**McGill University**
**Faculty of Engineering**

**FINAL EXAM**

# COURSE ECSE 353
# ELECTROMAGNETIC FIELDS AND WAVES

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| Examiner: | J. P. Webb | Co-Examiner: | S. McFee |
| Signature: | _____________ | Signature: | _____________ |
| Date: | Monday, December 17, 2007 | Time: | 14:00 |

- This is a closed book examination. No books or notes are permitted, except for the Formula Sheet attached.
- The Faculty Standard Calculator (Casio fx-115, Casio fx-991, Casio fx-570ms, Sharp EL-520, or Sharp EL-546) only is permitted.
- All units are SI unless otherwise stated.
- Unless otherwise stated: $x,y,z$ are rectangular (Cartesian) coordinates; $r, \phi, z$ are cylindrical coordinates; and $R, \theta, \phi$ are spherical coordinates.
- This is a 180 minute exam
- The marks indicated in square brackets at the start of each question are out of 180.

**INSTRUCTIONS:**

- Answer all questions.
- Put your name and student ID also on the Answer Sheet provided.
- **Part A** is multiple choice. There is one correct answer for each question. Mark your answers on the Answer Sheet, not on this examination paper. Only the answers on the Answer Sheet will be considered.
- **Part B**: Put your answers in the spaces provided on the Answer Sheet.

**WHEN INSTRUCTED, TURN TO NEXT PAGE AND START THE EXAM**

# PART A

Part A is multiple choice. There is one correct answer for each question. Mark your final answers on the Answer Sheet. Only the answers on the Answer Sheet will be considered.

1. [5] In a circularly-polarized plane electromagnetic wave, at a fixed point in space:
   - A the electric field always points in one direction
   - B at any instant, the magnetic and electric fields point in the same direction
   - C the magnitude of the electric field goes to zero periodically
   - D the magnitude of the electric field does not change in time

2. [5] The *directivity* of an antenna, expressed in dB,
   - A is independent of the frequency of operation
   - B depends on the direction of radiation $(\theta,\phi)$
   - C can be positive or negative, depending on the antenna
   - D is no less than the antenna gain

3. [5] A two-port device consists of a resistor and a capacitor, connected as shown. Which of the following relationships between its scattering parameters must be true?
   - A $S_{11} = S_{22}$
   - B $S_{12} = S_{21}$
   - C $S_{11} = S_{12}$
   - D $|S_{11}|^2 + |S_{21}|^2 = 1$

4. [5] Which of the following statements about the self and mutual inductance of coils is true?

A Self inductance can be positive or negative, depending on the positive orientation chosen for the coil.

B Mutual inductance can be positive or negative, depending on the positive. orientations chosen for the two coils.

C Mutual inductance is always positive.

D Mutual inductance is always negative.

5. [5] Which of the following equations tells you how much heat is generated in a conductor carrying a current?

A $\mathbf{J} = \sigma(\mathbf{E}+\mathbf{E}_i)$

B $\text{EMF} = \oint_C \left(\mathbf{E} + \mathbf{E}_i\right) \cdot d\mathbf{l}$

C $\nabla \cdot \mathbf{J} = 0$

D $P = \int_V \mathbf{E} \cdot \mathbf{J} \mathrm{d}v$

6. [5] A linearly polarized plane wave is travelling through air and is normally incident on an infinite half space filled with perfect conductor. Which of the following is true?

A The reflection coefficient is –1 and the transmission coefficient is 0.

B The reflection coefficient is +1 and the transmission coefficient is 0.

C The reflection coefficient is 0 and the transmission coefficient is –1.

D The reflection coefficient is 0 and the transmission coefficient is +1.

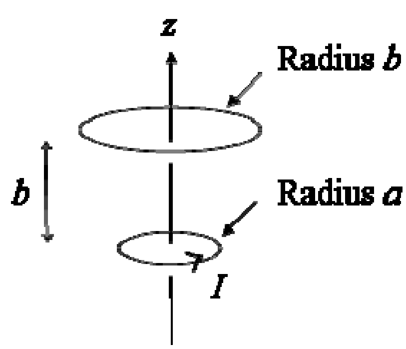
# PART B

In Part B, put your answers in the spaces provided on the Answer Sheet.

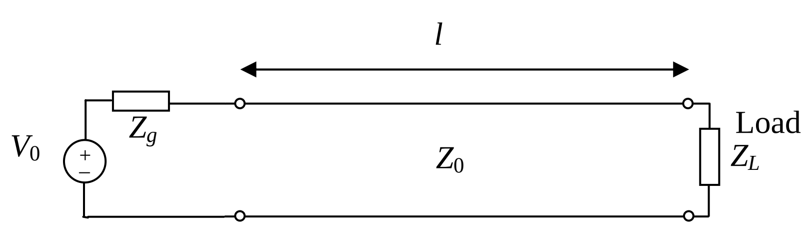
7. [30] A circular loop of wire of radius *a* carries a steady current *I*. The loop lies in the *x*-*y* plane, with its centre at the origin. The current flows clockwise around the positive *z* axis.

(a) Find the magnetic dipole moment of the loop.

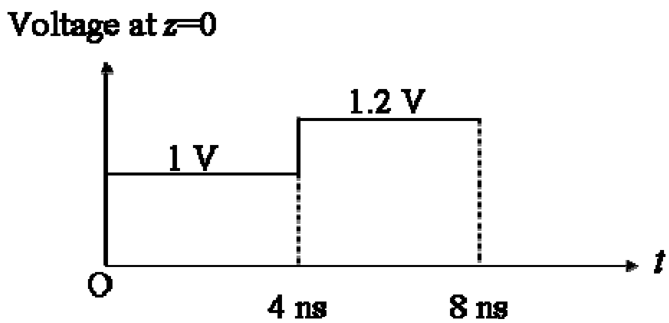
(b) A second circular loop of wire of radius *b* is parallel with the first, and distance *b* away, where *b* is much greater than *a*. Find the magnetic flux through the second loop, in the +z direction. [Hint: Get the flux by integrating over a surface that has the second loop as its boundary and is part of the surface of a sphere.]




8. [30] A lossless coaxial cable is filled with a dielectric with relative permittivity 2.25. It has a characteristic impedance $Z_0$=50Ω and length *l*=21cm. The source generates a sinusoidal signal with phasor voltage $V_0$=5+j0 V, at frequency 1GHz. It has an internal impedance $Z_g$=j75Ω. Find the phasor voltage across the load, $Z_L$=j75Ω.



9. [30] A lossless, air-filled transmission line has a characteristic resistance of $R_0$. The load is a resistor with resistance $R_L$. The generator is a 3V battery in series with a 100Ω resistor, connected to the transmission line via a switch that closes at times $t$=0. The line extends from $z$=0 (generator end) to $z$=$L$ (load end). The voltage measured at $z$=0 is shown in this diagram:



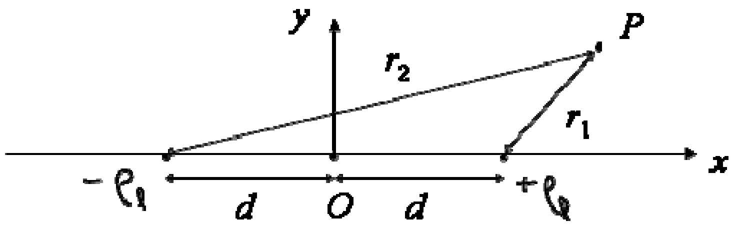

Calculate the numerical values of the following quantities:

(a) the characteristic resistance, $R_0$;

(b) the length of the line, $L$; and

(c) the load resistance, $R_L$.

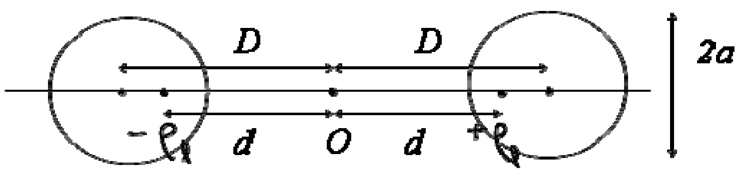
10. [30] Two infinite, static line charges $+\rho_l$ and $-\rho_l$ are parallel to the $z$ axis and distance 2$d$ apart, as shown. The medium around them is air.




(a) A point $P$ is distance $r_1$ from $+\rho_l$ and distance $r_2$ from $-\rho_l$. Find the electrostatic potential difference between $P$ and the origin $O$.

Any equipotential of the above charge system is a circle with centre $(x,y) = (d\beta, 0)$ and radius $d\sqrt{\beta^2 - 1}$, where $\beta$ depends on the potential.

(b) Consider two circles that are each of radius $a$. Their centres are distance $2D$ apart, as shown. Find the value of $d$ that makes these two circles equipotentials.




11. [30] In a free-space region, the phasor electric field in spherical coordinates is:

$$\mathbf{E} = \mathbf{a}_\phi \frac{e^{-jk_0 R}}{R} \cos\phi$$

where $k_0$ is the free space wavenumber.

(a) Find an exact expression for the corresponding phasor magnetic field, **H**.

(b) Suppose that the **E** given above is the electric field in the far-zone of an antenna. What is **H** in the far-zone?

(c) Find the time-averaged Poynting vector in the far-zone.

(d) What is the total power radiated by the antenna?
[You may use the formula: $\int_{x=0}^{u} \cos^2 x \, dx = \frac{u}{2} + \frac{\sin 2u}{4}$ ]

**END OF QUESTIONS. FORMULAS FOLLOW.**