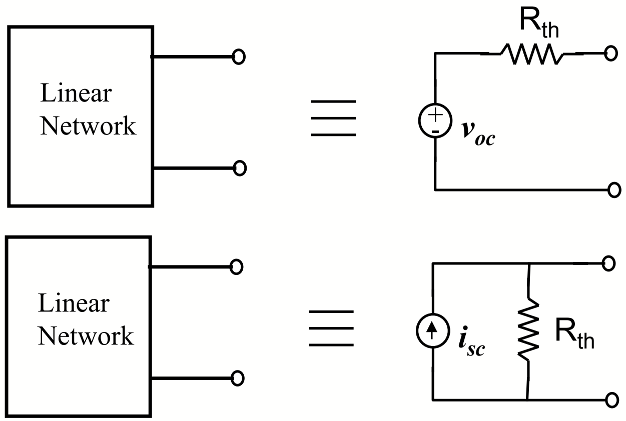
ECSE 210: Circuit Analysis

Lecture #5: First Order Circuits

Thevenin/Norton Equivalent

- See textbook Section 2.6
- Review course notes of ECE 200



Some Important Points

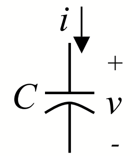
1. The part of the circuit to be replaced by a Thevenin or Norton equivalent must be **linear**. The remainder of the circuit can be linear or nonlinear.
2. If dependent sources appear in the part of a circuit to be transformed, their control variables must also be present in that part of the circuit.
3. The Thevenin and Norton circuits are wholly equivalent to each other.
4. v_{oc} and i_{sc} are related by $v_{oc}=i_{sc}R_{th}$

Applying Norton and Thevenin

1. If only **independent sources** are present, calculate v_{oc} or i_{sc} and R_{th} using standard techniques.
2. If **both independent and controlled sources** belong to the network, determine v_{oc} and i_{sc} first, and then calculate $R_{th} = v_{oc}/i_{sc}$ afterwards.
3. In **no independent sources** are present, then both i_{sc} and v_{oc} are zero. In this case the equivalent Thevenin/Norton circuit is R_{th} alone. To find R_{th}, apply an arbitrary test source v to the network; determine the generated input current i ; then calculate $R_{th}=v/i$.

Capacitors/Inductors

•See textbook Chapter 5
•Review course notes of ECE 200

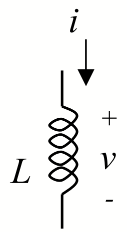


$$i = C\frac{dv}{dt}$$

$$E_c = \frac{1}{2}Cv^2$$

Open circuit in dc steady state

Voltage is continuous



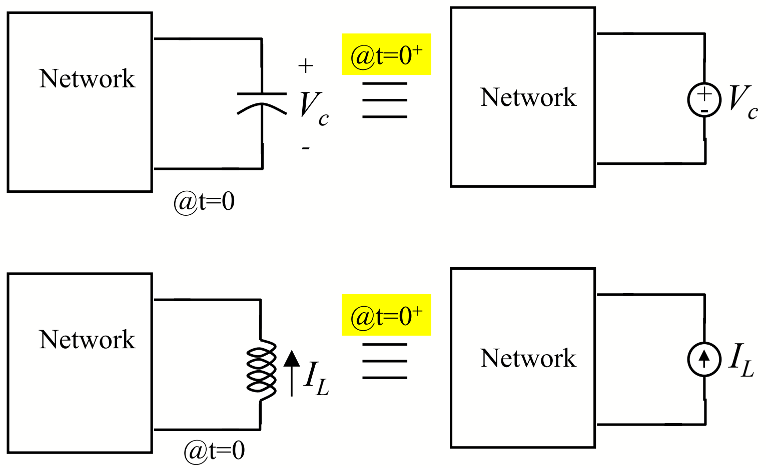
$$v = L\frac{di}{dt}$$

$$E_L = \frac{1}{2}Li^2$$

Short circuit in dc steady state

Current is continuous

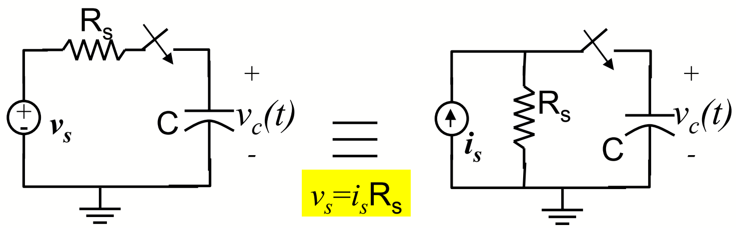
Capacitors/Inductors



RC / RL Circuits

1. Contain energy storage elements (C or L).
2. RC/RL circuits have “memory.” The response depends on the ***current input and*** the ***history*** of the circuit.
3. So far we examined dc steady-state analysis (after all transients have died out and all signals are constant). For this case capacitors are open circuits and inductors are short circuits.
4. Now we take a look at transients.

RC Circuits



• Switch closes at t=0.
• Assume initial voltage on the capacitor $v_c(0^-)=V_o$

<u>KCL</u>

$$\frac{v_c(t)}{R_s} + C\frac{dv_c(t)}{dt} = i_s \quad \Rightarrow \quad \frac{dv_c(t)}{dt} + \frac{1}{R_sC}v_c(t) = \frac{i_s}{C}$$

Solution of D.E.

$$\frac{dx(t)}{dt} + ax(t) = A \quad \Rightarrow \quad \begin{cases} \text{Solution of the form:} \\ x(t) = C_1 + C_2 e^{-at} \end{cases}$$

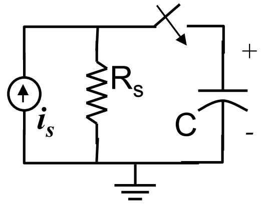
Constant

•The constants C_1 and C_2 can be determined from the boundary conditions.

$$\frac{dv_c(t)}{dt} + \frac{1}{R_s C} v_c(t) = \frac{i_s}{C} \quad \Rightarrow \quad \boxed{\begin{array}{l} v_c(t) = C_1 + C_2 e^{-\frac{t}{T_C}} \\ T_C = R_s C \end{array}}$$

RC Circuits

$$v_c(t) = C_1 + C_2 e^{-\frac{t}{T_C}}$$



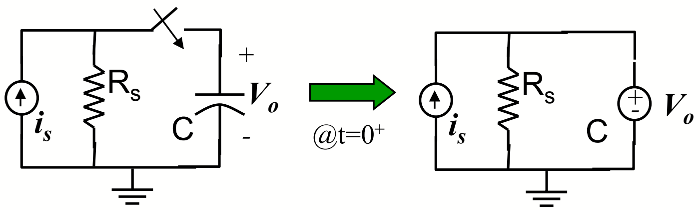
Find C_1 from boundary condition at $t = \infty$

From above, $v_c(\infty) = C_1$

From circuit, $v_c(\infty) = i_s R_s$

$$\Rightarrow \quad C_1 = i_s R_s$$

RC Circuits



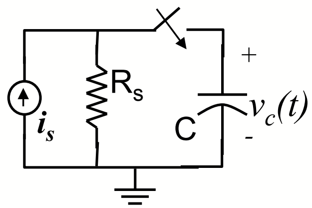
Find C_2 from boundary condition at $t=0^+$

From above, $v_c(0^+) = C_1 + C_2$

From circuit, $v_c(0^+) = V_o$

$$\Rightarrow \quad \begin{array}{l} C_1 + C_2 = V_o \\ C_2 = V_o - i_s R_s \end{array}$$

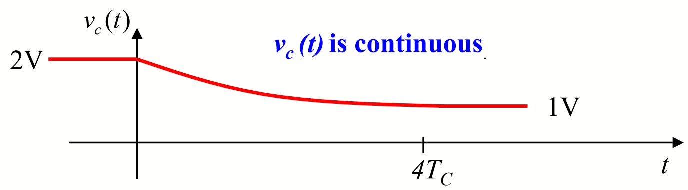
RC Circuits



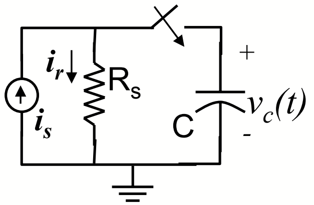
$$v_c(t) = i_s R_s + (V_o - i_s R_s) e^{-\frac{t}{T_C}}$$

$$T_C = R_s C$$

Suppose $R_s = 1\Omega \quad i_s = 1A \quad V_o = 2V$



RC Circuits

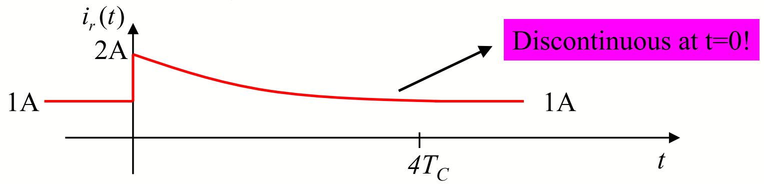


$$v_c(t) = i_s R_s + (V_o - i_s R_s) e^{-\frac{t}{T_C}}$$

$$R_s = 1\Omega \quad i_s = 1A \quad V_o = 2V$$

Determine i_r

$$\begin{cases} i_r(t) = i_s + \left(\dfrac{V_o}{R_s} - i_s\right) e^{-\frac{t}{T_C}} & ; t > 0 \\ i_r(t) = i_s & ; t < 0 \end{cases}$$



RC Circuits

1. The solution for **all** circuit variables has the form:

$$x(t) = C_1 + C_2 e^{-\frac{t}{T_C}}$$

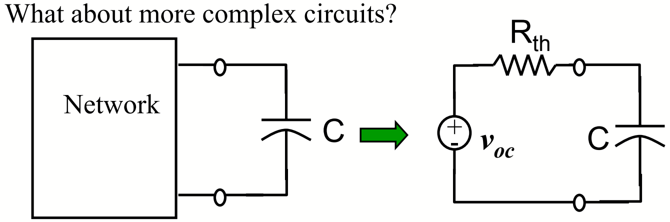
2. The time constant T_c is **common** for all circuit variables

$$T_C = R_s C$$

3. The value of $x(t)$ for $t<0$, is found from the dc solution before the switch is closed (or opened).
4. C_1 is found from the dc steady state solution at $t = \infty$
5. Find voltage V_o across capacitor **before** switch is closed/opened.
6. C_2 is found from the **"initial-state"** solution at t=0+ (replace capacitor with voltage source V_o).

RC Circuits

What about more complex circuits?



→ Network can be replaced by it Thevenin or Norton equivalent.

→ All circuit variables have the same form as before.

→ All circuit variables have the same time constant $T_C = R_{th} C$

→ Find C_1 and C_2 using same procedure as before.

Summary

1. Assume the unknown circuit variable has the form:

$$x(t) = C_1 + C_2 e^{-\frac{t}{T_C}}$$

2. Consider the equilibrium circuit that is valid at t=0^-,
   → Capacitor replaced by an open circuit.
   → Inductor replaced by a short circuit.
   Calculate the steady-state capacitor voltage $V_o = v_c(0^-)$ or inductor current $I_o = i_L(0^-)$.
3. Consider the circuit that is valid at t=0^+,
   → Capacitor replaced by voltage source V_o.
   → Inductor replaced by current source I_o.
   Calculate the "initial-state" solution value $x(0^+)$.

Summary

4. Consider the circuit valid at $t = \infty$

→ Capacitor replaced by an open circuit.

→ Inductor replaced by a short circuit.

Calculate the steady-state solution value

5. Calculate the transient solution constants

→. $C_1 = x(\infty)$

→ $C_2 = x(0^+) - x(\infty)$

6. Calculate the time constant of the circuit:

→ Determine the Thevenin resistance seen by the terminals of the storage element.

Then $T_C = R_{th}C$ for an RC circuit, and $T_C = L/R_{th}$ for an RL circuit.