Electrostatics, section 05

Dielectrics

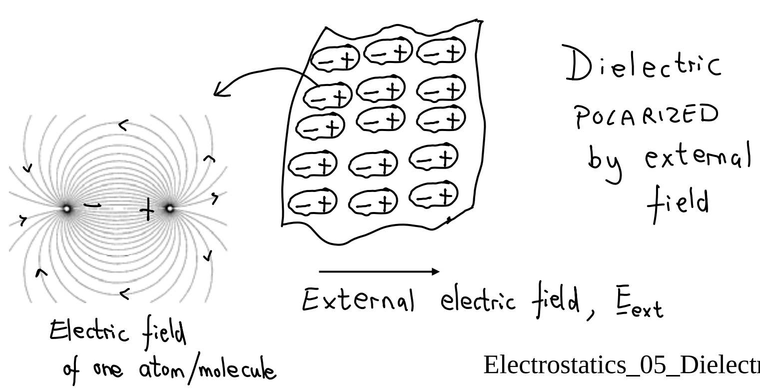
Electric flux density

Electrostatics_05_Dielectrics: 1

DIELECTRIC: Material with no movable (free) charge
— an insulator

Unless charge is added: $\rho = 0$, $\rho_s = 0$

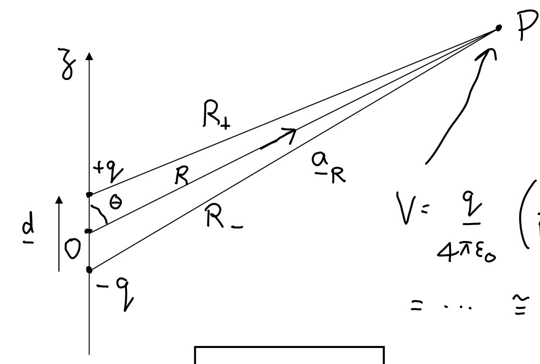
BUT there is BOUND charge:



Electrostatics_05_Dielectrics: 2

Consider $+q$ and $-q$ with separation $d \ll R$.

This is known as a DIPOLE, with DIPOLE MOMENT $\boxed{\underline{p} = q\,\underline{d}}$ (C m)



$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_+} - \frac{1}{R_-}\right)$$

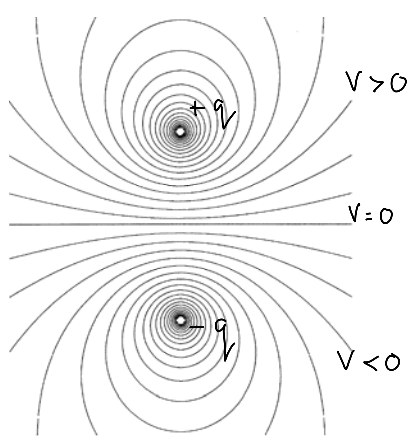
$$= \cdots \cong \frac{q\,d\cos\theta}{4\pi\varepsilon_0 R^2}$$

or $$\boxed{V = \frac{\underline{p}\cdot\underline{a}_R}{4\pi\varepsilon_0 R^2}}$$ POTENTIAL OF DIPOLE $\underline{p}$

Electrostatics_05_Dielectrics: 3

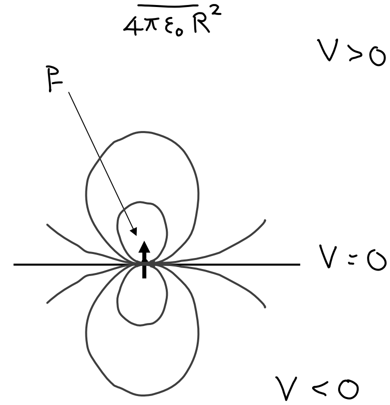
Pair of equal and opposite charges, +q and -q:

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R_+} - \frac{q}{R_-}\right)$$

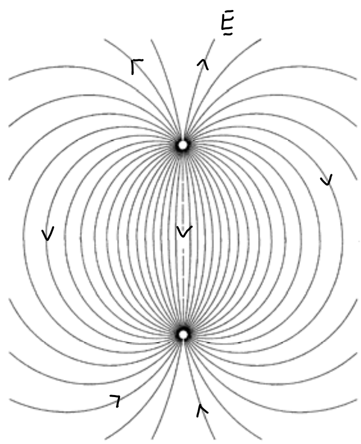


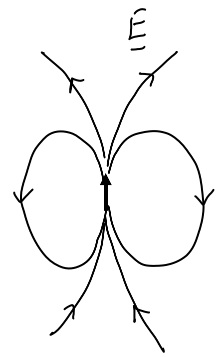
Dipole, $\underline{p}$:

$$V = \frac{\underline{p}\cdot\underline{a}_R}{4\pi\varepsilon_0 R^2}$$

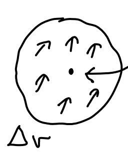


Electrostatics_05_Dielectrics: 4





Electrostatics_05_Dielectrics: 5



$$\underline{P} = \lim_{\Delta v \to 0} \frac{\sum_{\underline{p}_i \in \Delta v} \underline{p}_i}{\Delta v}$$

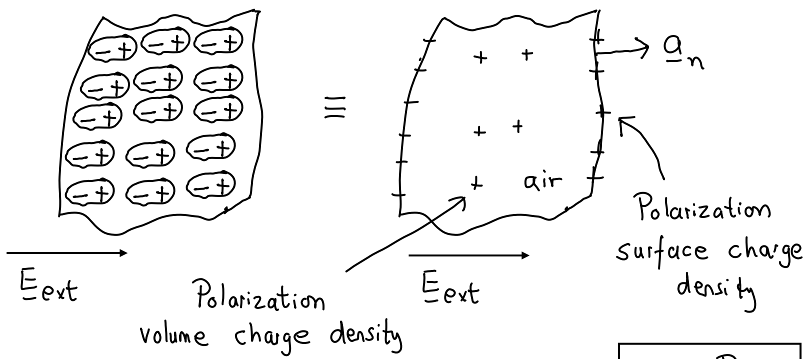
DEF^{N} of POLARIZATION (Dipole moment density)

UNITS:

Field of a dielectric is the field of its polarization

Electrostatics_05_Dielectrics: 6

EQUIVALENT SYSTEM with same electric field:

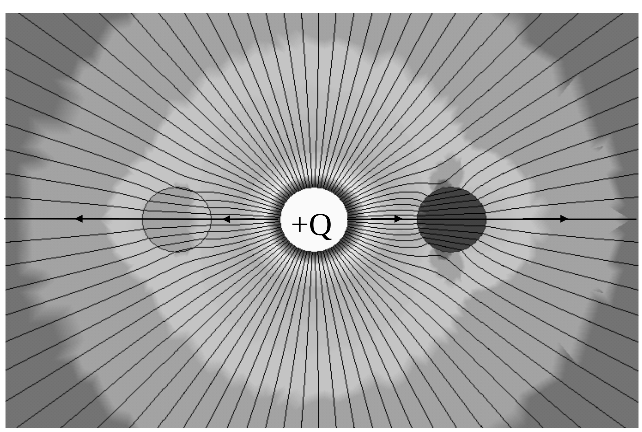


$$\rho_p = -\nabla \cdot \underline{P}$$

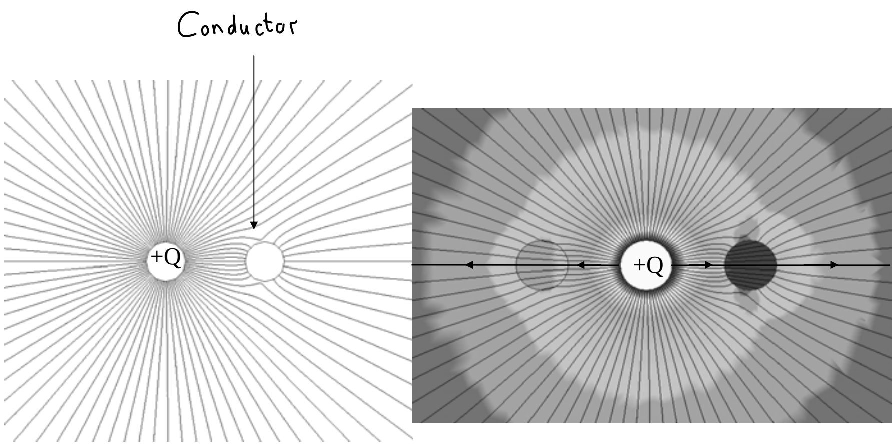
$$\rho_{ps} = \underline{P} \cdot \underline{a}_n$$

Electrostatics_05_Dielectrics: 7

Conducting Cylinder and Two Dielectric Cylinders



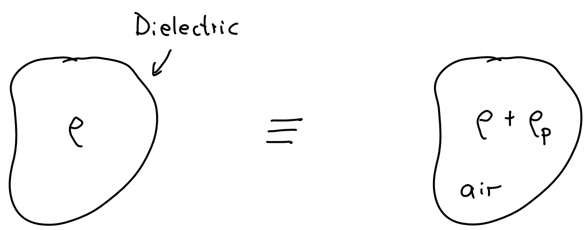
Electrostatics_05_Dielectrics: 8



Electrostatics_05_Dielectrics: 9

In a cloud of charge in free space: $\nabla \cdot \underline{E} = \rho / \varepsilon_0$

What happens if the charge is inside a dielectric?



So in a dielectric, $\nabla \cdot \underline{E} = \frac{1}{\varepsilon_0}(\rho + \rho_p) = \frac{1}{\varepsilon_0}(\rho - \nabla \cdot \underline{P})$

$$\Rightarrow \quad \nabla \cdot (\varepsilon_0 \underline{E} + \underline{P}) = \rho$$

Electrostatics_05_Dielectrics: 10

$$\nabla \cdot (\varepsilon_0 \underline{E} + \underline{P}) = \rho$$

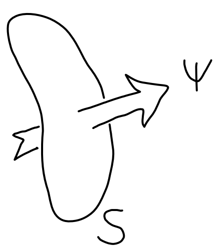
Define $$\boxed{\underline{D} = \varepsilon_0 \underline{E} + \underline{P}}$$ ELECTRIC FLUX DENSITY

Units = units of $\underline{P}$ = $C\,m^{-2}$

Then $$\boxed{\nabla \cdot \underline{D} = \rho}$$ Generalizes $\nabla \cdot \underline{E} = \rho / \varepsilon_0$

$\Rightarrow$ $$\boxed{\oint_S \underline{D} \cdot d\underline{s} = Q}$$ GAUSS' LAW (general form)

Electrostatics_05_Dielectrics: 11

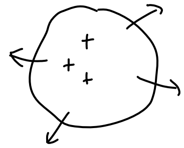


$$\boxed{\Psi = \int_S \underline{D} \cdot d\underline{s}}$$ ELECTRIC FLUX THROUGH S

UNITS :

So:

$$\oint_S \underline{D} \cdot d\underline{s} = Q \quad \equiv$$ " Flux out of S = Charge inside S "



Electrostatics_05_Dielectrics: 12

In many materials: $\underline{P} \propto \underline{E}$

Then $\underline{D} = \varepsilon_0 \underline{E} + \underline{P} \propto \underline{E}$

$$\Rightarrow \boxed{\underline{D} = \varepsilon \underline{E}}$$

$\uparrow$ constant called the PERMITTIVITY of the material

Units: $\dfrac{Cm^{-2}}{Vm^{-1}} = Fm^{-1}$

For free-space: $\underline{D} = \varepsilon_0 \underline{E} + \underline{P}$

$= \varepsilon_0 \underline{E}$ (why?)

So $\varepsilon = \varepsilon_0$ for free-space

Electrostatics_05_Dielectrics: 13

For other materials, $\varepsilon > \varepsilon_0$

Define $\boxed{\varepsilon_r = \dfrac{\varepsilon}{\varepsilon_0}}$ RELATIVE PERMITTIVITY, or DIELECTRIC CONSTANT

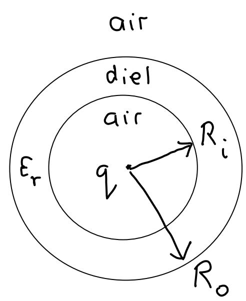
eg paper: $\varepsilon_r = 2 \rightarrow 4$

glass: $\varepsilon_r = 4 \rightarrow 10$

air: $\varepsilon_r =$

Electrostatics_05_Dielectrics: 14

EXAMPLE: SPHERICAL DIELECTRIC SHELL

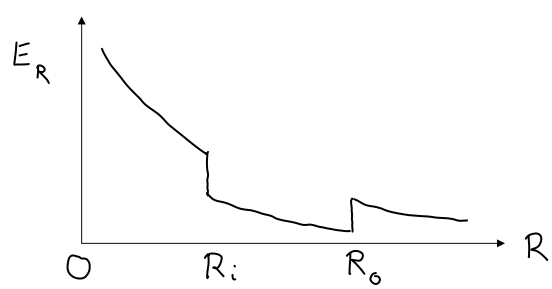


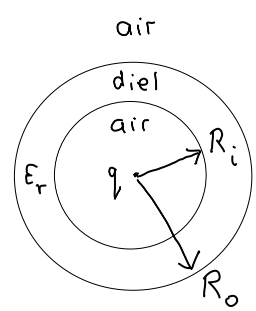
Find $\underline{E}, V, \underline{D}, \underline{P}$ as function of R

Spherical symmetry: $\underline{E} = \underline{a}_R E_R(R)$

Gauss' Law on sphere R:

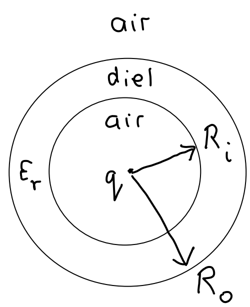
Electrostatics_05_Dielectrics: 15





$\underline{D} = \varepsilon_0 \underline{E} + \underline{P} \Rightarrow \underline{P} = \underline{D} - \varepsilon_0 \underline{E}$

Electrostatics_05_Dielectrics: 16



Electrostatics_05_Dielectrics: 17