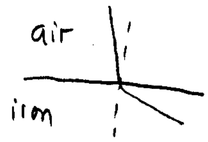
VERSION A

1. D
2. A
3. D ($\underline{E} \times \underline{H}$ in propagation direction)
4. C ($\underline{p} = q\,\underline{d}$)
5. D
6. C
7. B
8. D
9. B

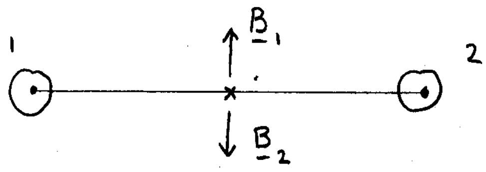


10. C

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 + j100 - 50}{50 + j100 + 50} = \frac{j}{1+j} \Rightarrow |\Gamma_L| = \frac{1}{\sqrt{2}}$$

$$SWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 1/\sqrt{2}}{1 - 1/\sqrt{2}} = \frac{\sqrt{2}+1}{\sqrt{2}-1} = (\sqrt{2}+1)^2 = 3 + 2\sqrt{2}$$

11. A



$$\underline{B}_{TOTAL} = \underline{B}_1 + \underline{B}_2 = 0$$

12. A

L_{int} for a single wire is $\frac{\mu_0}{8\pi}$ Hm^{-1} (e.g. from coax formula)

So for loop, radius b: $L_{int} = \frac{2\pi b \mu_0}{8\pi} = \frac{b\mu_0}{4} = \underline{2\pi \text{ nH}}$

13. B

Potential is same as point charge, ie $\frac{Q}{4\pi\varepsilon_0 R}$, for $R \geq a$

So potential of sphere, $V_0 = \frac{Q}{4\pi\varepsilon_0 a} \Rightarrow Q = 4\pi\varepsilon_0 a V$

14. B $|E|$ is 0 at the surface and maximum $\lambda/4$ away.

ie. $\frac{\lambda}{4} = 0.1 \Rightarrow \lambda = 0.4\text{ m} \Rightarrow f = \frac{c}{\lambda} = \frac{3\times10^8}{0.4} = 750\text{ MHz}$

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15 A

$$\rho = \nabla\cdot \underline{D} = \nabla\cdot\varepsilon\underline{E} = \varepsilon_0\nabla\cdot\underline{E} = \varepsilon_0\nabla\cdot\frac{1}{\sigma}\underline{J} = \varepsilon_0\nabla\cdot\left[\frac{J_0\,a_x}{\sigma_0(1+x)}\right]$$

$$= \frac{\varepsilon_0 J_0}{\sigma_0}\frac{\partial}{\partial x}\left(\frac{1}{1+x}\right) = -\frac{\varepsilon_0 J_0}{\sigma_0}\frac{1}{(1+x)^2}$$

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16 D

$$\underline{H} = \frac{1}{\mu_0}\nabla\times\underline{A} = \frac{1}{\mu_0}\nabla\times\left[\underline{a}_R A_R(R,\theta)\right] = -\frac{1}{\mu_0}\underline{a}_\varphi\frac{1}{R}\frac{\partial A_R}{\partial\theta}$$

$$\underline{E} = \frac{1}{j\omega\varepsilon_0}\nabla\times\underline{H} = -\frac{1}{j\omega\varepsilon_0\mu_0}\nabla\times\left[\underline{a}_\varphi\frac{1}{R}\frac{\partial A_R}{\partial\theta}\right]$$

$$\Rightarrow\quad E_R = -\frac{1}{j\omega\varepsilon_0\mu_0}\frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A_R}{\partial\theta}\right)$$

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17 A

$$\text{EMF} = -j\omega\Phi\ ;\quad I = \frac{\text{EMF}}{R} = \frac{-j\omega\Phi}{R} \Rightarrow |I| = \frac{\omega|\Phi|}{R}$$

$$\Rightarrow |\Phi| = \frac{R|I|}{\omega} = \frac{(100)(10^{-3})}{(2\pi)(60)} = 0.265\text{ mWb}$$

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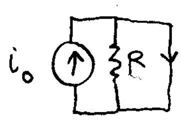
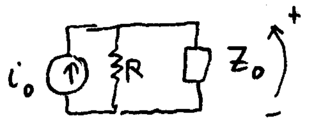
18. C



$i = 1\text{mA}$

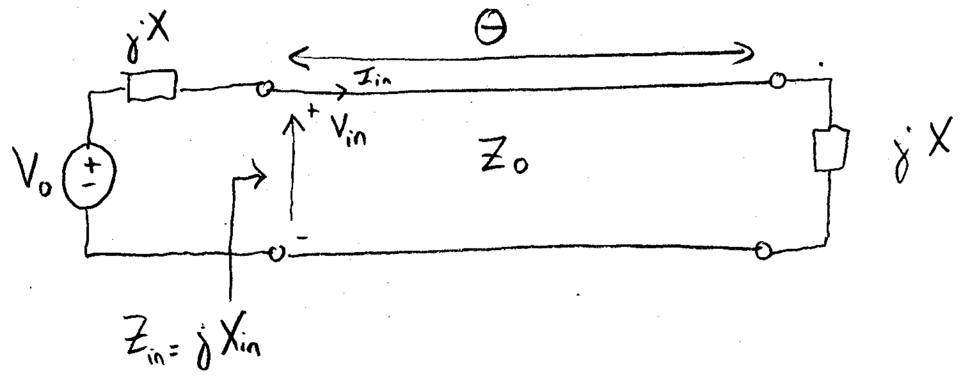
So $i_o = 1\text{mA}$. Then



$$\Rightarrow V_1^+\left(\frac{1}{Z_0}+\frac{1}{R}\right) = i_o \Rightarrow V_1^+ = \frac{Z_0 R}{Z_0+R}\,i_o$$

$$= \frac{(50)(100)}{(50+100)}(10^{-3}) = 33.3\text{ mV}$$

19.



(a) $$Z_{in} = Z_0 \left(\frac{jX + Z_0 j\tan\theta}{Z_0 + jX\, j\tan\theta} \right) = jZ_0 \left(\frac{X + Z_0 \tan\theta}{Z_0 - X\tan\theta} \right)$$

$$\Rightarrow \quad X_{in} = \underline{\underline{Z_0 \left(\frac{X + Z_0 \tan\theta}{Z_0 - X\tan\theta} \right)}}$$

(b) $$V_{in} = \frac{jX_{in}}{jX + jX_{in}} V_0 = \frac{X_{in}}{X + X_{in}} V_0 \quad ; \quad I_{in} = \frac{V_{in}}{jX_{in}} = -j\frac{V_{in}}{X_{in}}$$

$$V_0^+ = \frac{1}{2}\left(V_{in} + Z_0 I_{in}\right) = \frac{V_{in}}{2}\left(1 - j\frac{Z_0}{X_{in}}\right)$$

$$V_0^- = \frac{1}{2}\left(V_{in} - Z_0 I_{in}\right) = \frac{V_{in}}{2}\left(1 + j\frac{Z_0}{X_{in}}\right)$$

$$V_L = V_0^+ e^{-j\theta} + V_0^- e^{+j\theta}$$

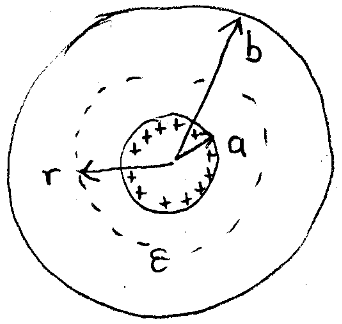
$$= \frac{V_{in}}{2}\left[\left(1 - j\frac{Z_0}{X_{in}}\right)\left(\cos\theta - j\sin\theta\right) + \left(1 + j\frac{Z_0}{X_{in}}\right)\left(\cos\theta + j\sin\theta\right) \right]$$

$$= \frac{V_{in}}{2}\left[2\cos\theta - 2\frac{Z_0}{X_{in}}\sin\theta \right].$$

$$\Rightarrow \quad V_L = V_{in}\left(\cos\theta - \frac{Z_0}{X_{in}}\sin\theta\right) = V_0 \frac{X_{in}}{X + X_{in}}\left(\frac{X_{in}\cos\theta - Z_0\sin\theta}{X_{in}}\right)$$

$$= \underline{\underline{V_0 \left(\frac{X_{in}\cos\theta - Z_0 \sin\theta}{X + X_{in}} \right)}}$$

20.



Cylindrical coords: r, φ, z

Assume charge ρ_ℓ (C/m) on inner conductor

Gauss: $D_r\, 2\pi r = \rho_\ell$

$\Rightarrow \quad D_r = \rho_\ell / 2\pi r \quad \Rightarrow \quad E_r = D_r/\varepsilon = \dfrac{\rho_\ell}{2\pi r(\varepsilon_0 + \varepsilon_1 r)}$

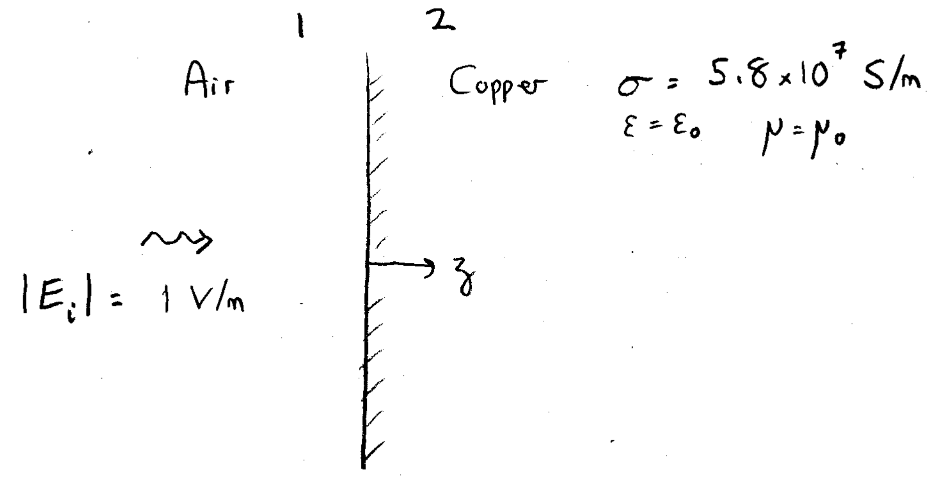
$$V = -\int_{-}^{+} \underline{E} \cdot d\underline{\ell} = -\int_{r=b}^{a} E_r\, dr = -\frac{\rho_\ell}{2\pi} \int_b^a \frac{dr}{r(\varepsilon_0 + \varepsilon_1 r)}$$

$$= -\frac{\rho_\ell}{2\pi} \int_b^a \left[\frac{1}{\varepsilon_0 r} - \frac{\varepsilon_1}{\varepsilon_0(\varepsilon_0 + \varepsilon_1 r)} \right] dr$$

$$= -\frac{\rho_\ell}{2\pi\varepsilon_0} \left[\ln r - \frac{\varepsilon_1}{\varepsilon_1} \ln(\varepsilon_0 + \varepsilon_1 r) \right]_b^a = -\frac{\rho_\ell}{2\pi\varepsilon_0} \ln\left(\frac{a}{b} \cdot \frac{\varepsilon_0 + \varepsilon_1 b}{\varepsilon_0 + \varepsilon_1 a} \right)$$

So $$C' = \frac{\rho_\ell}{V} = \underline{\underline{\frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a} \cdot \frac{\varepsilon_0 + \varepsilon_1 a}{\varepsilon_0 + \varepsilon_1 b} \right)}}} \quad (\text{F/m})$$

21.



Transmission coeff $\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}$

In copper: $\varepsilon = \varepsilon_0 = \dfrac{1}{36\pi}\times10^{-9}$ F/m

$\dfrac{\sigma}{\omega} = \dfrac{5.8\times10^{7}}{2\pi\times10^{3}} = 9.2\times10^{3} >> \varepsilon$ • ie. GOOD CONDUCTOR

So $\eta_2 \approx (1+j)\dfrac{\alpha}{\sigma} = (1+j)\dfrac{1}{\delta\sigma}$ where $\delta = \dfrac{1}{\sqrt{\pi f \mu_0 \sigma}} = 2.0898$ mm

So $\eta_2 \approx \dfrac{(1+j)}{2.0898\times10^{-3}\times 5.8\times10^{7}} = (1+j)\times 8.250\times10^{-6}\ \Omega$

and $\tau = \dfrac{2\eta_2}{\eta_2+\eta_1} \simeq \dfrac{2\eta_2}{\eta_0} = \dfrac{2\times(1+j)\times 8.250\times10^{-6}}{377} = (1+j)\times 4.377\times10^{-8}$

So $|E_t| = |\tau|\,|E_i|$

$= \sqrt{2}\times 4.377\times10^{-8} = 6.190\times10^{-8}$ V/m

After 1mm, amplitude is $6.190\times10^{-8}\times e^{-10^{-3}/\delta} = 6.190\times10^{-8}\times 0.6197$

$= \underline{\underline{3.84\times10^{-8}\ \text{V/m}}}$