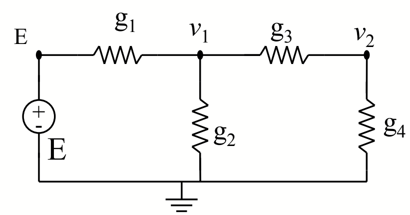
ECSE 210: Circuit Analysis

Lecture #3: Nodal & Mesh Analysis

Example with Voltage Source

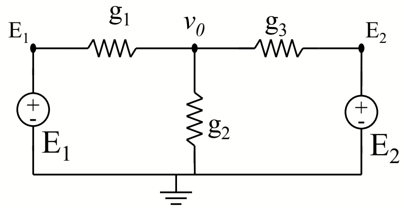


KCL at node 1: $(v_1\text{-E})g_1 + v_1 g_2 + (v_1\text{-}v_2)g_3 = 0$

KCL at node 2: $(v_2\text{-}v_1)g_3 + v_2 g_4 = 0$

- 2 equations, 2 unknowns, → solve
- The voltage source allowed us to save one equation (but only if we chose the right reference node or ground).

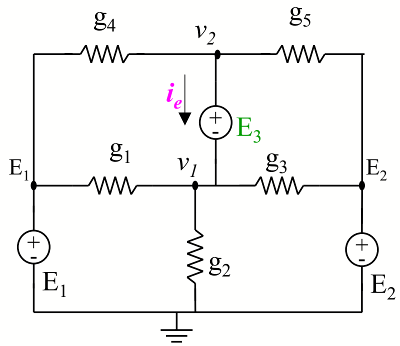
Example With Two Voltage Sources



KCL at node 0: $(v_0\text{-E}_1)g_1 + v_0\, g_2 + (v_0\ \text{-E}_2)g_3 = 0$

- 1 equations, 1 unknowns, → solve
- Try to choose the reference at a point where two or more voltages sources intersect. This eliminates more unknowns and therefore more equations.

Voltage Sources With No Common Nodes



→ One extra variable (the current in the voltages source).

→ One extra equation (the voltage relation across the voltage source).

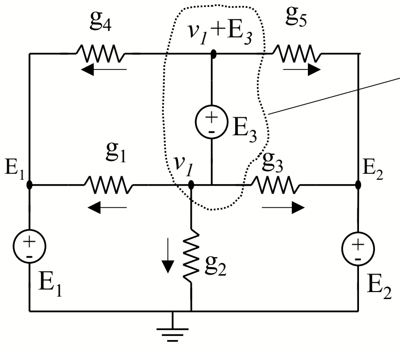
KCL at node 1: $(v_1\text{-E}_1)g_1 + v_1 g_2 + (v_1\text{-E}_2)g_3 - i_e = 0$

KCL at node 2: $(v_2\text{-E}_1)g_4 + (v_2\text{-E}_2)g_5 + i_e = 0$

Voltage source relation: $v_2 = v_1 + \text{E}_3$

3 equations
3 unknowns

Supernodes



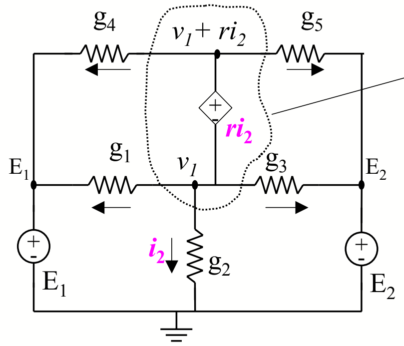
Supernode!

→ Apply KCL at supernode!
That is, sum the currents leaving the supernode.

KCL at supernode:

$$(v_1-E_1)g_1 + v_1g_2 + (v_1-E_2)g_3 + (v_1+E_3-E_1)g_4+(v_1+E_3-E_2)g_5=0$$

Dependent Voltage Source



Supernode!

→ Apply KCL at super node!
That is, sum the currents leaving the super node.

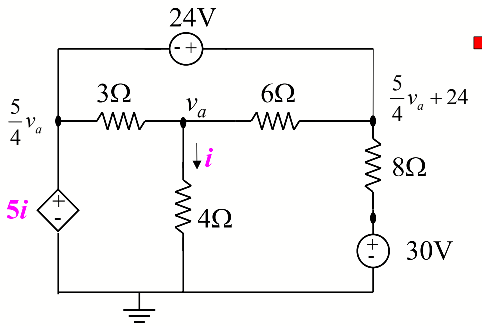
Note: $i_2=v_1g_2$; Controlled source = $ri_2 = rg_2v_1$

One unknown v_1 → solve!

KCL at super node:

$$(v_1-E_1)g_1 + v_1g_2 + (v_1-E_2)g_3 + (v_1+rg_2v_1-E_1)g_4+(v_1+rg_2v_1-E_2)g_5=0$$

Dependent Voltage Source: Example



Find i

Note: $i=\frac{v_a}{4}$

$$5i=\frac{5}{4}v_a$$

$$\frac{v_a-\frac{5}{4}v_a}{3}+\frac{v_a}{4}+\frac{v_a-(\frac{5}{4}v_a+24)}{6}=0$$

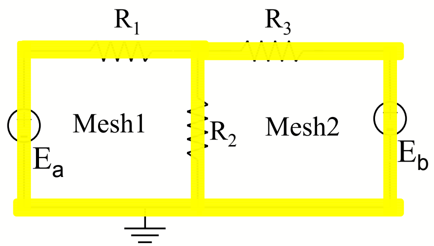
$$-\frac{1}{12}v_a+\frac{v_a}{4}-\frac{1}{24}v_a=4$$

$$3v_a=96$$

$$v_a=32V$$

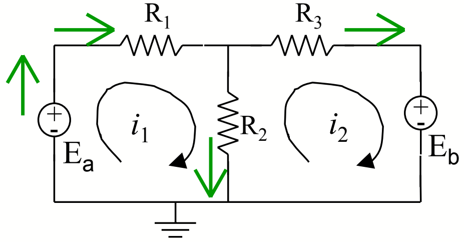
$$i=\frac{32}{4}=8A$$

What is a Mesh?



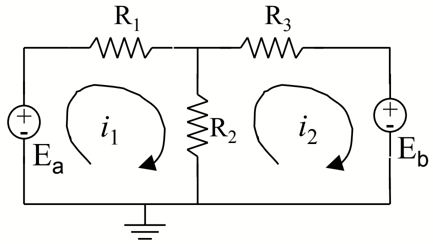
1. The circuit is divided into a collection of "smallest possible" loops.
2. Each small loop is a mesh.
3. A mesh is a loop that cannot be made smaller (i.e., divided into separate loops).
4. A mesh should not contain any elements inside it.
5. In this course we are restricted to "planar circuits".

Mesh Current



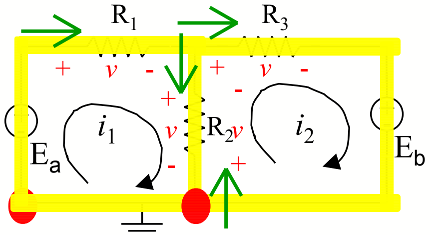
1. Each mesh is assigned a mesh current.
2. We will arbitrarily choose to define all mesh currents in a clockwise direction.
3. Do not confuse “mesh currents” and “branch currents.”

KVL equations



1. We choose to start from the lower left node and move clockwise.
2. We choose to add “voltages drops” across elements.
3. Be careful not to violate passive sign convention.
4. Again, do not confuse mesh currents with branch currents.

KVL equations



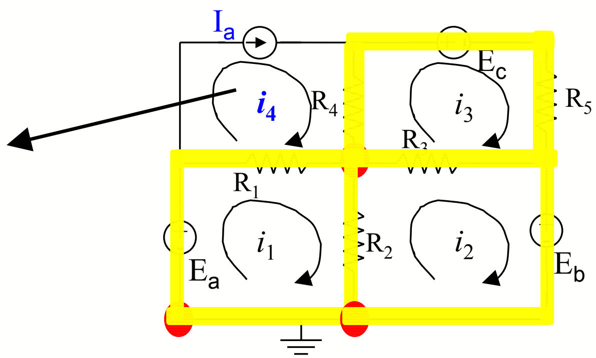
$-E_a + R_1 i_1 \quad + R_2(i_1 - i_2) \ = 0$

$R_2(i_2 - i_1) \quad + R_3 i_2 \quad + E_b \quad = 0$

➡ Two equations, two unknowns

Current Source

$i_4 = I_a$



KVL equations

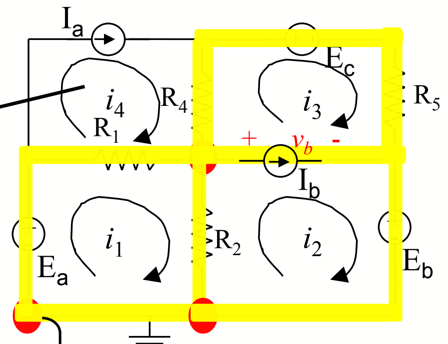
$-E_a + R_1(i_1 - i_4) \quad + R_2(i_1 - i_2) \ = 0$

$R_2(i_2 - i_1) \quad + R_3(i_2 - i_3) \quad + E_b \quad = 0$

$R_4(i_3 - i_4) \quad + E_c + R_5 i_3 \quad + R_3(i_3 - i_2) = 0$

3 equations, 3 unknowns
(i_4 is not an unknown!)

Current Source: Example 2



$i_4 = I_a$

KVL equations

$-E_a + R_1(i_1 - i_4) + R_2(i_1 - i_2) = 0$

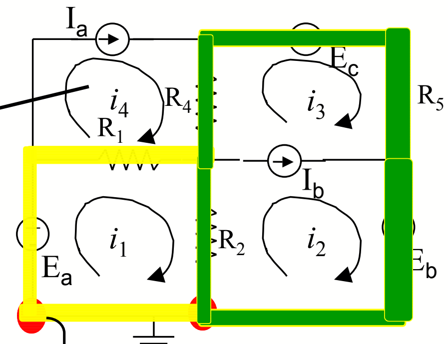
$R_2(i_2 - i_1) + v_b + E_b = 0$

$R_4(i_3 - i_4) + E_c + R_5 i_3 - v_b = 0$

$i_2 - i_3 = I_b$

4 equations
4 unknowns: i_1, i_2, i_3, v_b
(i_4 is not an unknown!)

Supermesh: Open Circuit Current Sources



$i_4 = I_a$

KVL equations

$-E_a + R_1(i_1 - i_4) + R_2(i_1 - i_2) = 0$

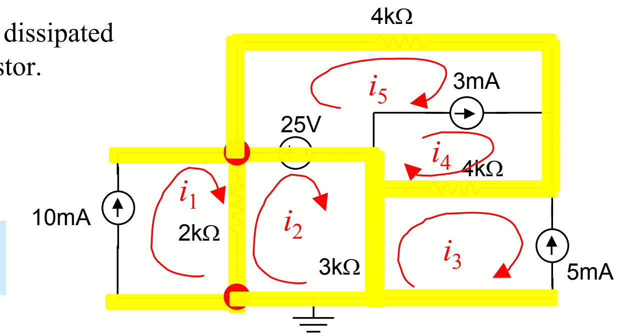
$R_2(i_2 - i_1) + R_4(i_3 - i_4) + E_c + R_5 i_3 + E_b = 0$

$i_2 - i_3 = I_b$

3 equations, 3 unknowns
(i_4 is not an unknown!)

Example

Find the power dissipated in the 3kΩ resistor.



Constraints
$i_1 = 10mA$
$i_3 = -5mA$
$i_4 - i_5 = 3mA$

$2(i_2 - i_1) + 25 + 3(i_2 - i_3) = 0$

$4i_5 + 4(i_4 - i_3) - 25 = 0$

$i_2 = -4mA$

$Power =$

$(i_2 - i_3)^2 R_3 = (1mA)^2 (3k\Omega) = 3mW$

Be careful with units.