ECSE 353 CLASS TEST 1 SOLUTIONS OCTOBER 2003

1. D

2. C

3. B

$$\text{Rise time} = CR = \frac{2\pi\varepsilon_0\varepsilon_r \ell R}{\ln b/a} < 0.2\,T = \frac{0.2}{f}$$

$$\Rightarrow \ell < \frac{0.2}{f}\,\frac{\ln b/a}{2\pi\varepsilon_0\varepsilon_r R} = \frac{(0.2)}{(10^9)}\,\frac{(\ln 5/0.5)}{(2\pi)\left(\frac{1}{36\pi}\right)(10^{-9})(2)(50)}$$

$$= 0.0829 \text{ m}$$

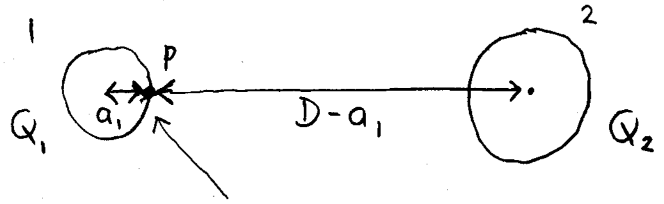
4. C

$$\nabla \times \underline{E} = 0 \Rightarrow \Big(\underbrace{\cancel{\tfrac{\partial E_z}{\partial y}}}_{0} - \underbrace{\cancel{\tfrac{\partial E_y}{\partial z}}}_{0}\Big)\underline{a}_x + \Big(\underbrace{\cancel{\tfrac{\partial E_x}{\partial z}}}_{0} - \underbrace{\cancel{\tfrac{\partial E_z}{\partial x}}}_{0}\Big)\underline{a}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\underline{a}_z = 0$$

$$\Rightarrow \frac{\partial}{\partial x}\left(x f(y)\right) - \frac{\partial}{\partial y}\left(y^2\right) = 0 \Rightarrow f(y) = 2y$$

$$\rho = \nabla \cdot \underline{D} = \varepsilon_0 \nabla \cdot \underline{E} = \varepsilon_0\Big[\underbrace{\cancel{\tfrac{\partial E_x}{\partial x}}}_{0} + \frac{\partial E_y}{\partial y} + \underbrace{\cancel{\tfrac{\partial E_z}{\partial z}}}_{0}\Big] = \varepsilon_0 \frac{\partial}{\partial y}(2xy) = \underline{\underline{2x\varepsilon_0}}$$

5. (a)



V due to Q_1 is $\dfrac{Q_1}{4\pi\varepsilon_0 R}$ (formula sheet)

So V_1 = Potential at P $= \dfrac{Q_1}{4\pi\varepsilon_0 a_1} + \dfrac{Q_2}{4\pi\varepsilon_0 (D - a_1)}$

$$= \frac{1}{4\pi\varepsilon_0 a_1}\left[Q_1 + \frac{a_1}{D}\,\frac{Q_2}{(1 - a_1/D)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 a_1}\left[Q_1 + \bar{a}_1 Q_2 (1 - \bar{a}_1)^{-1}\right] \qquad \text{where } \bar{a}_1 = \frac{a_1}{D} \ll 1$$

$$= \frac{1}{4\pi\varepsilon_0 a_1}\left[Q_1 + \bar{a}_1 Q_2 (1 + \bar{a}_1 + \dots)\right]$$

$$\simeq \frac{1}{4\pi\varepsilon_0 a_1}\left[Q_1 + \bar{a}_1 Q_2\right]$$

Similarly for V_2

So

$$\left.\begin{aligned} Q_1 + \bar{a}_1 Q_2 &= 4\pi\varepsilon_0 a_1 V_1 \\ Q_2 + \bar{a}_2 Q_1 &= 4\pi\varepsilon_0 a_2 V_2 \end{aligned}\right\} \text{Solve for } Q_1, Q_2$$

$$\Rightarrow \quad Q_1 - \bar{a}_1\bar{a}_2 Q_1 = 4\pi\varepsilon_0\left[a_1 V_1 - \bar{a}_1 a_2 V_2\right]$$

$$\Rightarrow \quad Q_1 = 4\pi\varepsilon_0 a_1\left[V_1 - \bar{a}_2 V_2\right]\left(1 - \bar{a}_1\bar{a}_2\right)^{-1}$$

$$\simeq 4\pi\varepsilon_0 a_1\left[V_1 - \frac{a_2}{D} V_2\right] \quad //$$

Similarly for Q_2.

(b) V halfway between centres $= \dfrac{Q_1}{4\pi\varepsilon_0 D/2} + \dfrac{Q_2}{4\pi\varepsilon_0 D/2}$

$$= \frac{1}{2\pi\varepsilon_0 D}\left[4\pi\varepsilon_0 a_1\left(V_1 - \bar{a}_2 V_2\right) + 4\pi\varepsilon_0 a_2\left(V_2 - \bar{a}_1 V_1\right)\right]$$

$$= \frac{2}{D}\left[a_1\left(1-\bar{a}_2\right)V_1 + a_2\left(1-\bar{a}_1\right)V_2\right]$$

$$= \frac{2}{10^{-2}}\left[(10^{-3})\left(1-\tfrac{2}{10}\right)(1) + (2)(10^{-3})\left(1-\tfrac{1}{10}\right)(1)\right]$$

$$= 0.2\left[0.8 + (2)(0.9)\right] = \underline{\underline{0.52\ V}}$$