Final exam, 2005

Solutions

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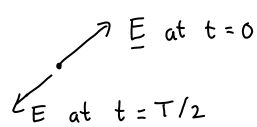
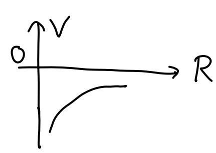
(Version A) 1. D 2. D

3. B



4. B $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ If $q<0$, $V \to -\infty$ as $R \to 0$

5. C



6. D $\Gamma_L = \frac{R_L - R_0}{R_L + R_0}$

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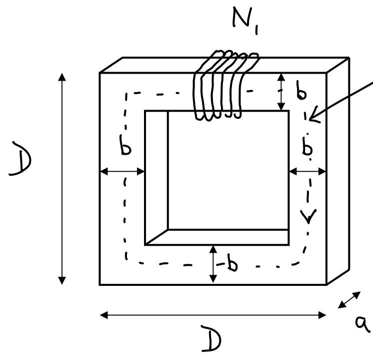
7. (a)



Contour C running through middle of core.

$\underline{H}$ is approximately uniform inside core

Apply Ampère's Law to C ; $\oint_C \underline{H} \cdot d\underline{l} = I_C \Rightarrow H\,L = N_1 I$

where L = length of C $\simeq 4D$ (remember, a and b << D)

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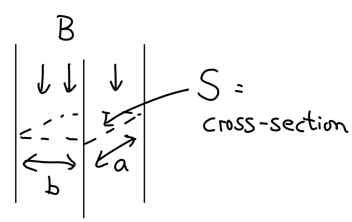
So $H = \dfrac{N_1 I}{4D}$

$\Rightarrow B = \mu \dfrac{N_1 I}{4D}$



$\Rightarrow$ Flux $\Phi = \int_S \underline{B} \cdot d\underline{s} = B\,S = \left(\mu \dfrac{N_1 I}{4D}\right) ab$

(b) Faraday: $EMF = -\dfrac{d\Lambda}{dt}$ For phasors: $EMF = -j\omega\Lambda$

So $EMF = -N_2\, j\omega\Phi = -j\omega\mu N_1 N_2 I\, ab\,/(4D)$

So $|\text{open-circuit voltage}| = \omega\mu \dfrac{N_1 N_2 |I|\, ab}{4D}$

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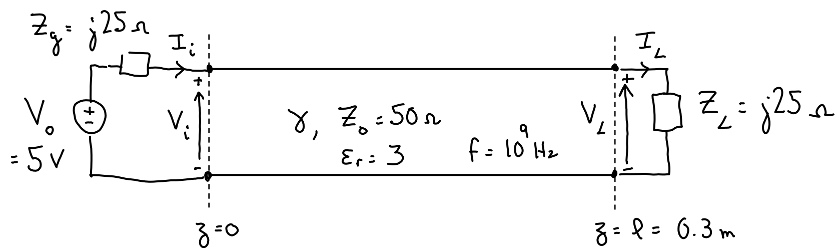

Lossless $\Rightarrow$ $\gamma = j\beta$ $\quad \beta = \omega\sqrt{LC} = \omega\sqrt{\varepsilon\mu} = 2\pi \frac{f}{c}\sqrt{\varepsilon_r} = \frac{20\pi}{3} = 36.28$ rads/m

$\Rightarrow \beta\ell = 2\pi\sqrt{3} = 10.88$ rads

$$Z_i = Z_o\left(\frac{Z_L + Z_o \tanh\gamma\ell}{Z_o + Z_L \tanh\gamma\ell}\right) = 50\left(\frac{j25 + j50\tan 10.88}{50 - 25\tan 10.88}\right)$$

$$= -j137.8\ \Omega$$

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$$V_i = \left(\frac{Z_i}{Z_g + Z_i}\right)V_o = \frac{-j137.8}{j25 - j137.8} = 6.108\ V$$

$$I_i = \frac{V_o}{Z_g + Z_i} = \frac{5}{j25 - j137.8} = j\,44.33\ mA$$

$$V_o^+ = \frac{1}{2}\left(V_i + Z_o I_i\right) = \frac{1}{2}\left(6.108 + j50 \times 44.33\times10^{-3}\right) = 3.054 + j1.108$$

$$V_o^- = \frac{1}{2}\left(V_i - Z_o I_i\right) = \frac{1}{2}\left(6.108 - j50 \times 44.33\times10^{-3}\right) = 3.054 - j1.108$$

$$I(z) = V_o^+/Z_o\, e^{-\gamma z} - V_o^-/Z_o\, e^{+\gamma z}$$

$$\Rightarrow I_L = \frac{V_o^+}{Z_o} e^{-j\beta\ell} - \frac{V_o^-}{Z_o} e^{+j\beta\ell}$$

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$$\Rightarrow \quad I_L = \left(\frac{3.054 + j\,1.108}{50}\right)\left(\cos 10.88 - j \sin 10.88\right)$$

$$- \left(\frac{3.054 - j\,1.108}{50}\right)\left(\cos 10.88 + j \sin 10.88\right)$$

$$= \frac{-2j}{50}\left[\, 3.054 \sin 10.88 \;-\; 1.108 \cos 10.88 \,\right]$$

$$= j\,117 \text{ mA} \qquad 1$$

$$\left(\text{Also } V_L = Z_L I_L = -2.91 \text{ V}\right)$$

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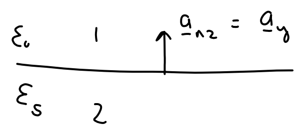
9 (a)



$$\underline{a}_{n2} \cdot \left(\underline{D}_1 - \underline{D}_2\right) = \rho_s$$

$$\Rightarrow \quad \underline{a}_y \cdot \left(\varepsilon_0 \underline{E}_1 - \varepsilon_s \underline{E}_2\right) = \rho_s$$

$$\text{But } \underline{E}_1 = \underline{E}_2 = E_0\left(\underline{a}_x - \underline{a}_y\right)$$

$$\text{So} \quad \left(\varepsilon_0 - \varepsilon_s\right)\left(-E_0\right) = \rho_s$$

$$\Rightarrow \quad \rho_s = \left(\varepsilon_s - \varepsilon_0\right) E_0$$

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(b) $\underline{a}_{n2} \cdot (\underline{J}_1 - \underline{J}_2) = -\dfrac{\partial \rho_s}{\partial t}$

$\Rightarrow \quad \underline{a}_{n2} \cdot (\underline{J}_1 - \underline{J}_2) = -j\omega \rho_s$ in phasor form

Here $\underline{J}_1 = 0$ (no current in air)

and $\underline{a}_{n2} = \underline{a}_y$

So $\quad -J_{2y} = -j\omega\rho_s = -j\omega\,(\varepsilon_s - \varepsilon_0) E_0$

$$\Rightarrow \quad J_{2y} = \underline{j\omega\,(\varepsilon_s - \varepsilon_0) E_0}$$

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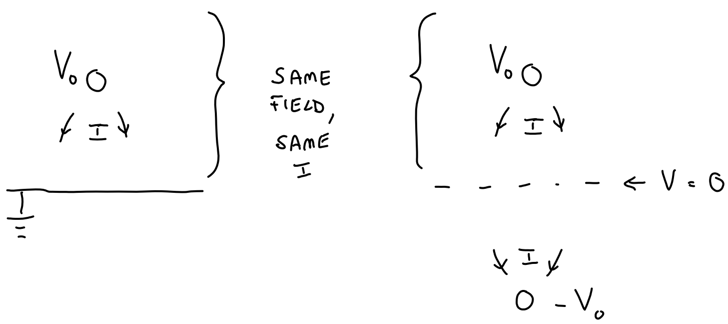
10.



So, we need G' for this two-wire problem

First, get C', then $\quad \dfrac{G'}{C'} = \dfrac{\sigma}{\varepsilon}$

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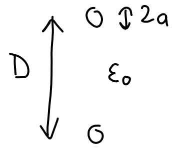

$$C' = \frac{\pi \varepsilon_0}{\cosh^{-1} D/2a}$$ (Formula Sheet)

Hence $G' = \frac{C' \sigma}{\varepsilon_0} = \frac{\pi \sigma}{\cosh^{-1} D/2a}$ = conductance/m between 2 wires

and $I' = (2V_0)\, G' = \frac{2 V_0 \pi \sigma}{\cosh^{-1} D/2a}$ A m^{-1}

$$= \frac{2 \cdot 10^3 \cdot \pi \cdot 4}{\cosh^{-1} (2\times 4)/(2\times 10^{-2})} = \frac{8 \cdot 10^3 \cdot \pi}{\cosh^{-1} 400} = 3.76 \text{ kA}$$

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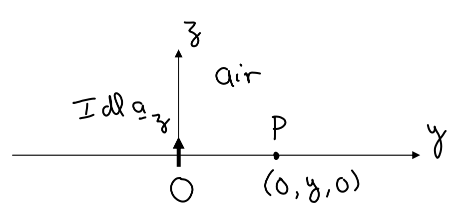

$$\underline{A} = \mu_0 I\, d\underline{l}\, \frac{e^{-jkR}}{4\pi R}$$ (from Formula Sheet)

$$\underline{H} = \frac{1}{\mu_0} \nabla \times \underline{A} = \frac{I}{4\pi} \nabla \times\, d\underline{l}\, \frac{e^{-jkR}}{R} = \frac{I}{4\pi} \left(\nabla \frac{e^{-jkR}}{R} \right) \times d\underline{l}$$

$$= \frac{I}{4\pi} \left(-\frac{1}{R^2} - \frac{jk}{R} \right) e^{-jkR}\, \nabla R \times d\underline{l}$$

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$$= \frac{\underline{I}}{4\pi}\left(-\frac{1}{R^2} - \frac{jk}{R}\right)e^{-jkR}\ \nabla R \times d\underline{l}$$

$$= \frac{\underline{I}}{4\pi}\left(-\frac{1}{R^2} - \frac{jk}{R}\right)e^{-jkR}\ \underline{a}_R \times d\underline{l}$$

At $(0,y,0)$: $\underline{a}_R = \underline{a}_y$ Also, $d\underline{l} = dl\ \underline{a}_z$

$R = y$ and $\underline{a}_y \times \underline{a}_z = -\underline{a}_x$

So $\underline{H} = \frac{-\underline{I}\,dl}{4\pi}\ \underline{a}_x \left(\frac{1}{y^2} + \frac{jk}{y}\right)e^{-jky}$

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