly through the volume of the conductor.
   - C uniformly over the surface of the conductor.
   - (D) non-uniformly over the surface of the conductor.

2. [4] The tangential part of the electric flux density is continuous across a surface, *S*. Which one of the following statements must be true?
   - A The surface charge density on *S* is zero.
   - B The surface charge density on *S* is non-zero.
   - (C) The dielectric constant is the same on both sides of *S*.
   - D The dielectric constant is the same on both sides of *S* and the surface charge density on *S* is zero.

$$\underline{E}_{1t} = \underline{E}_{2t}$$

$$\underline{D} = \varepsilon \underline{E}$$

3.



$$\boxed{V_p = \frac{q}{4\pi\varepsilon_o R}}$$

$$V_P = \frac{q}{4\pi\varepsilon_o (d/4)} + \frac{q}{4\pi\varepsilon_o (3d/4)} = \frac{q}{4\pi\varepsilon_o d}\left(4 + \frac{4}{3}\right) = \frac{q}{4\pi\varepsilon_o d}\,\frac{16}{3}$$

$$V_O = \frac{q}{4\pi\varepsilon_o (d/2)} + \frac{q}{4\pi\varepsilon_o (d/2)} = \frac{q}{4\pi\varepsilon_o d}\left(2 + 2\right) = \frac{q}{4\pi\varepsilon_o d}\,4$$

So $V_{PO} = \frac{q}{4\pi\varepsilon_o d}\left(\frac{16}{3} - 4\right) = \frac{q}{4\pi\varepsilon_o d}\,\frac{4}{3} = \frac{q}{3\pi\varepsilon_o d}$ (A)

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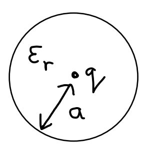
4.



$$\boxed{\underline{D} = \varepsilon \underline{E}} \quad \boxed{\underline{D} = \varepsilon_o \underline{E} + \underline{P}}$$

$$\boxed{\rho_{ps} = \underline{P}\cdot \underline{a}_n} \quad \boxed{\oint_S \underline{D}\cdot d\underline{s} = Q}$$

Gauss' Law: $4\pi R^2 D_R = q$

$\Rightarrow D_R = \frac{q}{4\pi R^2}$ and $\underline{D} = \underline{a}_R \frac{q}{4\pi R^2}$

In sphere: $\underline{E} = \frac{1}{\varepsilon}\underline{D} = \frac{1}{\varepsilon_o \varepsilon_r}\underline{a}_R \frac{q}{4\pi R^2}$

At $R = a$: $\underline{P} = \underline{D} - \varepsilon_o \underline{E} = \underline{a}_R \frac{q}{4\pi a^2} - \varepsilon_o \underline{a}_R \frac{1}{\varepsilon_o \varepsilon_r}\frac{q}{4\pi a^2} = \frac{\underline{a}_R q}{4\pi a^2}\left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)$

and $\rho_{ps} = \underline{P}\cdot\underline{a}_n = \underline{P}\cdot\underline{a}_R = \frac{q}{4\pi a^2}\left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)$ (D)

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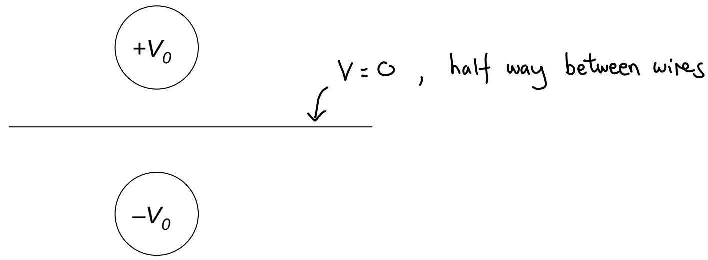
5. (a)



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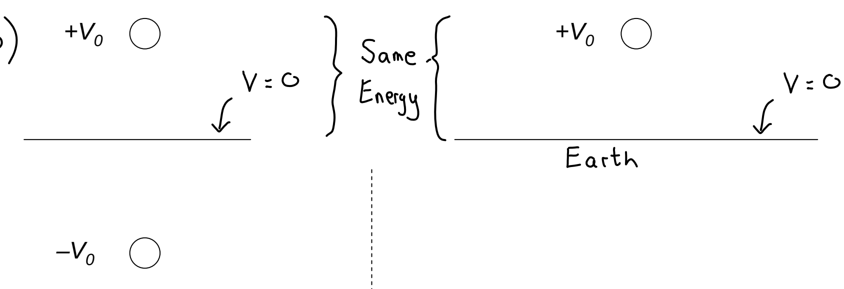
(b)



Energy stored $= W_e$

$= \frac{1}{2} C V_{12}^2$

$= \frac{1}{2} C (2V_0)^2$

$= 2 C V_0^2 \quad (Jm^{-1})$

So energy stored $= W_e/2$

$= C V_0^2 \quad (Jm^{-1})$

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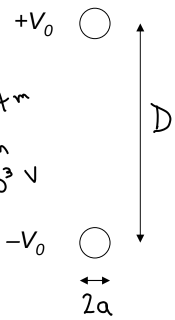

$D = 2 \times 4\ \text{m}$

$a = 1\ \text{cm}$

$V_0 = 10^3\ \text{V}$

From F.S.:

$$C = \frac{\pi \varepsilon}{\cosh^{-1}\left(D/2a\right)} \qquad \left(\text{or: } C \cong \frac{\pi \varepsilon}{\ln\left(D/a\right)}\right)$$

$$= \pi \left(\frac{1}{36\pi} \times 10^{-9}\right) \frac{1}{\cosh^{-1}\left(\frac{8}{2 \times 0.01}\right)}$$

$$= 4.155\ \text{pF m}^{-1}$$

Finally, energy $= C V_0^2$

$$= 4.155 \times 10^{-12} \times (10^3)^2$$

$$= 4.155\ \mu\text{J m}^{-1}$$

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