ECSE 353 FINAL EXAM SOLUTIONS FALL 2003

For VERSION A:

1. B
2. D
3. D
4. C
5. D
6. B
7. C
8. C
9. A

10. D

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{10-50}{10+50} = -\frac{40}{60} = -\frac{2}{3} \Rightarrow V^- = -\frac{2}{3}V^+$$

$$V_L = V^+ + V^- = V^+\left(1-\frac{2}{3}\right) = \frac{1}{3}V^+ \quad \text{So } |V^+| = 3|V_L| = 3V_0$$

11. C

$$C_{new} = \frac{2\pi\varepsilon}{\ln(b_{new}/a)} = 2C = 2 \cdot \frac{2\pi\varepsilon}{\ln(b/a)}$$

$$\Rightarrow \ln\frac{b_{new}}{a} = \frac{1}{2}\ln\frac{b}{a} = \ln\sqrt{\frac{b}{a}} \Rightarrow \frac{b_{new}}{a} = \sqrt{\frac{b}{a}} \Rightarrow b_{new} = \sqrt{ab}$$

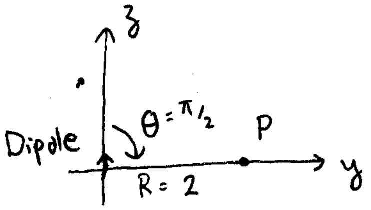
12. A

In $y<0$: $H_x = H_{x0}$, so $B_x = \mu_0\mu_r H_{x0}$

$B_y = \mu_0 H_{y0}$, so $H_y = \frac{1}{\mu_r}H_{y0}$

Then
$$\underline{M} = \frac{1}{\mu_0}\underline{B} - \underline{H} = \left(\mu_r H_{x0}\,\underline{a}_x + H_{y0}\,\underline{a}_y\right) - \left(H_{x0}\,\underline{a}_x + \frac{1}{\mu_r}H_{y0}\,\underline{a}_y\right)$$
$$= (\mu_r - 1)\left(H_{x0}\,\underline{a}_x + \frac{1}{\mu_r}H_{y0}\,\underline{a}_y\right)$$

13. A



$V = \dfrac{p\cos\theta}{4\pi\varepsilon_0 R^2}$ $\quad \underline{E} = -\nabla V$

At P, $E_z = -E_\theta$

$$= +\frac{1}{R}\frac{\partial V}{\partial \theta} = -\frac{p\sin\theta}{4\pi\varepsilon_0 R^3} = \frac{-p}{32\pi\varepsilon_0} \quad //$$

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14. C

$\eta_1 = \eta_0$ (air) | $\eta_2 = \sqrt{\dfrac{\mu_0}{\varepsilon_r\varepsilon_0}} \leq \eta_0$

$\Gamma \rightarrow$ | $\leftarrow$ Interface

$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, real and ≤ 0

But $S = \dfrac{1+|\Gamma|}{1-|\Gamma|} \Rightarrow (1-|\Gamma|)S = 1+|\Gamma|$

$$\Rightarrow |\Gamma| = \frac{S-1}{S+1} = \frac{4-1}{4+1} = \frac{3}{5}$$

So $\Gamma = -3/5 = -0.6 \quad //$

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15. C

Steady currents $\Rightarrow \oint_S \underline{J}\cdot d\underline{s} = 0$. But current through bottom $(z=0) = 0$

So current out through sides = current <u>in</u> through top $(z=h)$

$$= \int_{Top} \underline{J}\cdot(-\underline{a}_z\, ds) = -\int_{\varphi=0}^{2\pi}\int_{r=0}^{a} J_z\, r\, d\varphi\, dr$$

$$= -2\pi h\int_{r=0}^{a} r^2\, dr = -2\pi h a^3/3 \quad //$$

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16. C

$\mathcal{P}_{av} = \dfrac{U}{R^2}$; $U = G_D U_{iso}$; $U_{iso} = \dfrac{P_r}{4\pi}$

Combining: $\mathcal{P}_{av} = \dfrac{G_D P_r}{4\pi R^2} = \dfrac{10^3 \cdot 1}{4\pi (10^2)^2} = 7.96\ \text{mW}/\text{m}^2 \quad //$

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17. B

In solenoid: $B_z = \mu_0 n I(t)$. So flux linking loop is $\Phi = B_z(\pi a^2)$

$$= \mu_0 n I(t)\pi a^2$$

Faraday: $|EMF| = \left|\dfrac{d\Phi}{dt}\right| = \left|\mu_0 n\pi a^2 \dfrac{dI}{dt}\right| = (4\pi)(10^{-7})(10^3)(\pi)(10^{-4})\dfrac{(10^{-3})}{(10^{-6})}$

$$= 0.395\ \text{mV} \quad //$$

18. D $S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$, so $S = \infty \Leftrightarrow |\Gamma_L| = 1$

But $|\Gamma_L| = \left|\frac{Z_L - R_0}{Z_L + R_0}\right| = \sqrt{\frac{(R_L-R_0)^2 + X_L^2}{(R_L+R_0)^2 + X_L^2}}$

For this to equal 1, R_L must be 0, or $Z_L = 0$, or $Z_L = \infty$.

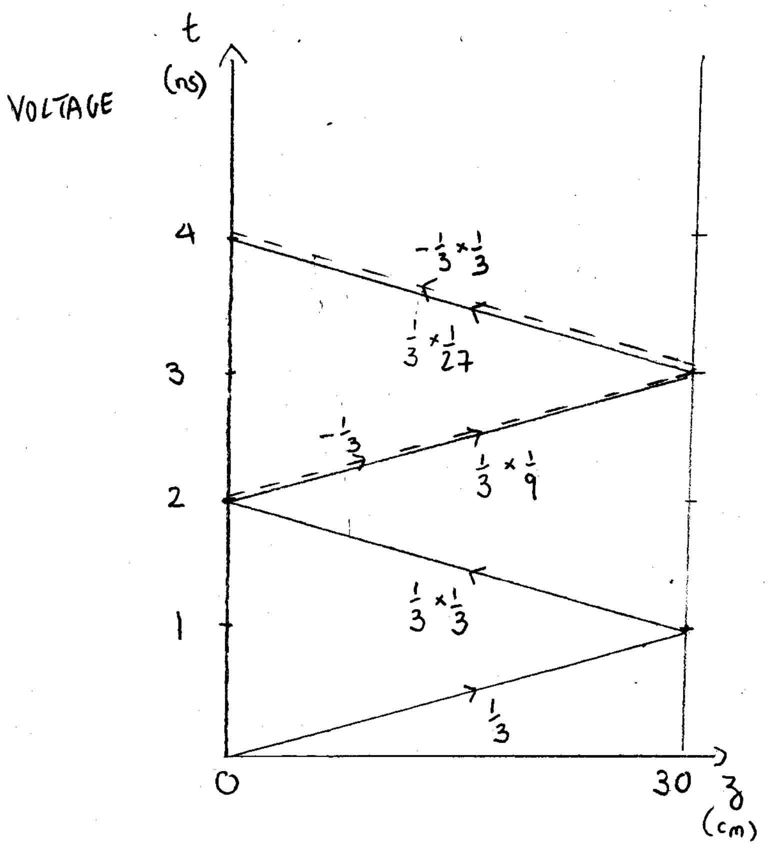
(Physically; $|\Gamma_L| = 1$ means load is lossless, i.e. S/C, O/C or reactive).

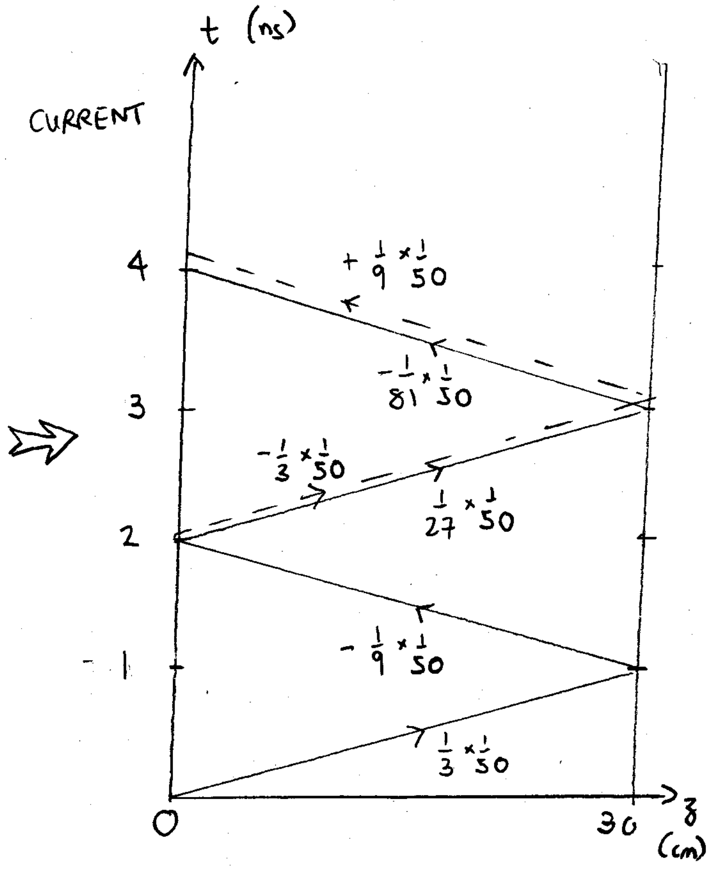
19.

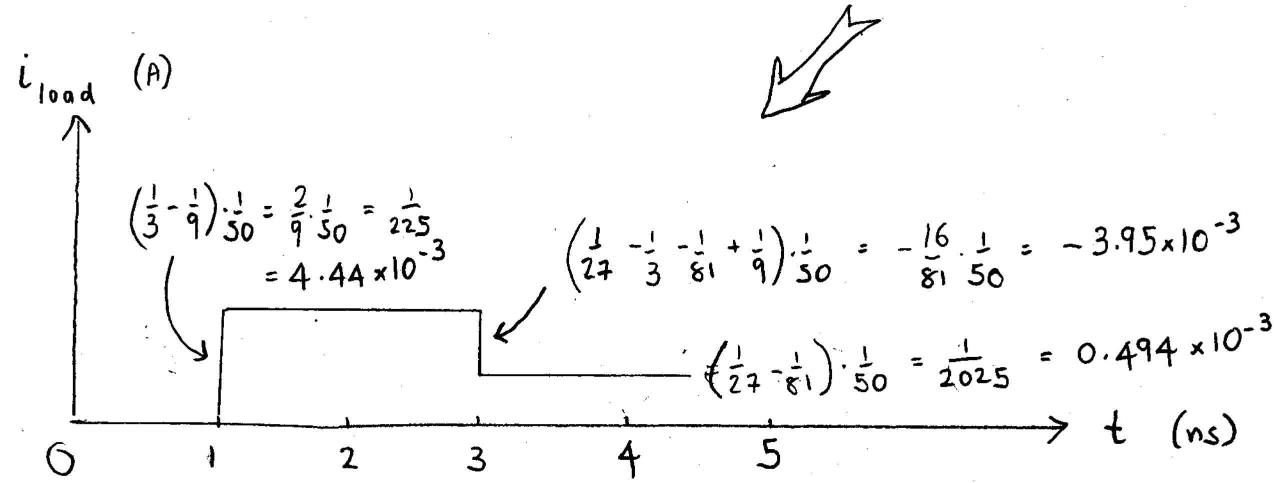
$$T = \frac{0.3}{3\times10^{8}} = 10^{-9} = 1\,\text{ns} \qquad T_0 = 2\,\text{ns}$$

$$\Gamma_g = \Gamma_L = \frac{100-50}{100+50} = \frac{50}{150} = \frac{1}{3}$$

$$V_1^+ = V_0\,\frac{50}{100+50} = \frac{1}{3}$$







[Alternative method: Use $i_{load} = V_{load}/100$ and get V_{load} from VOLTAGE diagram directly]

20.

(a)

$$I = \int_{\substack{\text{Wire}\\\text{cross-section}}} \underline{J}\cdot d\underline{s} = \int_{\varphi=0}^{2\pi}\int_{r=0}^{a} J_z \, r\,d\varphi\,dr$$

$$= 2\pi J_0 \int_{r=0}^{a} r\,e^{(r-a)/\delta}\,dr = 2\pi J_0 \Big[\delta\, e^{(r-a)/\delta}(r-\delta)\Big]_0^a$$

$$= 2\pi J_0\,\delta\left[(a-\delta) + \delta e^{-a/\delta}\right]$$

(b)

$$P = \int_{\substack{\text{Unit}\\\text{length}}} \underline{J}\cdot\underline{E}\,dv = 1.\int_{\varphi=0}^{2\pi}\int_{r=0}^{a} \frac{1}{\sigma} J_z^2 \, r\,d\varphi\,dr$$

$$= \frac{2\pi J_0^2}{\sigma}\int_{r=0}^{a} r\,e^{2(r-a)/\delta}\,dr = \frac{2\pi J_0^2}{\sigma}\Big[\frac{\delta}{2} e^{2(r-a)/\delta}\left(r-\frac{\delta}{2}\right)\Big]_0^a$$

$$= \frac{2\pi J_0^2}{\sigma}\,\frac{\delta}{2}\left[\left(a-\frac{\delta}{2}\right) + \frac{\delta}{2} e^{-2a/\delta}\right]$$

(c)

$$R_{eff} = \frac{P}{I^2}$$

If $\delta \ll a$: $P \simeq \frac{2\pi J_0^2 \delta a}{2\sigma}$

$I \simeq 2\pi J_0 \delta a$

So $R_{eff} \simeq \frac{2\pi J_0^2 \delta}{2\sigma}\,\frac{1}{(2\pi)^2 J_0^2 \delta^2 a^2} = \frac{1}{4\pi\sigma\delta a}$ //

[For <u>uniform current in skin δ</u>, $R = \frac{1}{\sigma S} = \frac{1}{\sigma\, 2\pi a\delta}$ Ω/m]

21. (a) $\underline{E}_1 = (\underline{a}_x - j\underline{a}_y)e^{-j\beta_1 z}$ $p + jq = -j$, prop. in $+z$ dirⁿ

$\Rightarrow$ RIGHTHAND CIRCULARLY POLARIZED //

$\underline{E}_2 = (\underline{a}_x + j\underline{a}_y)e^{-j\beta_2 z}$ $p + jq = +j$, prop. in $+z$ dirⁿ

$\Rightarrow$ LEFTHAND CIRCULARLY POLARIZED //

(b) At $z = 0$, $\underline{E} = c_1(\underline{a}_x - j\underline{a}_y) + c_2(\underline{a}_x + j\underline{a}_y)$

$= (c_1 + c_2)\underline{a}_x - j(c_1 - c_2)\underline{a}_y$

If this is lin. pol. in x-dirⁿ, $c_1 - c_2 = 0$, i.e. $c_1 = c_2$

Then $\underline{E} = 2c_1\underline{a}_x$

If $|\underline{E}| = 1$ V/m, $2|c_1| = 1 \Rightarrow$ $c_1 = c_2 = \frac{1}{2}e^{j\theta}$ V/m, any real θ

(c) $\underline{E} = \frac{e^{j\theta}}{2}(\underline{a}_x - j\underline{a}_y)e^{-j\beta_1 z} + \frac{e^{j\theta}}{2}(\underline{a}_x + j\underline{a}_y)e^{-j\beta_2 z}$

$= \frac{e^{j\theta}}{2}\underline{a}_x\left(e^{-j\beta_1 z} + e^{-j\beta_2 z}\right) - \frac{e^{j\theta}}{2}j\underline{a}_y\left(e^{-j\beta_1 z} - e^{-j\beta_2 z}\right)$

For $E_x = 0$, $e^{-j\beta_1 z} + e^{-j\beta_2 z} = 0$

$\Rightarrow e^{-j\beta_1 z} = -e^{-j\beta_2 z}$

$= e^{j(\pi - \beta_2 z)}$

$\Rightarrow -\beta_1 z = \pi - \beta_2 z + 2n\pi$ $n = 0, \pm 1, \pm 2, \ldots$

$\Rightarrow (\beta_1 - \beta_2)z = -\pi + 2n\pi$ $n = 0, \pm 1, \pm 2, \ldots$

Since $\beta_1 - \beta_2 > 0$, smallest positive soln. is for $n = -1$

Then $z = \frac{\pi}{\beta_1 - \beta_2} = \frac{\pi}{2.1 - 2.0} = 31.4$ m