Electrostatics, section 04

Conductors

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Conductor : material with plentiful supply of movable charge

INSIDE a conductor under static conditions :

No charge is moving $\rightarrow$ $\boxed{\underline{E} = 0}$

$\rho = \varepsilon_0 \nabla \cdot \underline{E}$ $\rightarrow$ $\boxed{\rho = 0}$

$V_{21} = -\int_1^2 \underline{E} \cdot d\underline{\ell}$ $\rightarrow$ $\boxed{V = \text{CONST}}$

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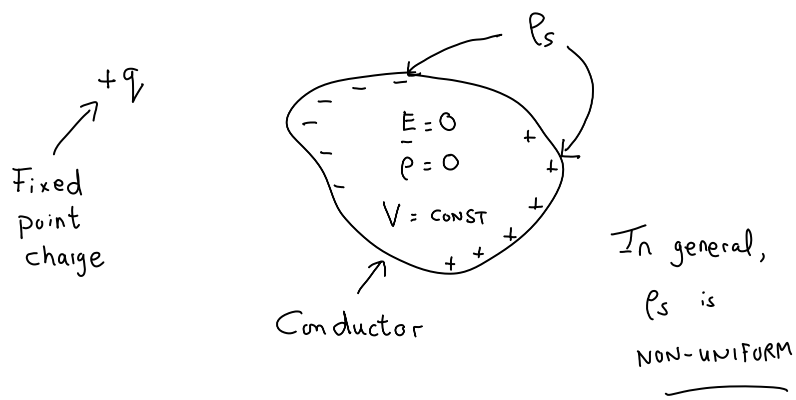
But there IS a layer of induced surface charge, ρ_s :



At SURFACE of conductor :

$$\boxed{\underline{E}_t = 0}$$

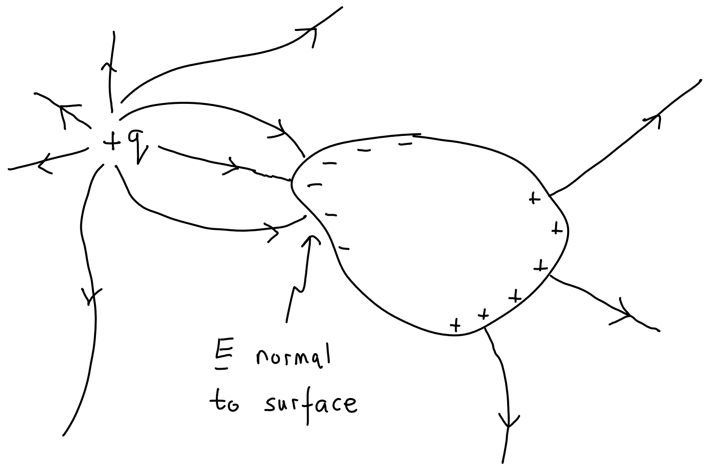
(Why?)

↑ tangential part

$$\boxed{E_n = \rho_s / \varepsilon_0}$$

(Proof by Gauss' Law)

↑ normal part



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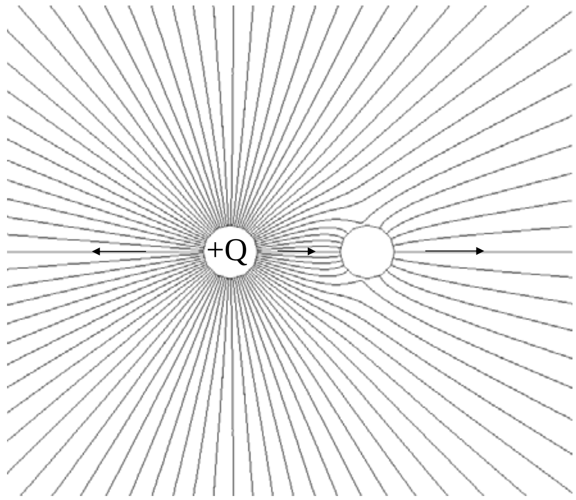
Two Conducting Cylinders



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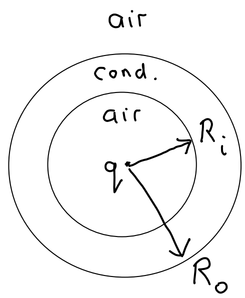
EXAMPLE: SPHERICAL CONDUCTING SHELL



Find $\underline{E}$ and $\underline{V}$ as function of R

Spherical symmetry: $\underline{E} = \underline{a}_R E_R(R)$

Gauss' Law on sphere $R < R_i$:

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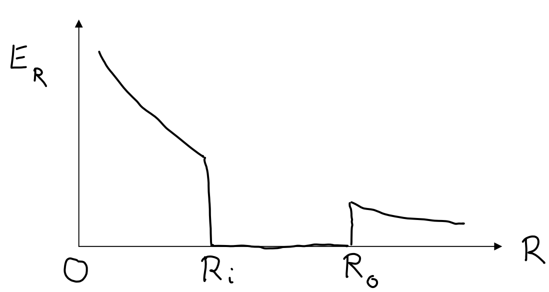
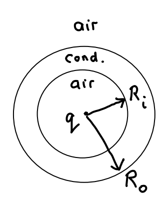
In $R_i < R < R_o$, $E_R =$

Gauss' Law on sphere $R > R_o$:





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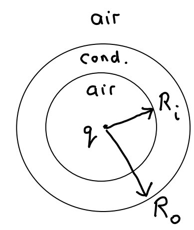
In $R > R_o$: $\quad V = -\int_{\infty}^{R} \left(E_R \underline{a}_R\right) \cdot \left(\underline{a}_R \, dR\right)$

$=$

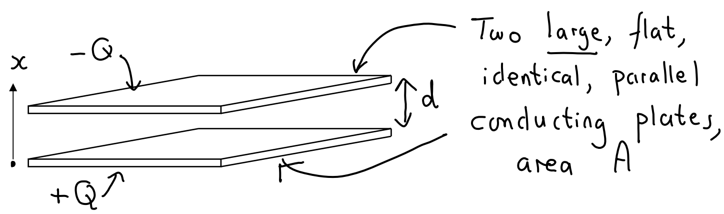


In $R_i < R < R_o$: $\quad V =$

In $R < R_i$: $\quad V = -\int_{\infty}^{R} E_R \, dR = -\int_{\infty}^{R_o} E_R \, dR - \int_{R_i}^{R} E_R \, dR$

$=$

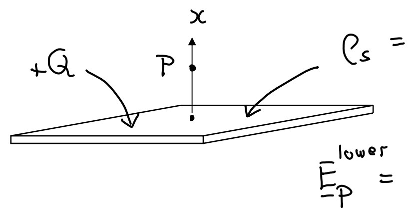
EXAMPLE : PARALLEL CONDUCTING PLATES



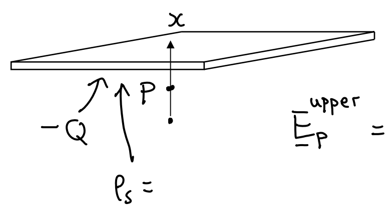
Neglect "edge effects"

(a) Find $\underline{E}$ between the plates.

Consider just lower plate:



Consider just upper plate:



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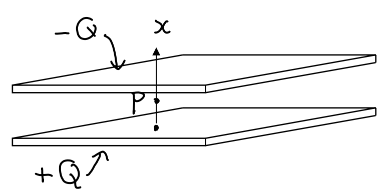

So $\quad \underline{E}_P = \underline{E}_P^{lower} + \underline{E}_P^{upper} =$

Note the $\underline{E}$ is <u>uniform</u> between the plates.

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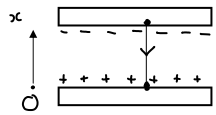
(b) Find the potential difference, V_o, between the plates.



$$V_o = -\int_{-}^{+} \underline{E} \cdot d\underline{l}$$

$$= -\int_{d}^{0} (E_x \underline{a}_x) \cdot (\underline{a}_x \, dx)$$

$$= -E_x \int_{d}^{0} dx$$

$$= E_x d =$$

Note also that

$$E_x = \frac{V_o}{d}$$

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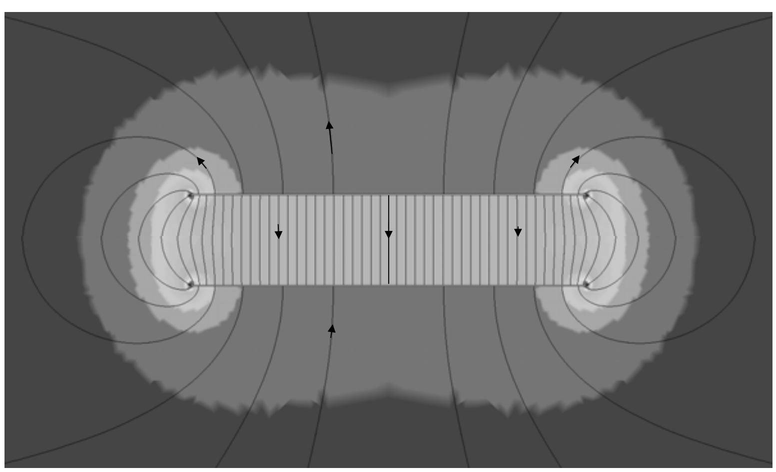
Two Parallel Conducting Plates



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