CIRCUIT ANALYSIS
304-210B
Test 2

Test 2 — Time: 10:30-11:30
Mon. 15 March, 1999
Professor: Martin D. Levine

- CLOSED BOOK TEST -

Name Solutions
Student # ____________

INSTRUCTIONS
1. Answer all questions.
2. Write your answers in the space at the bottom of each page.

QUESTION #	Q1	Q2	Q3	Q4	TOTAL
MARKS	3 Points	4.5 Points	4.5 Points	3 Points	15 Points

Remarks:

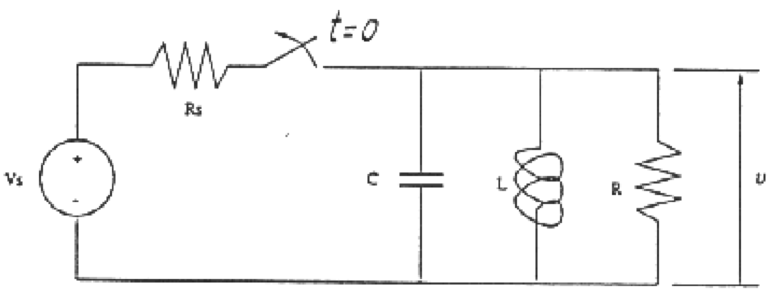
Question 1

For the circuit shown below, assume steady-state for $t < 0$:

A. Find the differential equation associated with the voltage v, for $t \geq 0$.

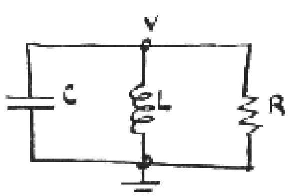
B. If $R = \infty$, find $v(t)$ for $t \geq 0$.

C. For the equation obtained in part A., find R such that $v(t)$ is critically damped.



A. $t \geq 0$

(1.4pt)



By KCL @ NODE (v):

$$i_c + i_L + i_R = 0$$

$$C\frac{dv}{dt} + i_L + \frac{v}{R} = 0 \qquad (1)$$

Voltage across the inductor: $v = L\frac{di_L}{dt}$ (2)

Differentiating (1): $C\ddot{v} + \frac{di_L}{dt} + \frac{\dot{v}}{R} = 0$ (3)

Replacing $\frac{di_L}{dt}$ from (2) in (3):

$$C\ddot{v} + \frac{v}{L} + \frac{\dot{v}}{R} = 0 \longrightarrow \boxed{\ddot{v} + \frac{1}{RC}\dot{v} + \frac{1}{LC}v = 0} \quad (4)$$

B. (i) For $t<0$

(1pt) Initial Conditions $V_c(0^-) = 0$ $I_L(0^-) = 0$

$V_L(0^-) = 0$ $V_L(0^-) = L\frac{dI_L(0^-)}{dt} = 0$ (since V_s = constant).

In steady-state:

- Capacitor → Open circuit
- Inductor → Short circuit (and the rate of change of the current through it is zero, i.e. $I_L(0^-)$ = constant).

2

(ii) From eq. (4) in A., with $R=\infty$.

$$\ddot{v} + \frac{1}{LC} v = 0$$

Roots of the characteristic equation: $s^2 + \frac{1}{LC} = 0 \rightarrow s = \pm j\frac{1}{\sqrt{LC}}$

$$\therefore \quad v(t) = A_1 \sin\left(\frac{1}{\sqrt{LC}} t\right) + A_2 \cos\left(\frac{1}{\sqrt{LC}} t\right)$$

$$\left.\begin{aligned} &v(0) = 0 = A_2 \\ &\frac{dv(0)}{dt} = \frac{i_L(0)}{C} = 0 = \frac{A_1}{\sqrt{LC}} \end{aligned}\right\} \therefore \; A_1 = A_2 = 0$$

$$\boxed{v(t) = 0 \text{ [V]}, \; t \geq 0}$$

C. (0.6 pt) Critically damped $\Rightarrow$ Discriminant $= 0$

From eq. (4) in A., the discriminant is:

$$\sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}} = 0 \quad \rightarrow \quad \frac{1}{RC} = \frac{2}{\sqrt{LC}} \rightarrow \boxed{R = \frac{1}{2}\sqrt{\frac{L}{C}} \text{ [}\Omega\text{]}}$$

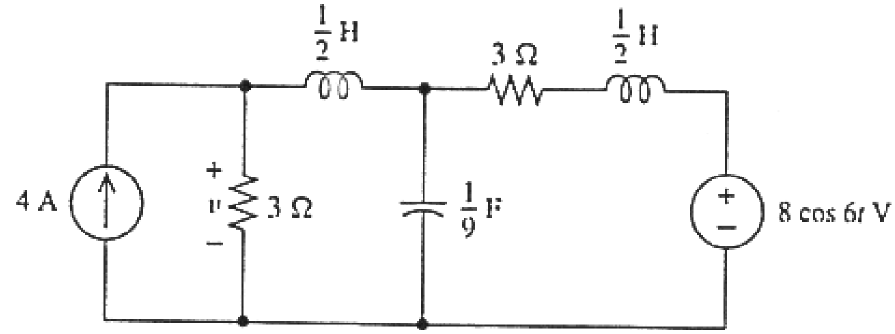
ANS.: A. $\ddot{v} + \frac{1}{RC}\dot{v} + \frac{1}{LC} v = 0$

B. $v(t) =$ 0 [V]

C. $R = \frac{1}{2}\sqrt{\frac{L}{C}}$ [Ω]

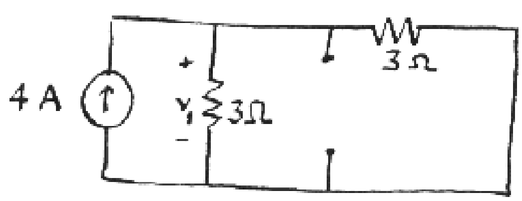
Question 2

Find the steady-state voltage $v(t)$.



Different frequency sources ⟹ Apply SUPERPOSITION (0.5p)

(i) Killing the ac-voltage source



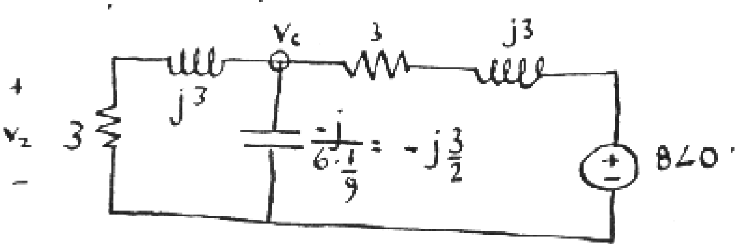
In steady-state (dc)

$$V_1 = 3\cdot\left(\frac{I}{2}\right) = 3\cdot 2 = 6 \ [V]$$

(1p)

(ii) Killing the dc-current source.

In steady-state (ac) with $\omega = 6$



By voltage divider

$$V_2 = \frac{3}{3+j3} V_c = \frac{1}{1+j} V_c$$

(1p)

And

$$V_c = \frac{\dfrac{(3+j3)\cdot(-j\frac{3}{2})}{3+j3-j\frac{3}{2}}}{\dfrac{(3+j3)(-j\frac{3}{2})}{3+j3-j\frac{3}{2}} + 3+j3} \cdot 8\angle 0 = \frac{\cancel{3(1+j)}(-j\frac{3}{2})}{\cancel{3(1+j)}(-j\frac{3}{2}) + \cancel{3(1+j)}(3+j\frac{3}{2})} 8 = \frac{-j\frac{3}{2}}{3} 8 = -j4$$

(1.5p)

4

$$\therefore \quad V_2 = \frac{1}{1+j} V_c = \frac{-j4}{1+j} = \frac{4\angle -90}{\sqrt{2}\angle 45^\circ} = 2\sqrt{2}\angle -135^\circ \ [V]$$

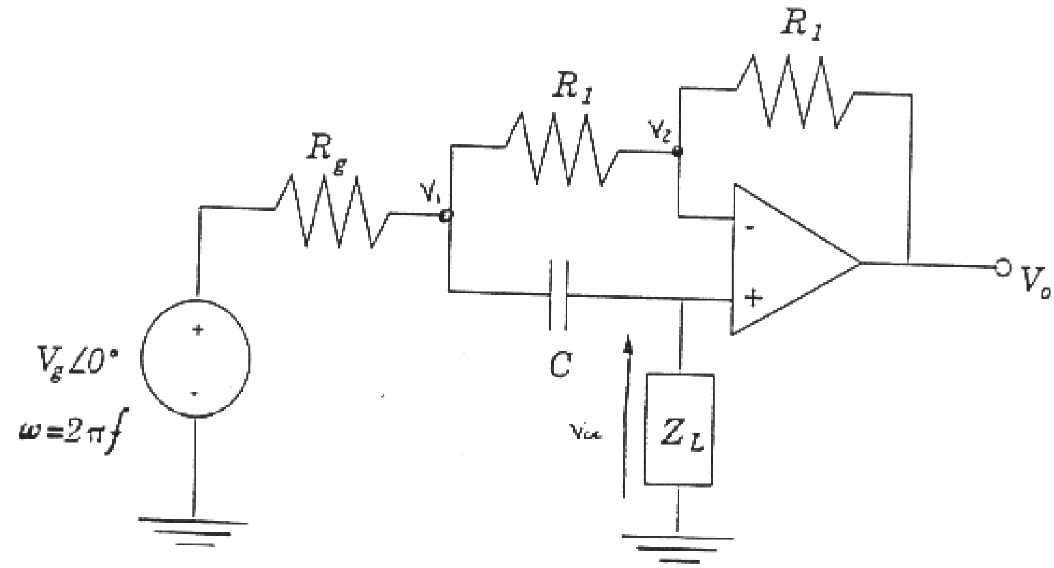
$$\therefore \quad v(t) = v_1(t) + v_2(t)$$

$$= 6 + 2\sqrt{2}\cos(6t - 135^\circ) \qquad (0.5p)$$

ANS.: $v(t) = \underline{6 + 2\sqrt{2}\cos(6t - 135^\circ)}$ [V]

Question 3

Find the load Z_L so that it absorbs maximum power.



OP.AMP. $\rightarrow$ Dependent Source $\rightarrow$ $Z_{TH} = \dfrac{V_{oc}}{I_{sc}}$ (0.3p)

(i) $\underline{V_{oc}}$

$V_{oc} = V_2$ (since $V^- = V^+$)

$I_+ = 0 \Rightarrow I_+ = \dfrac{V_1 - V_2}{\frac{-j}{\omega C}} = 0 \Rightarrow V_1 = V_2 \Rightarrow I_{R_1} = 0$

Then, since $I_{source} = I_{R_1} + I_+ = 0 \rightarrow V_2 = V_g\angle 0^\circ$

$\therefore \boxed{V_{oc} = V_g}$ (1.2p)

(ii) $\underline{I_{sc}}$

$V^+ = V_2 = 0$

$I_{sc} = \dfrac{V_1}{\frac{-j}{\omega C}}$ (since $I_+ = 0$)

$\underline{\text{KCL @ } V_1}$ $\dfrac{V_g - V_1}{R_g} = \dfrac{V_1 - 0}{R_1} + \dfrac{V_1}{\frac{-j}{\omega C}}$ (eq. 1)

6

From eq. (1):

$$\frac{V_g}{R_g} = \left(\frac{1}{R_g} + \frac{1}{R_1} + j\omega C \right) V_1$$

$$V_1 = \frac{V_g}{R_g \left(\frac{1}{R_g} + \frac{1}{R_1} + j\omega C \right)}$$

$$I_{sc} = \frac{V_1}{\frac{-j}{\omega C}} = j\omega C V_1 = \frac{j\omega C\, V_g}{R_g \left(\frac{1}{R_g} + \frac{1}{R_1} + j\omega C \right)} \qquad (1.8p).$$

$$Z_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{R_g}{j\omega C} \left(\frac{1}{R_g} + \frac{1}{R_1} + j\omega C \right) = \frac{(R_1 + R_g)}{j\omega C R_1} + R_g$$

$$Z_{TH} = R_g - j\,\frac{(R_1 + R_g)}{\omega C R_1}$$

For maximum power

$$\boxed{Z_L = Z_{TH}^{*} = R_g + j\,\frac{(R_1 + R_g)}{\omega C R_1}} \qquad (1.2p)$$

ANS.: $Z_L = \underline{R_g + j\,\frac{(R_1 + R_g)}{\omega C R_1}}$

7

Question 4

A 7.46 kW motor operates from a 480 V_{rms}, 60 Hz source with 0.8 lagging power factor. Determine the parallel capacitor C required so that the power factor is changed to 0.922 lagging.

$P = 7.46$ kW

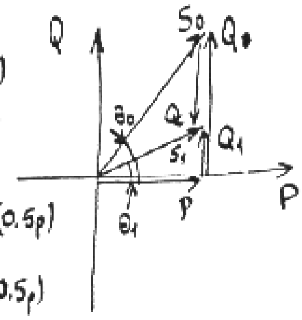
$V = 480$ [V_{rms}]

$p.f._0 = 0.8$ (lagging) $\longrightarrow \theta_0 = a\cos(p.f._0) = 36.8699°$ (0.5p)

$p.f._1 = 0.922$ (") $\longrightarrow \theta_1 = a\cos(p.f._1) = 22.7798°$ (0.5p)

$Q_0 = P\tan(\theta_0) = 7.46 \cdot 0.75 = 5.595$ KVAR (0.5p)

$Q_1 = P\tan(\theta_1) = 7.46 \cdot 0.4199 = 3.133$ KVAR (0.5p)



P-Q Diagram

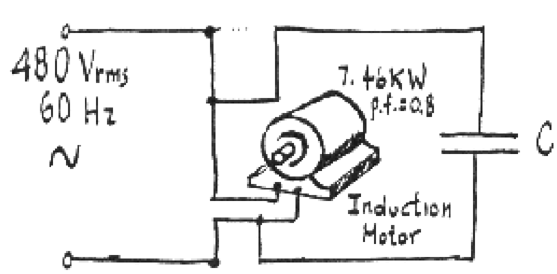
$Q_c = Q_1 - Q_0 = -2.4622$ kVAR (0.5p)

$$S_c = V_{c_{rms}} \cdot I_{c_{rms}}^* = V_c \cdot \left(\frac{V_c}{Z_c}\right)^* = \frac{V_{c_{rms}}^2}{Z_c^*}, \quad Z_c = \frac{-j}{wC}$$

$$S_c = jQ_c \Rightarrow -j V_{c_{rms}}^2 \cdot C \cdot w = jQ_c$$

$$\therefore C = \frac{-Q_c}{2\pi f V_{rms}^2} = \frac{2.4622 \cdot 10^3}{2\pi 60 \cdot (480)^2} \quad (0.5p)$$

$$\boxed{C = 28.3472\ \mu F}$$



ANS.: $C =$ 28.3472 µF

8