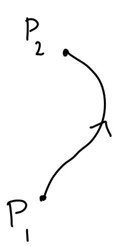
Electrostatics, section 03

Potentials

Electrostatics_03_Potential: 1

For any scalar f: $\nabla \times \nabla f = 0$

So: $\nabla \times \underline{E} = 0 \Rightarrow \boxed{\underline{E} = -\nabla V}$ Def^{n} of ELECTRIC POTENTIAL, V



Then:

$$-\int_{P_1}^{P_2} \underline{E} \cdot d\underline{\ell} = \int_{P_1}^{P_2} (\nabla V) \cdot d\underline{\ell} = \int_{P_1}^{P_2} dV$$

$$= V_2 - V_1 = V_{21}$$

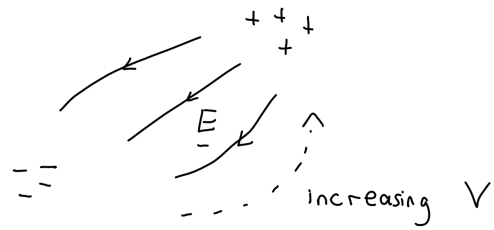
So:

$$\boxed{V_{21} = -\int_{P_1}^{P_2} \underline{E} \cdot d\underline{\ell}}$$

POTENTIAL DIFFERENCE between P_2 and P_1

Electrostatics_03_Potential: 2

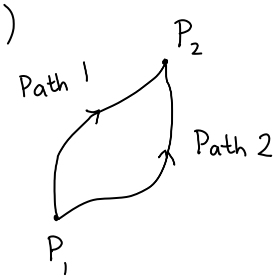
(a) Moving against $\underline{E}$ increases V:



$$\underline{E} = -\nabla V$$

Electrostatics_03_Potential: 3

(b)



$$-\int_{P_1}^{P_2} \underline{E} \cdot d\underline{\ell} = V_{21}$$

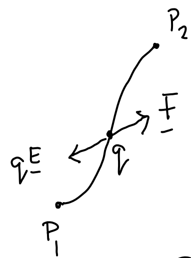
= same value, whichever path is taken

$\underline{E}$ is CONSERVATIVE

Another physical example of a conservative field is:

Electrostatics_03_Potential: 4

(c) To move q from P_1 to P_2 :



$$\text{Work done} = W = \int_{P_1}^{P_2} \underline{F} \cdot d\underline{l}$$

$$= \int_{P_1}^{P_2} (-q\underline{E}) \cdot d\underline{l} = q\, V_{21}$$

So: $\boxed{V_{21} = \dfrac{W}{q}}$

UNIT of V_{21} is $\frac{J}{C}$ which is the VOLT by definition

Also $\underline{E} = -\nabla V \Rightarrow$ Unit of $\underline{E}$ is $\frac{V}{m}$

Electrostatics_03_Potential: 5

(d) If a REFERENCE POINT, N, is specified or implied, then V_P is taken to mean V_{PN} :

$$V_P = V_{PN} = -\int_N^P \underline{E} \cdot d\underline{l}$$

Then it follows that

$$V_N = V_{NN} = 0$$ ie potential is zero at the ref. point.

Electrostatics_03_Potential: 6

The default reference point is infinity:

i.e. $V_P = V_{P\infty} = -\int_{\infty}^{P} \underline{E} \cdot d\underline{l}$

Electrostatics_03_Potential: 7

(e) LAPLACE & POISSON EQUATIONS

$$\left.\begin{array}{l} \nabla \cdot \underline{E} = \rho/\varepsilon_0 \\ \underline{E} = -\nabla V \end{array}\right\} \Rightarrow \nabla \cdot (-\nabla V) = \rho/\varepsilon_0$$

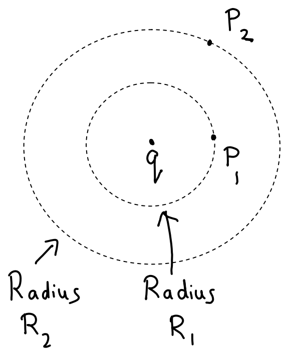
or $\boxed{\nabla^2 V = -\rho/\varepsilon_0}$ POISSON'S EQUATION

When $\rho = 0$:

$\boxed{\nabla^2 V = 0}$ LAPLACE'S EQUATION

Electrostatics_03_Potential: 8

POTENTIAL DUE TO POINT CHARGE



$$V_{21} = -\int_{P_1}^{P_2} \underline{E} \cdot d\underline{l}$$

$$\underline{E} = \underline{a}_R \frac{q}{4\pi\varepsilon_0 R^2}$$

Electrostatics_03_Potential: 9

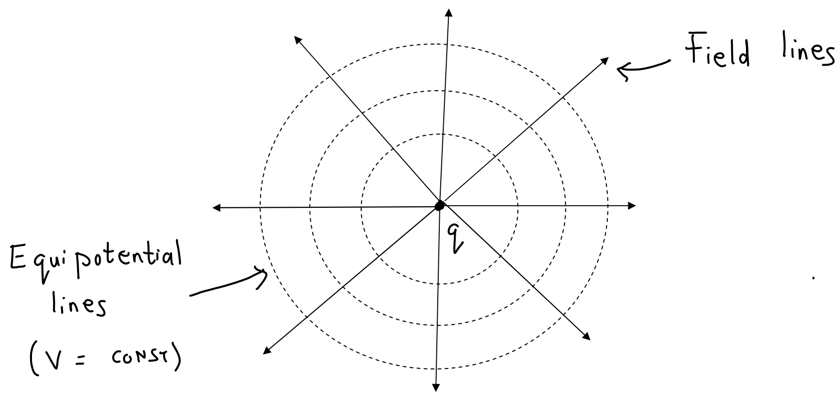
If P_1 is at ∞ and we set $P_2 = P$, $R_2 = R$:

$$V_P = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

i.e. $\boxed{V_P = \frac{q}{4\pi\varepsilon_0 R}}$ Potential due to point charge

Electrostatics_03_Potential: 10

$$V = \frac{q}{4\pi\varepsilon_0 R} \qquad \underline{E} = \underline{a}_R \frac{q}{4\pi\varepsilon_0 R^2}$$



Electrostatics_03_Potential: 11

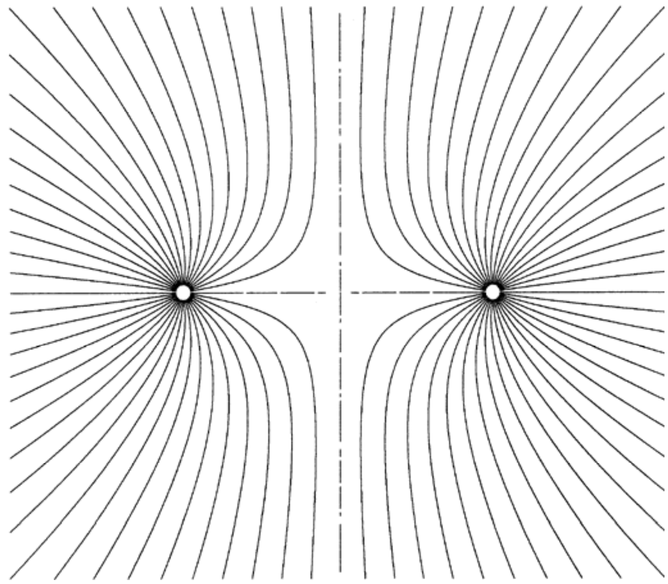
Two equal charges

$q \qquad\qquad q$

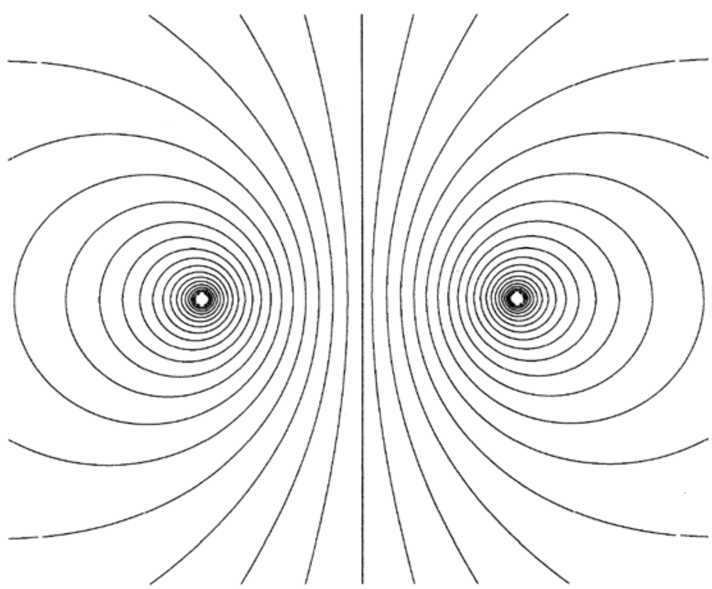
Two opposite charges

$q \qquad\qquad -q$

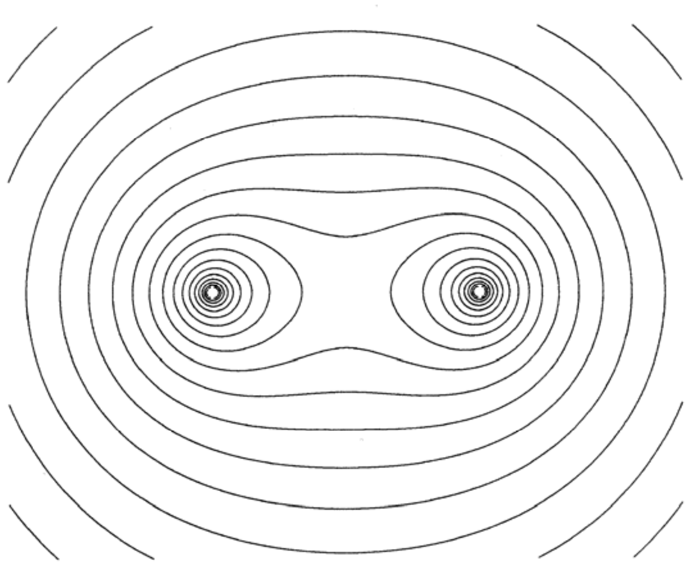
Electrostatics_03_Potential: 12



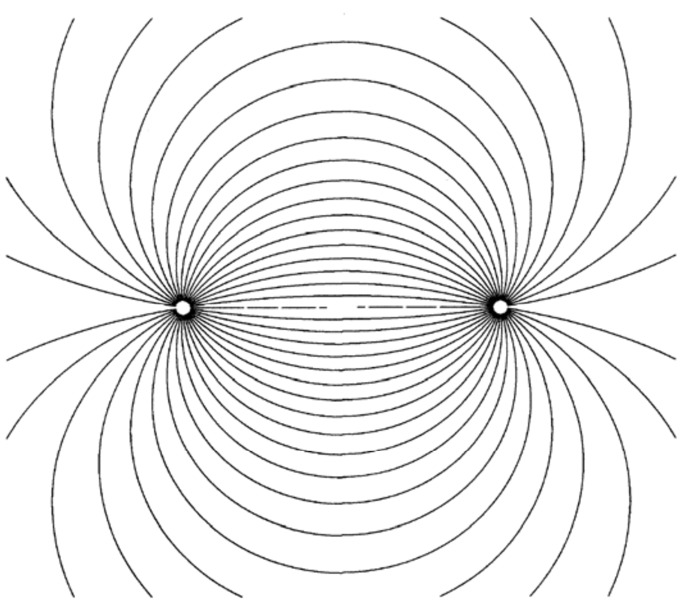
Electrostatics_03_Potential: 13



Electrostatics_03_Potential: 14



Electrostatics_03_Potential: 15



Electrostatics_03_Potential: 16