# COURSE ECSE 353 ELECTROMAGNETIC FIELDS AND WAVES 

| Examiner: | J. P. Webb | Co-Examiner: None |  |
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| Signature: |  | Signature: |  |
|  |  |  |  |
| Date: | October 10, 2003 | Time: | $08: 35-09: 25$ |

- This is a closed book examination. No books or notes are permitted, except for the Formula Sheet attached.
- The Faculty Standard Calculator (Casio fx-991 or Sharp EL-546L or R or V (VB) or G) only is permitted.
- All units are SI unless otherwise stated
- This is a 50 minute exam
- The marks indicated in square brackets at the start of each question are out of 50 .


## INSTRUCTIONS:

- Answer all questions.
- Put your name and student ID also on the Answer Sheet provided.
- Part A is multiple choice. There is one correct answer for each question. Mark your answers on the Answer Sheet, not on this examination paper. Only the answers on the Answer Sheet will be considered.
- Part B: Put your answers in the spaces provided on the Answer Sheet.


## PART A

Part A is multiple choice. There is one correct answer for each question. Mark your final answers on the Answer Sheet. Only the answers on the Answer Sheet will be considered.

1. [4] What is the unit of permittivity?

| A | C |
| :--- | :--- |
| B | F |
| C | $\mathrm{Cm}^{-1}$ |
| D | $\mathrm{Fm}^{-1}$ |

2. [4] The electric potential difference between points P 1 and P 2 is positive, i.e., P 1 is at a higher electric potential than P 2 . Which of the following statements is true?
A The potential energy of a negative point charge is higher at P1 than at P2.
B In moving from P2 to P1, on average you are moving in the same direction as the electric field vector.
C You have to do mechanical work to move a positive point charge from P 2 to P 1 .
D A positive point charge placed exactly half way between P1 and P2 will necessarily move towards P2.
3. [6] A coaxial cable is used to connect one computer to another. The radius of the inner conductor of the cable is 0.5 mm , the radius of the outer conductor is 5 mm , the dielectric constant is 2 , the input resistance of the computers is $50 \Omega$, and the clock frequency of the computers is 1 GHz . If the rise time $(C R)$ of the pulses is not to exceed $20 \%$ of the period of the clock, how long can the cable be? [Take into account only the capacitance of the cable, not its resistance, conductance and inductance].
A $\quad 3.60 \mathrm{~cm}$
B $\quad 8.29 \mathrm{~cm}$
C $\quad 16.6 \mathrm{~cm}$
D $\quad 4.15 \mathrm{~m}$
4. [6] Throughout a region with permittivity $\varepsilon_{o}$, the static electric field is of the form $\mathbf{E}=y^{2} \mathbf{a}_{x}+x f(y) \mathbf{a}_{y}$ (in rectangular coordinates $x, y, z$ ), where $f(y)$ is a function of $y$ only. By first finding $f(y)$, determine the volume charge density.
A $\quad 2 x y \varepsilon_{o}$
B $x y \varepsilon_{o}$
C $2 x \varepsilon_{o}$
D $x \varepsilon_{o}$

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## PART B

In Part B, put your answer in the spaces provided on the Answer Sheet.
5. [30] For an isolated conducting sphere of radius $a$ carrying a charge $Q$, the potential at a distance $R$ from the centre of the sphere is:

$$
\begin{equation*}
V=\frac{Q}{4 \pi \varepsilon_{o} R} \tag{1}
\end{equation*}
$$

The figure below shows two conducting spheres: sphere 1 is at potential $V_{1}$ and sphere 2 is at potential $V_{2}$, both potentials relative to infinity. The centres of the spheres are distance $D$ apart, where $D$ is much greater than either radius. Assume that the charge on each sphere is spread uniformly over its surface, so that the potential due to each sphere is given by equation (1), above.

(a) Show that the charges, $Q_{1}$ and $Q_{2}$, on the two spheres are:

$$
Q_{1} \cong 4 \pi \varepsilon_{o} a_{1}\left[V_{1}-\left(\frac{a_{2}}{D}\right) V_{2}\right] \quad \text { and } \quad Q_{2} \cong 4 \pi \varepsilon_{o} a_{2}\left[V_{2}-\left(\frac{a_{1}}{D}\right) Y_{1}\right]
$$

Make use of the fact that $D$ is much greater than $a_{1}$ and $a_{2}$. Ignore second and higher powers of $a_{1} / D$ and $a_{2} / D$. [You will find it useful to use the approximation $(1+x)^{-1} \cong 1-x$, which ignores second and higher powers of $x$ and is valid when the magnitude of $x$ is much less than 1.]
(b) Using the expressions given in (a), and with no further approximation, find the potential (relative to infinity) at a point exactly half way between the centres of the spheres, when $V_{1}=V_{2}=1 \mathrm{~V}, a_{1}=1 \mathrm{~mm}, a_{2}=2 \mathrm{~mm}$ and $D=1 \mathrm{~cm}$.

