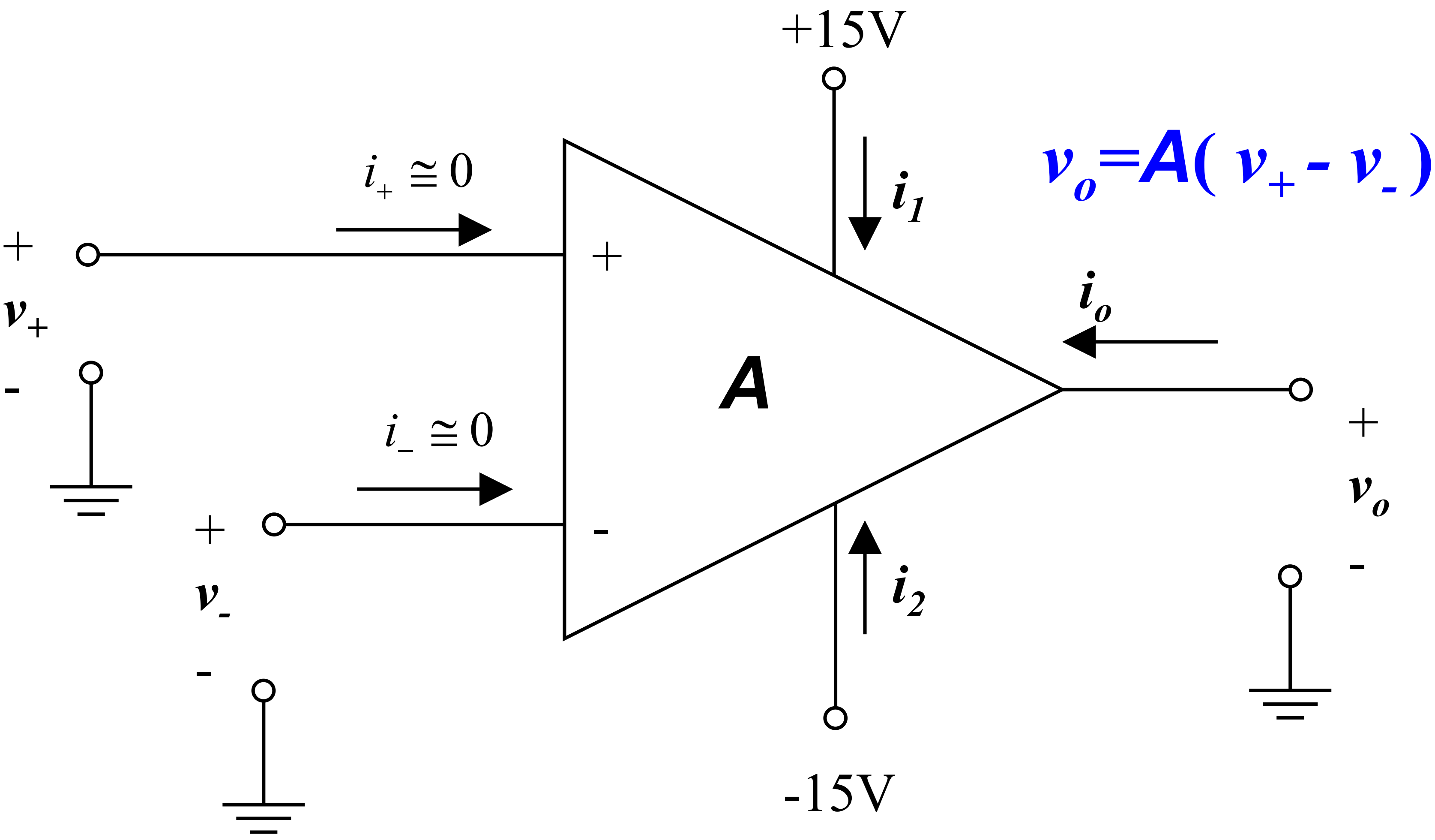
ECSE 210: Circuit Analysis

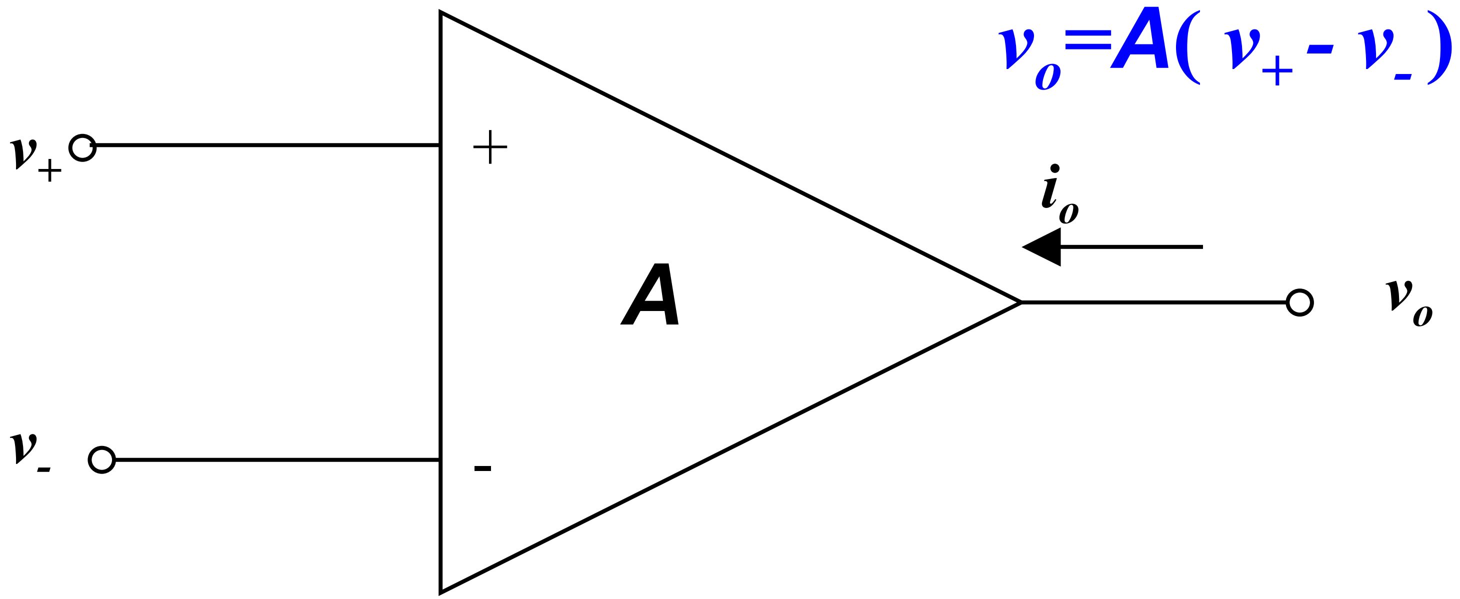
Lecture #4: Operational Amplifiers

OpAmp Symbol



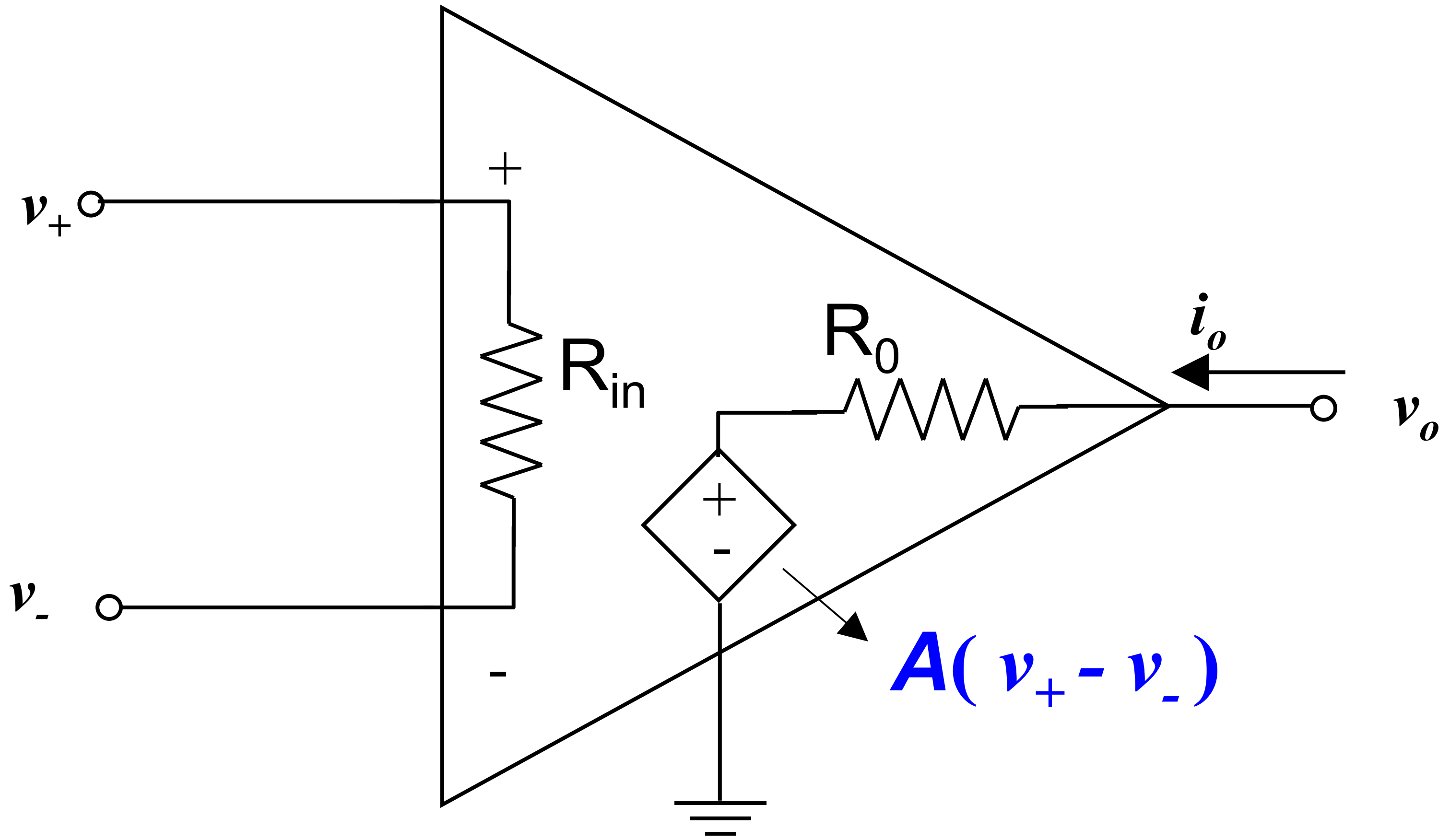
A *differential* amplifier.

Simplified OpAmp Symbol

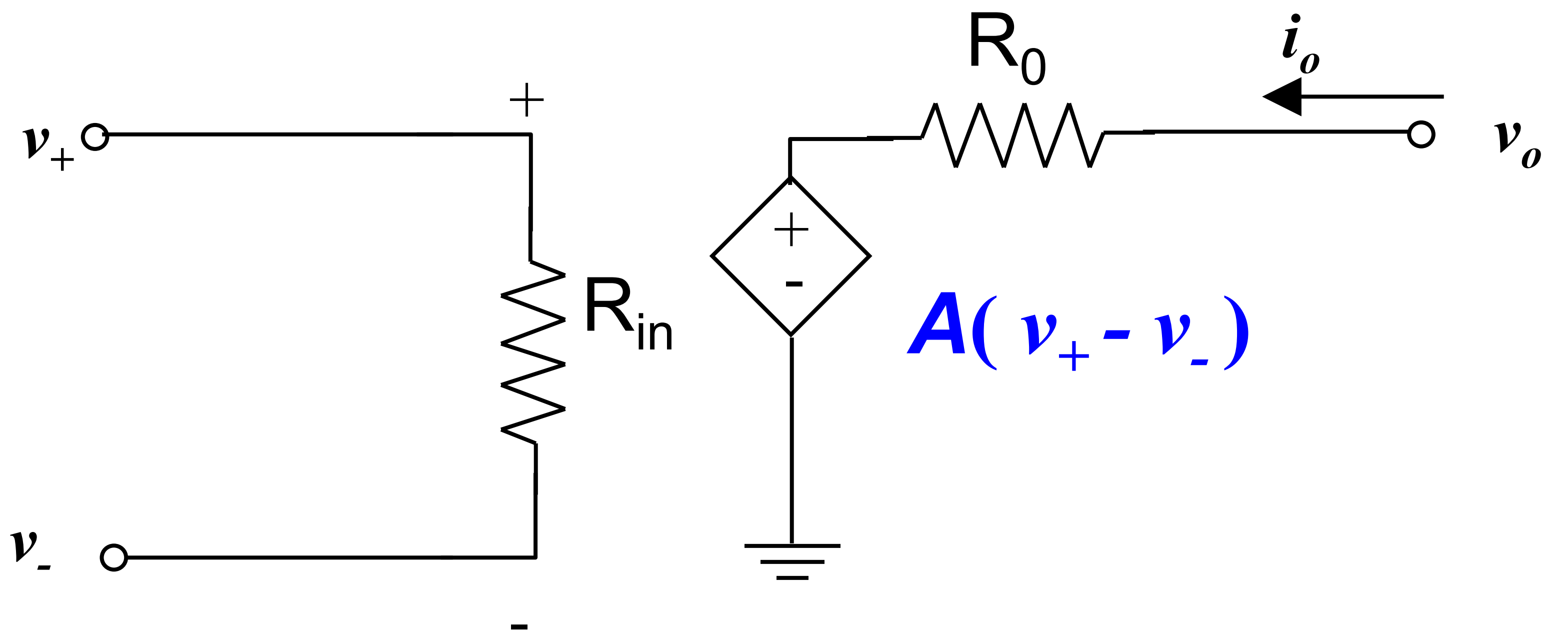


1. Supply voltages are not shown.
2. Supply currents are not shown.
   (Be careful with KCL)
3. Never do KCL at the reference node (otherwise need to include supply currents).
4. KCL at output node v_o is never done unless we want to calculate i_o.

OpAmp Model



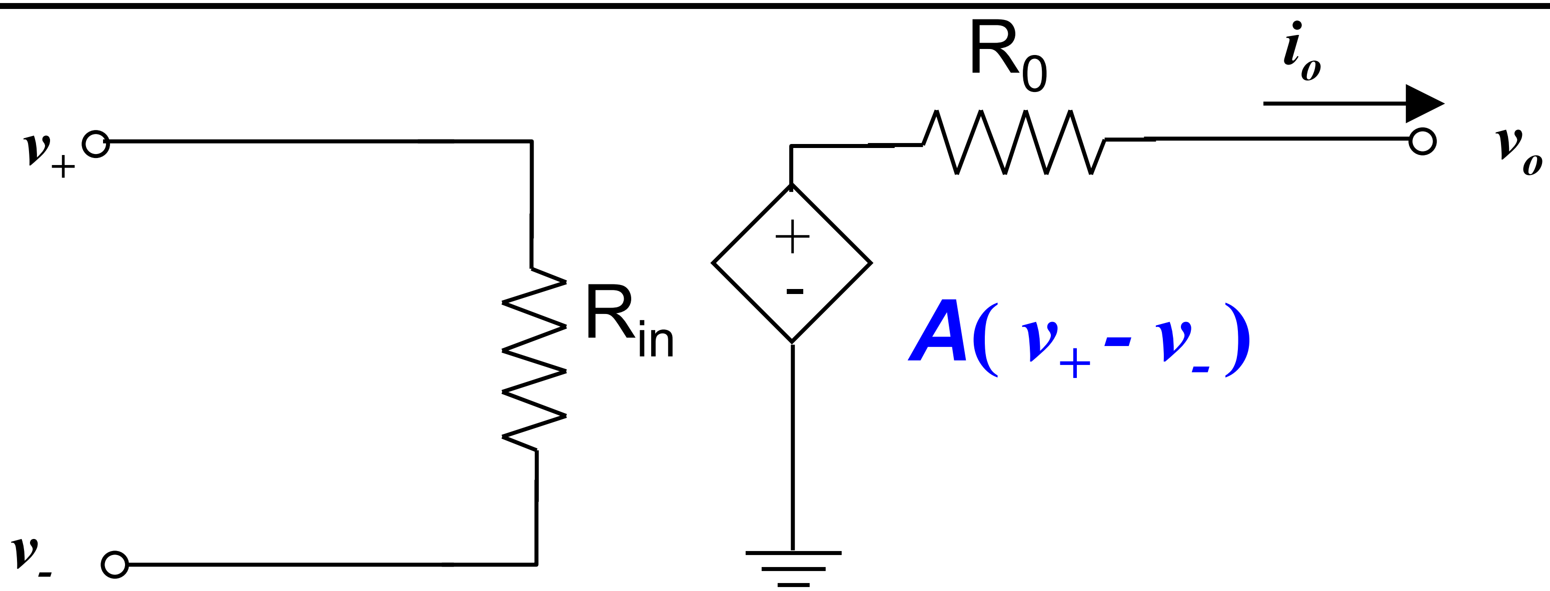
Op-Amp Equivalent Circuit



➡ Based on this circuit, the op-amp is a unilateral device.

→ The output voltage is determined as a function of the input, BUT ***the input voltages are not affected by the output voltage***.

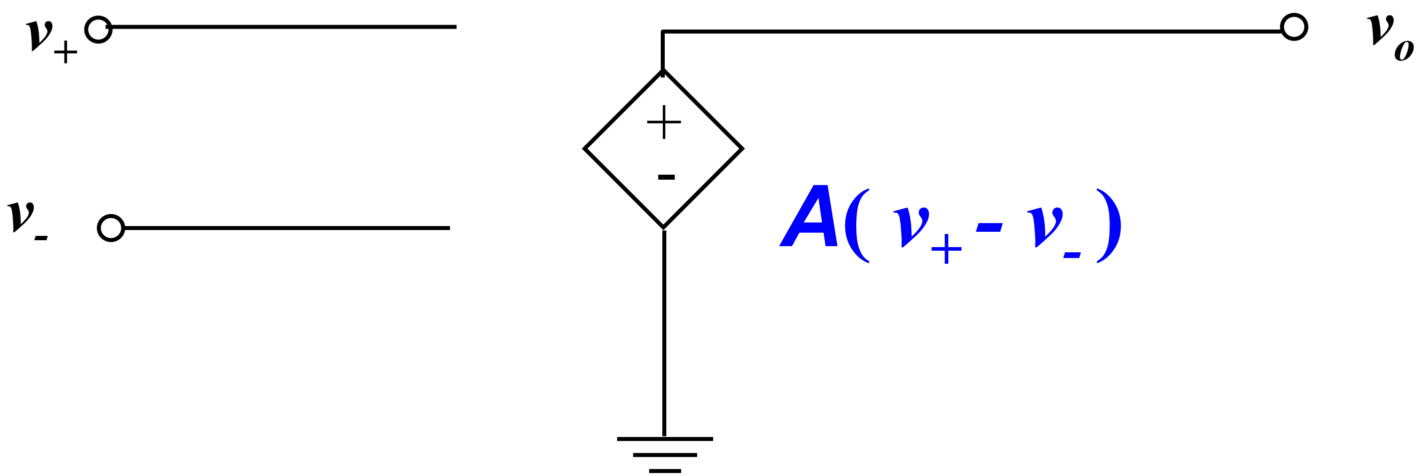
Some Practical Information



1. The input resistance R_{in} is very large, typically in the range of 100kΩ → 1TΩ.
2. The voltage gain ***A*** is very high, typically $10^5 \rightarrow 10^7$.
3. The output resistance R_o is very small ***compared*** to the recommended output load. Typically R_o is in the range 1Ω→75Ω.

⇨ These parameters suggest a simpler model!

Simpler Op-Amp Model



1. Since R_{in} is very large, assume R_{in} = infinity.
2. Since Ro is very small assume R_o = zero
3. This model appears on page 84 of the text, Figure 3.7. It depends only on the ***open loop*** gain ***A***.

The Ideal OpAmp Model

Assume the ***"OpAmp Equivalent Circuit"*** :

1. Since R_{in} is very large, assume R_{in} = infinity.
2. Since R_o is very small assume R_o = zero.
3. Since A is very large, assume A = infinity.

} *Same as previous slide*

This gives the ***"Ideal OpAmp Model"***

Condition of Linearity

- The ***ideal*** op-amp is linear.

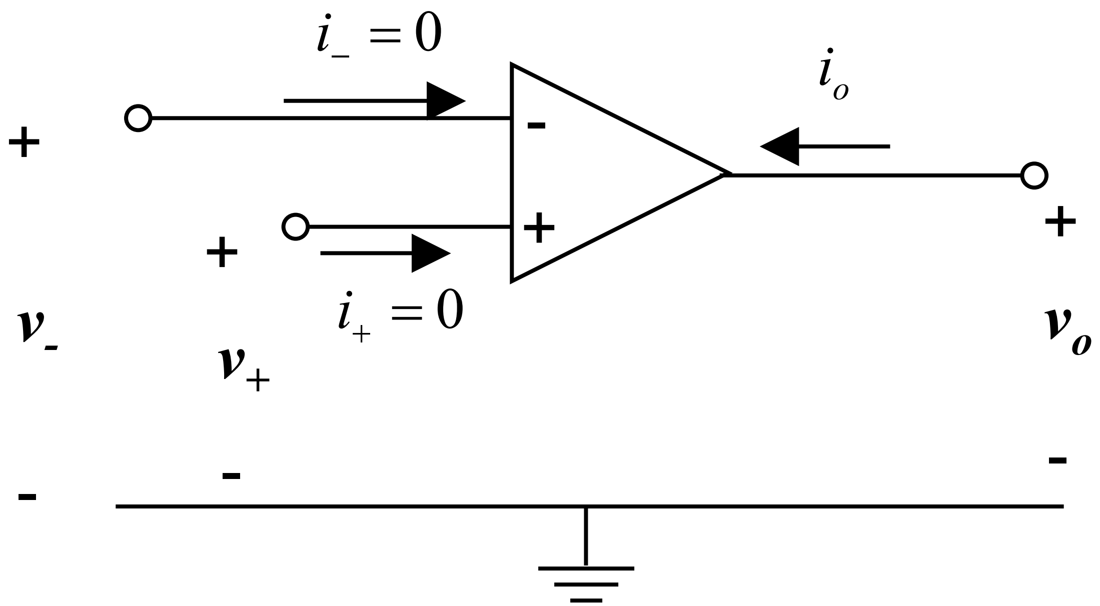
- For an op-amp to be linear:

$$|v_o| \le v_{sat} \qquad |i_o| \le i_{sat} \qquad \left|\frac{dv_o}{dt}\right| \le SR$$

  - What is v_{sat}?
  - What is SR (slew rate)?

Virtual Short / Virtual Open

- Virtual short / virtual open principles *(Textbook p.151)*.



1. Virtual open: $i_- = 0, i_+ = 0$
2. Virtual short: $v_+ = v_-$

- Note: i_o is not equal to zero!
- Op-Amp must function in the linear region.

Op-Amp Properties

→ **Good:**

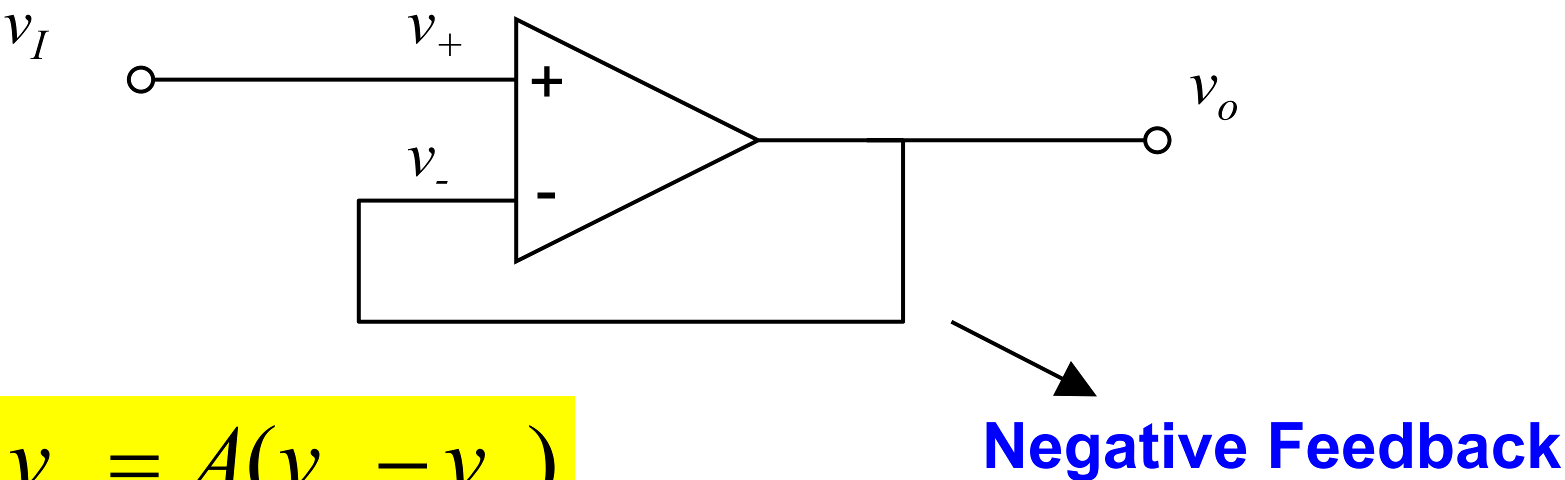
1. High input impedance.
2. Low output impedance.
3. Large gain.

→ **Bad:**

The properties of the op-amp (such as its gain A) are strongly dependent on process variations, temperature variations, etc.

➡ Use closed loop, ***negative feedback*** configuration.

Voltage Follower



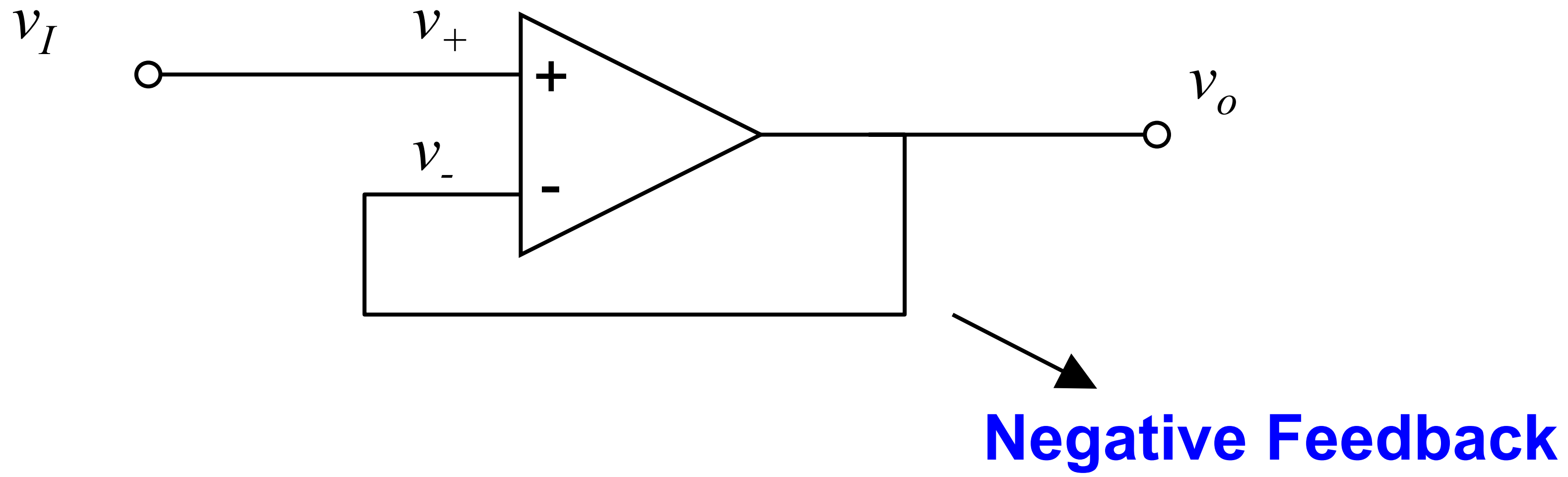
$$v_o = A(v_+ - v_-)$$

$$v_+ = v_I \qquad v_- = v_o$$

$$v_o = A(v_I - v_o) = Av_I - Av_o$$

$$\frac{v_o}{v_I} = \frac{A}{A+1} \cong 1$$

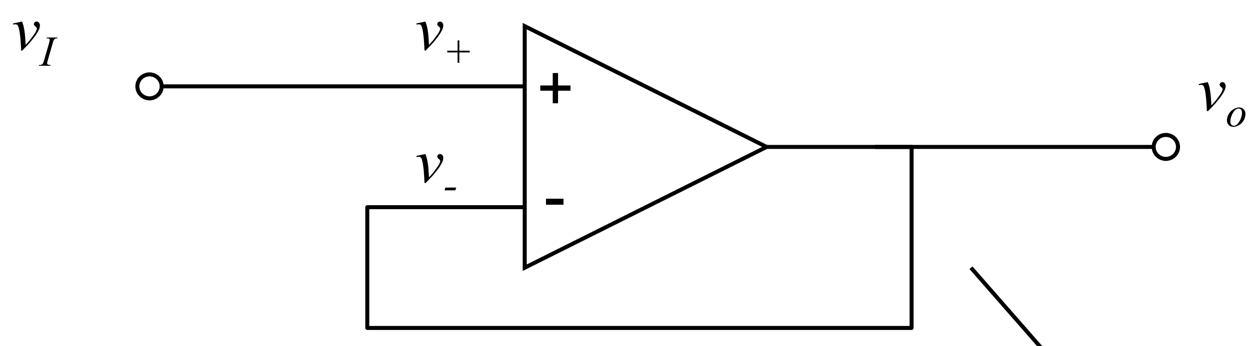
Voltage Follower



Alternatively use virtual short principle for ideal op-amps:

$$v_+ = v_- \Rightarrow v_o = v_I$$

Effect of Feedback on Noise



Noise

Negative Feedback

$$v_o = A(v_+ - v_-) + v_N$$

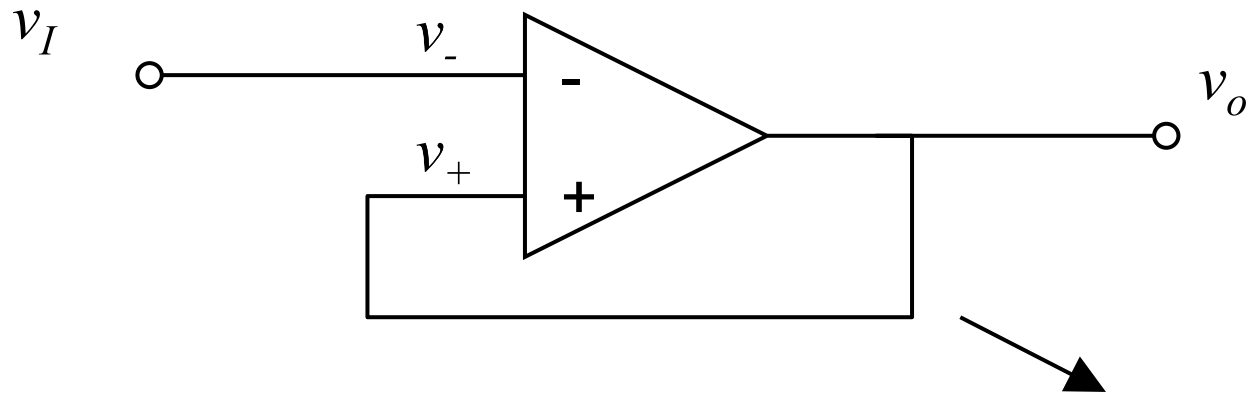
$$v_+ = v_I \qquad v_- = v_o$$

$$v_o = A(v_I - v_o) + v_N = Av_I - Av_o + v_N$$

$$v_o = \frac{A}{A+1} v_I + \frac{v_N}{A+1}$$

Noise reduced by a factor of 1+A

Positive Feedback



Positive Feedback!

$$v_o = A(v_+ - v_-)$$

$$v_+ = v_o \qquad v_- = v_I$$

$$v_o = A(v_o - v_I) = Av_o - Av_I$$

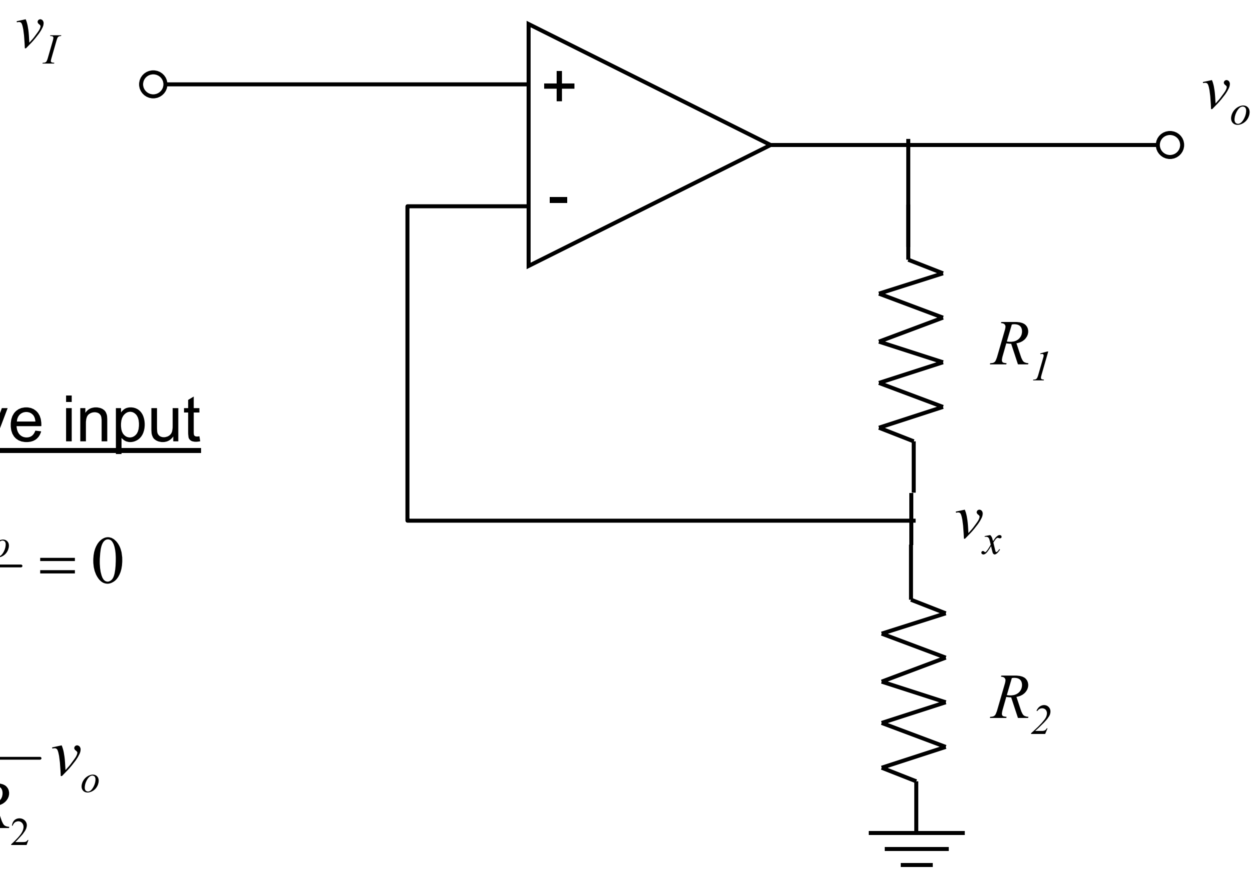
$$\frac{v_o}{v_I} = \frac{-A}{1-A} \cong 1$$

➡ Can be shown to be an unstable configuration.

Nodal Analysis of Op-Amp Circuits

1. Make use of the virtual short/virtual open principles.
2. Node voltages at the input are equal so one of them can be eliminated.
3. The currents at the input are zero and are involved in KCL equations at the ***input*** nodes.
4. The output current is ***not*** zero.
5. Make sure op-amp is in linear region so that virtual open/short principles can be applied (What happens when we have positive feedback?).

Example: Non-Inverting Amp



KCL at negative input

$$\frac{v_x}{R_2} + \frac{v_x - v_o}{R_1} = 0$$

$$v_x = \frac{R_2}{R_1 + R_2} v_o$$

Virtual short

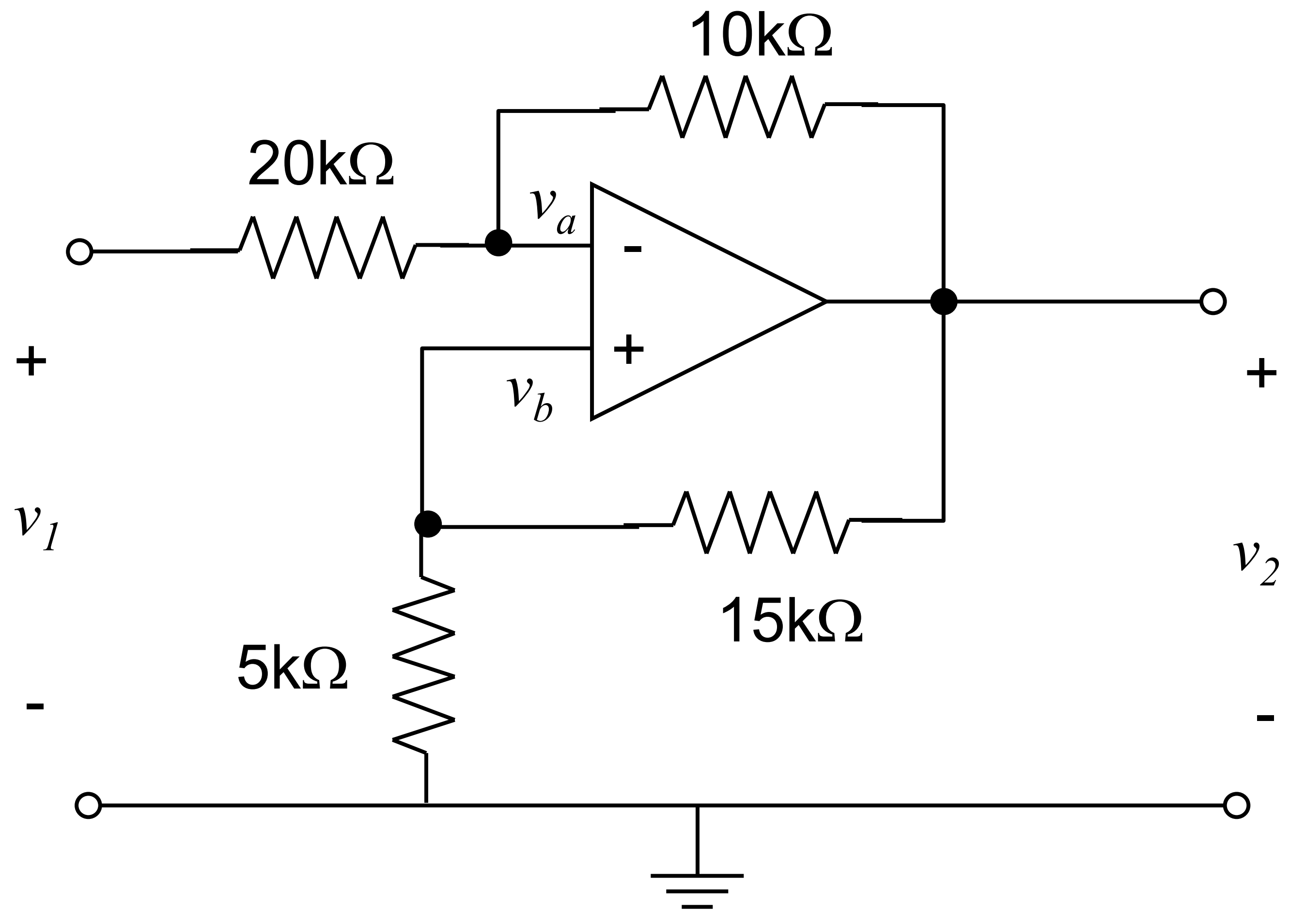
$$v_x = v_I \quad \Rightarrow \quad v_I = \frac{R_2}{R_1 + R_2} v_o$$

Typical Uses of OpAmps

1. Buffer circuit.
2. Voltage scaling.
3. Analog Computers (solution of differential equations.). OpAmp circuits can be used to perform mathematical operations (addition, subtraction, integration,etc.).
4. Negative resistor (active component).
5. Active filters.

Example: Inverting Amplifier

Textbook Exercise 4.8.3 (p.154)



Example: Inverting Amplifier

KCL at negative input

$$\frac{v_a - v_1}{20k} + \frac{v_a - v_2}{10k} + i_- = 0$$

$\rightarrow$ $\left\{ \begin{array}{l} i_- = 0 \\ \text{(Virtual open principle)} \end{array} \right.$

KCL at positive input

$$\frac{v_b}{5k} + \frac{v_b - v_2}{15k} + i_+ = 0$$

$\rightarrow$ $\left\{ \begin{array}{l} i_+ = 0 \\ \text{(Virtual open principle)} \end{array} \right.$

$v_a = v_b$ $\longrightarrow$ Virtual short principle

Example: Inverting Amplifier

First equation: → $$\frac{v_a}{20}+\frac{v_a}{10}=\frac{v_1}{20}+\frac{v_2}{10}=\frac{3v_a}{20}$$

Second equation: → $$\frac{v_b}{5}+\frac{v_b}{15}=\frac{v_2}{15}$$

$$4v_b=v_2$$

Third equation: → $$v_a=v_b$$

$$\frac{v_1}{20}+\frac{v_2}{10}=\frac{3v_2}{80}$$

$$v_2=-\frac{4}{5}v_1$$

Hints

1. Apply KCL at the input of the op-amp and take advantage of the virtual open principle.
2. Never apply KCL at the output node of the op-amp unless you are asked to calculate its output current.
3. Never apply KCL at the reference node. Remember we do not show the currents into the power supplies of the op-amp.
4. Make use of the virtual short principle (the input voltages of the op-amp are equal).