McGill University
Faculty of Engineering

FINAL EXAM
DECEMBER 2003

VERSION A

COURSE ECSE 353
ELECTROMAGNETIC FIELDS AND WAVES

Examiner:	J. P. Webb	Co-Examiner:	S. McFee
Signature:	_____________	Signature:	_____________
Date:	Friday, December 12, 2003	Time:	09:00

- This is a closed book examination. No books or notes are permitted, except for the Formula Sheet attached.
- The Faculty Standard Calculator (Casio fx-991 or Sharp EL-546L or R or V (VB) or G) only is permitted.
- All units are SI unless otherwise stated
- This is a 180 minute exam
- The marks indicated in square brackets at the start of each question are out of 180.

INSTRUCTIONS:

- Answer all questions.
- Put your name and student ID also on the Answer Sheet provided.
- **Part A** is multiple choice. There is one correct answer for each question. Mark your answers on the Answer Sheet, not on this examination paper. Only the answers on the Answer Sheet will be considered.
- **Part B**: Put your answers in the spaces provided on the Answer Sheet.

WHEN INSTRUCTED, TURN TO NEXT PAGE AND START THE EXAM

PART A

Part A is multiple choice. There is one correct answer for each question. Mark your final answers on the Answer Sheet. Only the answers on the Answer Sheet will be considered.

1. [4] Only one of the following quantities is *conserved*, i.e., the integral of it over a closed surface is always zero, under all circumstances (not just for the static case). Which is it?
   A electric flux density
   B magnetic flux density
   C conduction current density
   D displacement current density

2. [4] The far-zone electric field of an elemental electric dipole is:
   A isotropic
   B strongest along the axis (i.e., the line through the dipole parallel to the current)
   C strongest in directions that are at an angle of 45° to the axis
   D strongest in the plane perpendicular to the axis

3. [4] A material has conductivity σ and permittivity ε. For wave propagation at angular frequency ω, the material is said to be a *good conductor* when:
   A σ is very large
   B $\sigma >> \varepsilon$
   C $\sigma >> \omega/\varepsilon$
   D $\sigma >> \omega\varepsilon$

4. [4] In the air just outside a piece of dielectric, the static electric field:
   A is always tangential to the air-dielectric interface
   B is always perpendicular to the air-dielectric interface
   C can be in any direction
   D is always zero

5. [4] Inside a battery, there is an impressed electric field $\mathbf{E}_i$ due to chemical processes. If the battery is perfectly short-circuited (i.e., a perfect conductor is connected from one terminal to the other), what can you say about the electric field $\mathbf{E}$ inside the battery in the steady state?
   A it is exactly equal and opposite to $\mathbf{E}_i$
   B it has approximately the same magnitude as $\mathbf{E}_i$, and is in the same direction
   C it has approximately the same magnitude as $\mathbf{E}_i$, but is in the opposite direction
   D its path integral from one terminal to the other, inside the battery, is zero

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6. [4] What is the unit of magnetic flux density?
   A A / m^2
   B T
   C A / m
   D Wb

7. [4] The *characteristic impedance* of a cable is:
   A the impedance measured at one end of the cable when nothing is connected to the other end (i.e., it is open-circuit)
   B the impedance measured at one end of the cable when a short-circuit load is placed across the other end
   C the ratio of voltage to current for a wave propagating in one direction
   D the ratio of electric to magnetic field for a wave propagating in one direction

8. [4] A coaxial cable with characteristic impedance Z_o connects a sinusoidal voltage V_o to a load impedance Z_L. Half way along the cable, the phasor voltage of the inner conductor relative to the outer conductor is V; the phasor current in the inner conductor, flowing towards the load, is I. In general, the ratio V/I is:
   A equal to Z_o
   B equal to Z_L
   C dependent on both Z_o and Z_L, but not on V_o
   D dependent on both Z_o, Z_L and V_o

9. [4] A thin sheet of steady current flows uniformly from the bottom of the page to the top of the page. What is the direction of the magnetic field that it produces, above the page?
   A left to right
   B right to left
   C top to bottom
   D bottom to top

10. [6] A 10Ω load terminates a lossless transmission line with characteristic impedance 50Ω. The amplitude of the sinusoidal voltage across the load is V_o. Find the amplitude of the voltage of the wave incident on the load.
   A $\frac{3}{5}V_o$
   B V_o
   C $2V_o$
   D $3V_o$

11. [6] A coaxial cable has inner radius a and outer radius b. What value would you set the outer radius to, in order to double the capacitance per unit length, leaving a unchanged?

A $b/2$

B $b/\sqrt{2}$

C $\sqrt{ab}$

D $\sqrt{b/a}$

12. [6] The half space $y<0$ is filled with a uniform, linear, magnetic material with relative permeability μ_r. The half space $y>0$ is filled with air. There is no surface current flowing on the interface. If the magnetic field at the interface $y=0$, just on the air side, is $\mathbf{H} = H_{xo}\mathbf{a}_x + H_{yo}\mathbf{a}_y$, find the magnetization in the magnetic material, at the interface.

A $(\mu_r - 1)\left(H_{xo}\mathbf{a}_x + \frac{1}{\mu_r}H_{yo}\mathbf{a}_y\right)$

B $(\mu_r - 1)\left(\frac{1}{\mu_r}H_{xo}\mathbf{a}_x + H_{yo}\mathbf{a}_y\right)$

C $(\mu_r - 1)\left(H_{xo}\mathbf{a}_x + H_{yo}\mathbf{a}_y\right)$

D $\left(\frac{\mu_r - 1}{\mu_r}\right)\left(H_{xo}\mathbf{a}_x + H_{yo}\mathbf{a}_y\right)$

13. [6] An electrostatic dipole $\mathbf{p} = p\mathbf{a}_z$ is located at the origin of a rectangular coordinate system (x,y,z), in an infinite expanse of air. Find the z-component of the electric field due to the dipole, at the point $(x,y,z) = (0,2,0)$.

A $-\frac{p}{32\pi\varepsilon_0}$

B $+\frac{p}{32\pi\varepsilon_0}$

C $-\frac{p}{16\pi\varepsilon_0}$

D $+\frac{p}{16\pi\varepsilon_0}$

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14. [6] A linearly polarized, plane wave, propagating in air, is normally incident on an infinite, plane interface. On the other side of the interface is a half-space filled with a uniform, non-magnetic, lossless, dielectric material. The electric field in the air forms a standing wave pattern and the maximum amplitude is 4 times greater than the minimum amplitude. Find the reflection coefficient at the interface.
   - A –0.25
   - B +0.25
   - C –0.60
   - D +0.60

15. [6] A cylinder is of length h and has circular cross-section, radius a. The origin of a cylindrical coordinate system (r,ϕ,z) is at the center of one end of the cylinder and the z-axis is parallel to the axis of the cylinder. If the z component of the steady current density within the cylinder is $J_z = rz$, find the total current passing *outwards* through the curved surface of the cylinder.
   - A $-\pi h a^2$
   - B $+\pi h a^2$
   - C $-\frac{2}{3}\pi h a^3$
   - D $+\frac{2}{3}\pi h a^3$

16. [6] An antenna radiates a total of 1W. Its directional gain in a certain direction is 30dB. Find the magnitude of the time-averaged Poynting vector in this direction, at a distance of 100m from the antenna.
   - A $23.9\ \mu\text{W/m}^2$
   - B $0.239\ \text{mW/m}^2$
   - C $7.96\ \text{mW/m}^2$
   - D $100\ \text{mW/m}^2$

17. [6] A long, air-filled, solenoid has 1000 turns of fine wire per meter. Placed inside it, and coaxial with it, is a single circular loop of wire, radius 1 cm. The current in the solenoid is ramped up linearly from zero to 1 mA over a period of 1 μs. Find the magnitude of the EMF induced in the circular loop of wire while this is happening.
   - A 0.126 mV
   - B 0.395 mV
   - C 1.26 V
   - D 3.95 V

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18. [6] If the voltage standing wave ratio on a lossless transmission line is infinite, what can you say about the load impedance?

A it is zero
B it is zero or infinite
C it is zero, infinite, or equal to the characteristic impedance
D it is zero, infinite, or purely imaginary

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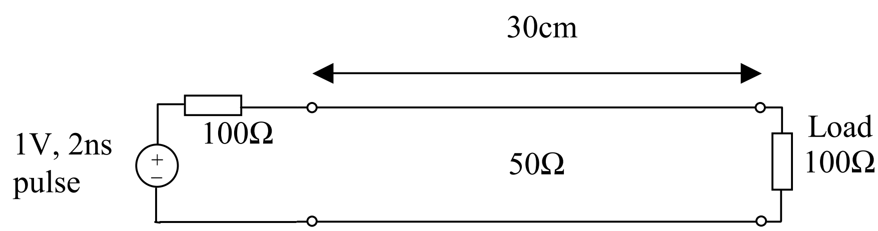
PART B

In Part B, put your answers in the spaces provided on the Answer Sheet.

19. [30] An air-filled, lossless coaxial cable has a characteristic resistance of 50Ω and length 30cm. The load is a 100Ω resistor. The source is a pulse generator, represented as shown by an ideal voltage source in series with a 100Ω resistor. The ideal voltage source produces a single voltage pulse, magnitude 1V, starting at time t=0 and lasting 2 ns.

    Plot the current through the load as a function of time from t=0 to t=4 ns. Calculate numerical current values and transition times and mark them on your diagram.



20. [30] A circular wire of radius a carries an AC current. The conductivity of the wire is σ. At one instant, the current density in the wire is:

$$J_o e^{(r-a)/\delta} \mathbf{a}_z$$

in cylindrical coordinates (r,ϕ,z) centered on the axis of the wire, where δ is the skin depth, a known quantity.

(a) Find I, the total current flow along the wire at this instant.
(b) Find P, the Ohmic power dissipation per unit length of the wire, at the same instant.
(c) From (a) and (b), obtain an approximation for the effective resistance, P/I^2, at the same instant, assuming that δ is much smaller than a and neglecting terms in δ/a, $e^{-a/\delta}$.

[You may find the following formula useful: $\int re^{(r-a)/c} dr = ce^{(r-a)/c}(r-c)$]

21. [30] In a certain medium, two plane waves can propagate at the same frequency, in the $+z$ direction, with different phase constants. The phasor electric fields of the two waves are:

$$\mathbf{E}_1 = \mathbf{a}_x \exp(-j\beta_1 z) - j\mathbf{a}_y \exp(-j\beta_1 z)$$
$$\mathbf{E}_2 = \mathbf{a}_x \exp(-j\beta_2 z) + j\mathbf{a}_y \exp(-j\beta_2 z)$$

where β_1=2.1 rads/m and β_2=2.0 rads/m.

(a) Describe the polarization of each of these two waves.

(b) Now suppose that the total electric field in the medium is a combination of these waves, i.e.,

$$\mathbf{E} = c_1\mathbf{E}_1 + c_2\mathbf{E}_2$$

If the total electric field at z=0 is linearly polarized in the x-direction and has a magnitude 1 V/m, find the coefficients c_1 and c_2.

(c) How far (in meters) does the wave in (b) have to propagate before the total electric field is linearly polarized in the y-direction?

END OF QUESTIONS **FORMULAS FOLLOW**