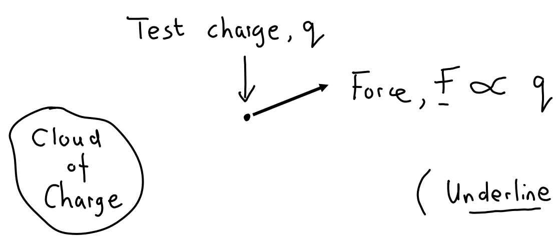
Electrostatics, section 02

Postulates of electrostatics

Gauss' Law

Electrostatics_02_Postulates: 1



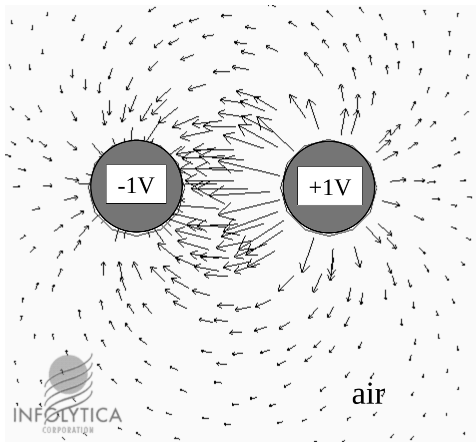
(Underline denotes vectors)

Definition of ELECTRIC FIELD: $\boxed{\underline{E} = \lim_{q \to 0} \frac{\underline{F}}{q}}$

Reversing: $\underline{F} = q\underline{E}$

Electrostatics_02_Postulates: 2

$\underline{E}$ is a force field around charged objects, e.g.:



Electrostatics_02_Postulates: 3

$$\underline{E} = \lim_{q \to 0} \frac{\underline{F}}{q}$$

$\Rightarrow$ UNIT of $\underline{E}$ is $\frac{\text{Newton}}{\text{Coulomb}}$, NC^{-1}

Actually, unit is $\boxed{\frac{\text{Volt}}{\text{Meter}}, \quad Vm^{-1}}$

— see later

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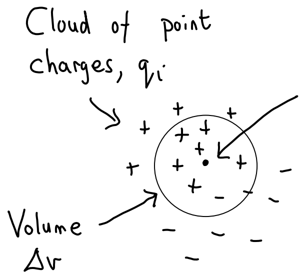


Volume charge density at this point is:

$$\rho = \lim_{\Delta v \to 0} \left(\frac{\sum_{q_i \in \Delta v} q_i}{\Delta v} \right)$$

Unit of volume charge density is

Reversing: Charge Q in volume V is

Electrostatics_02_Postulates: 5

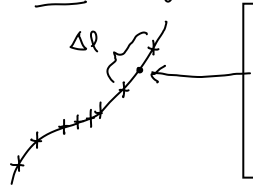
If charge is spread out in a thin layer over a surface:

Surface charge density is:

$$\rho_s = \lim_{\Delta s \to 0} \left(\frac{\sum_{q_i \in \Delta s} q_i}{\Delta s} \right)$$

Unit =

Line charge density is:



$$\rho_l = \lim_{\Delta l \to 0} \left(\frac{\sum_{q_i \in \Delta l} q_i}{\Delta l} \right)$$

Unit =

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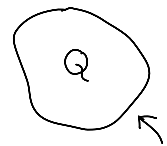
TWO POSTULATES OF ELECTROSTATICS IN FREE SPACE :

(1)

GAUSS' LAW

$$\boxed{\nabla \cdot \underline{E} = \rho / \varepsilon_0} \quad \Rightarrow \quad \boxed{\oint_S \underline{E} \cdot d\underline{s} = \frac{\underline{Q}}{\varepsilon_0}}$$



Closed surface, S

where $\varepsilon_0 \cong \frac{1}{36\pi} \times 10^{-9} \ Fm^{-1}$

$= 8.84 \ pF \ m^{-1}$

PERMITTIVITY OF FREE SPACE

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(2)

$$\boxed{\nabla \times \underline{E} = 0} \quad \Rightarrow \quad \boxed{\oint_C \underline{E} \cdot d\underline{l} = 0}$$

Closed contour, C

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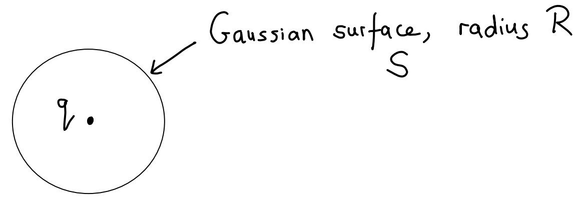
EXAMPLE: FIND $\underline{E}$ DUE TO ISOLATED POINT CHARGE q



Spherical coords: (R, θ, φ)

Assume: $\underline{E} = \underline{a}_R \, E_R(R)$

↑ i.e. no variation with θ, φ

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Gauss' Law: $\oint_S \underline{E} \cdot d\underline{s} = Q/\varepsilon_0$

Applied to sphere $\oint_{\text{Sphere}} (E_R \, \underline{a}_R) \cdot (\underline{a}_R \, ds) = q/\varepsilon_0$

=>

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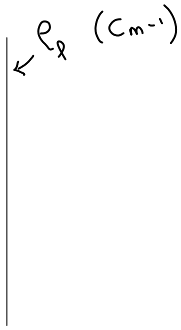
EXAMPLE: FIND $\underline{E}$ DUE TO INFINITE LINE OF CHARGE



Coord system:

Assume:

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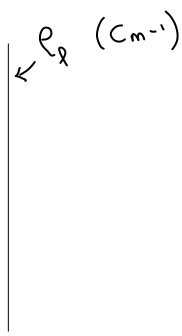

Gauss' Law: $\oint_S \underline{E} \cdot d\underline{s} = Q/\varepsilon_0$

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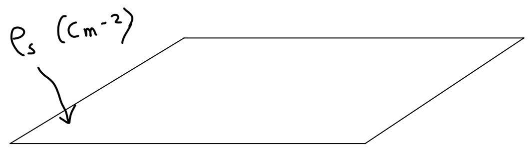
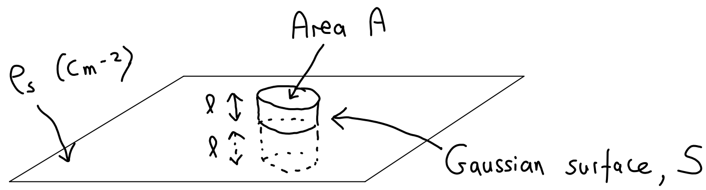
EXAMPLE : FIND $\underline{E}$ DUE TO INFINITE PLANE OF SURFACE CHARGE

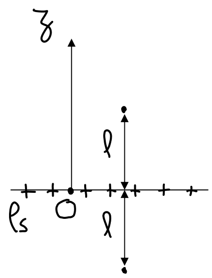


Coord system:

Assume:



Gauss' Law ; $\oint_S \underline{E} \cdot d\underline{s} = Q/\varepsilon_0$



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