McGill University
Faculty of Engineering

FINAL EXAM
DECEMBER 2006

VERSION A

COURSE ECSE 353
ELECTROMAGNETIC FIELDS AND WAVES

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Signature: _____________ Signature: _____________

Date: Thursday, December 21, 2006 Time: 14:00

- This is a closed book examination. No books or notes are permitted, except for the Formula Sheet attached.
- The Faculty Standard Calculator (Casio fx-991 or Sharp EL-546L or R or V (VB) or G) only is permitted.
- All units are SI unless otherwise stated.
- Unless otherwise stated: x,y,z are rectangular (Cartesian) coordinates; r,ϕ,z are cylindrical coordinates; and R,θ,ϕ are spherical coordinates.
- This is a 180 minute exam
- The marks indicated in square brackets at the start of each question are out of 180.

INSTRUCTIONS:

- Answer all questions.
- Put your name and student ID also on the Answer Sheet provided.
- **Part A** is multiple choice. There is one correct answer for each question. Mark your answers on the Answer Sheet, not on this examination paper. Only the answers on the Answer Sheet will be considered.
- **Part B**: Put your answers in the spaces provided on the Answer Sheet.

WHEN INSTRUCTED, TURN TO NEXT PAGE AND START THE EXAM

PART A

Part A is multiple choice. There is one correct answer for each question. Mark your final answers on the Answer Sheet. Only the answers on the Answer Sheet will be considered.

1. [5] Which of the following is *not* a true statement about the field in the far zone of any antenna?
   - A the electric and magnetic fields are perpendicular to one another
   - B the magnitude of the electric field is inversely proportional to the distance from the antenna
   - C the radial (*R*) components of the electric and magnetic field are in-phase with each other
   - D the time-averaged Poynting vector only has a radial (*R*) component

2. [5] The *skin depth* or *depth of penetration* of an electromagnetic wave into a conductor is:
   - A the distance beyond which the electric field in the conductor is zero
   - B the distance through which the magnitude of the Poynting vector decreases by a factor of $e^{-1} = 0.368$
   - C the distance through which the amplitude of the electric field of the wave decreases by a factor of $e^{-1} = 0.368$
   - D the distance through which the magnitude of the Joule heating ($\mathbf{E} \cdot \mathbf{J}$) decreases by a factor of $e^{-1} = 0.368$

3. [5] **A** is the phasor magnetic vector potential and *V* is the phasor electric scalar potential. Which of these is the Lorentz condition?
   - A $\nabla \cdot \mathbf{A} + j\omega\varepsilon\mu V = 0$
   - B $-\nabla V - j\omega\mathbf{A} = 0$
   - C $\nabla^2\mathbf{A} + \omega^2\varepsilon\mu\,\mathbf{A} = 0$
   - D $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

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4. [5] A steady (DC) voltage is applied to a parallel-plate capacitor. In the steady state, which of the these statements is true, concerning the displacement current density between the plates?
   - A it is zero when the material between the plates is a perfect insulator
   - B it is zero, whatever the material is between the plates
   - C it is non-zero, but uniform (ignoring fringing at the edge of the plates)
   - D it is non-zero, but non-uniform (even ignoring fringing)

5. [5] What is the unit of surface current density?
   - A A m^{-2}
   - B A m^{-1}
   - C A m
   - D A m^{2}

6. [5] The phasor electric field of a plane wave in a uniform medium is $\mathbf{E} = \mathbf{a}_z e^{-jkx}$, where k is the wavenumber. The intrinsic impedance of the medium is η. Which of these is the corresponding magnetic field?
   - A $\mathbf{H} = \frac{1}{\eta}\mathbf{a}_y e^{-jkx}$
   - B $\mathbf{H} = -\frac{1}{\eta}\mathbf{a}_y e^{-jkx}$
   - C $\mathbf{H} = \eta\mathbf{a}_y e^{-jkx}$
   - D $\mathbf{H} = -\eta\mathbf{a}_y e^{-jkx}$

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PART B

In Part B, put your answers in the spaces provided on the Answer Sheet.

7. [30] Point P has coordinates $(x,y)=(d,0)$ and is in an infinite, uniform medium. Consider two cases:

Case A. The medium has relative permittivity ε_r, and a static point charge αq is located at point $(x,y)=(0,h)$.

Case B. The medium is free space, and there are two static charges present: q located at point $(x,y)=(0,h)$ and βq located at point $(x,y)=(0,–h)$.

(a) Find the electric field at P in Case A. Give the answer in terms of α, q, h, d, ε_r and the distance $r=\sqrt{h^2+d^2}$.

(b) Find the electric field at P in Case B. Give the answer in terms of β, q, h, d and r.

(c) Now suppose that, at point P:
- the x-component of the electric field (**E**) in Case A matches the x-component of the electric field in Case B AND
- the y-component of the electric flux density (**D**) in Case A matches the y-component of the electric flux density in Case B.

Find the values of α and β needed to achieve this. Give the answers in terms of ε_r.

8. [30] A lossless transmission line has a characteristic resistance of 50Ω. The load is a 10Ω resistor. The generator is a 10V battery in series with 100Ω, connected directly to the transmission line (no switch). The line extends from $z=0$ (generator end) to $z=L$ (load end). Then, at $t=0$, a short-circuit fault drives the voltage at $z=0$ to zero and it stays at that value.

(a) Plot the voltage as a function of z along the transmission line, at time $t<0$.

(b) At $t=0$, just after the short-circuit fault happens, a wave with voltage V_1^+ starts out from $z=0$ and propagates towards the load. Give the value of V_1^+.

(c) SEE NEXT PAGE

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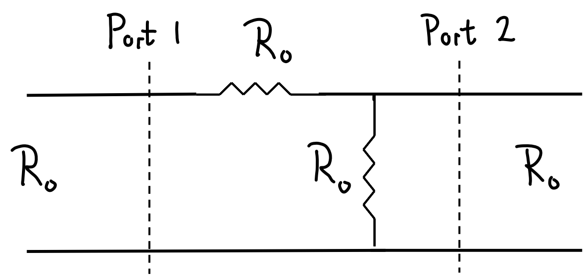
(c) The time it takes for a signal to go from one end of the transmission line to the other is T. Plot the voltage across the load as a function of time from t=0 to t=3.5T. Calculate numerical voltage values and transition times (in terms of T) and mark them on your diagram.

9. [30] A two-port device consists of two resistors, each with a resistance equal to R_0, as shown. Each port of the device is connected to a lossless transmission line with characteristic resistance equal to R_0. Find the scattering matrix of the device.



10. [30] On the curved surface of an infinitely-long, air-filled cylinder, radius a, there is the following steady surface current density:

$$\mathbf{J}_s = J_{s\phi}\mathbf{a}_\phi + J_{sz}\mathbf{a}_z$$

(a) Find the magnetic flux density at a point (r, ϕ, z) inside the cylinder.

(b) Find the magnetic flux density at a point (r, ϕ, z) outside the cylinder.

(c) Show that this solution satisfies the interface conditions across the surface of the cylinder.

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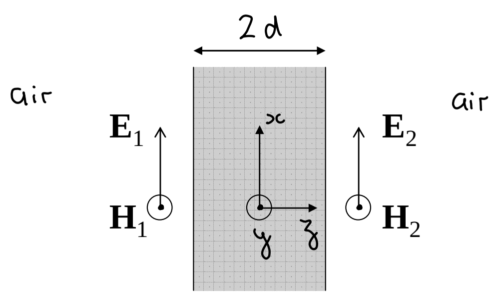
11. [30] An infinite slab of lossless material has thickness $2d$ as shown. The intrinsic impedance of the material is η and the wavenumber in the material is k. In the air just outside the material, on either side, the electric and magnetic field are given by:

$$\mathbf{E}_1 = E_1\,\mathbf{a}_x$$
$$\mathbf{H}_1 = H_1\,\mathbf{a}_y$$
$$\mathbf{E}_2 = E_2\,\mathbf{a}_x$$
$$\mathbf{H}_2 = H_2\,\mathbf{a}_y$$



(a) Assume that inside the slab there are two plane waves, one propagating in the +z direction and one propagating in the –z direction. The electric field is of the form:

$$\mathbf{E} = \mathbf{a}_x\left(E^{+}e^{-jkz} + E^{-}e^{+jkz}\right)$$

Give the expression for the magnetic field, **H**, inside the slab, in terms of E^{+}, E^{-}, k and η.

(b) Find E_2 and H_2 in terms of E_1, H_1, k, d and η only.

END OF QUESTIONS **FORMULAS FOLLOW**