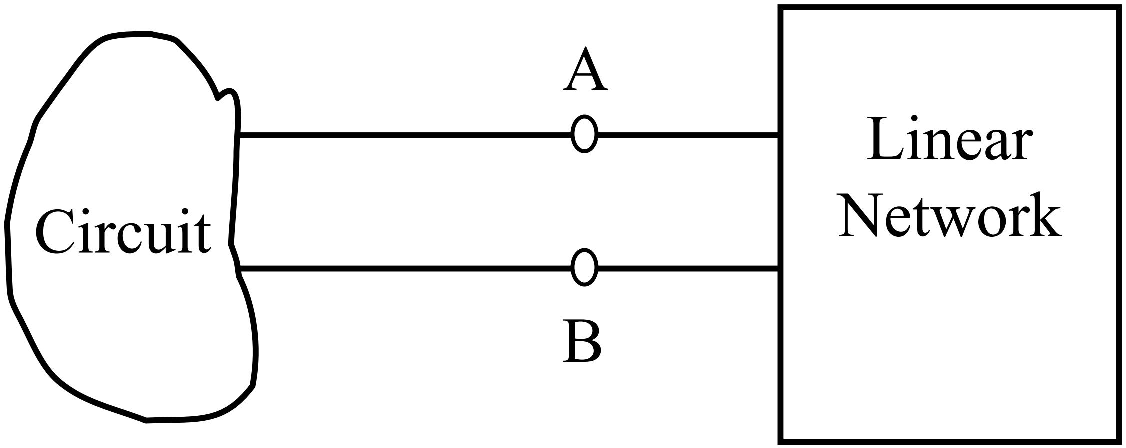
ECSE 210: Circuit Analysis

Lecture #29:

Two-Port Networks

Single-Port Network

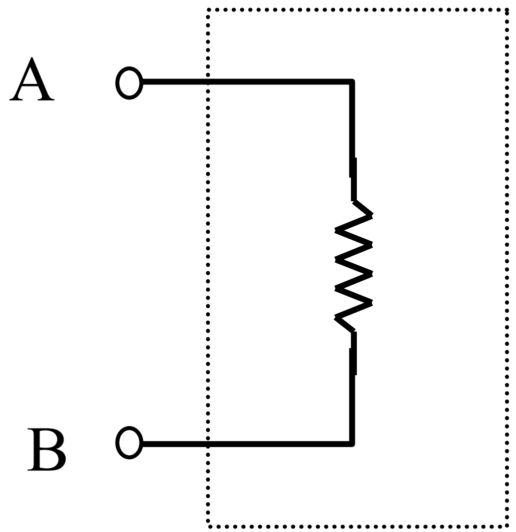


The linear network is connected to the circuit through a *single pair of terminals* (A-B), called a ***port***.

→ Network is called a ***single-port*** or ***one-port*** network.

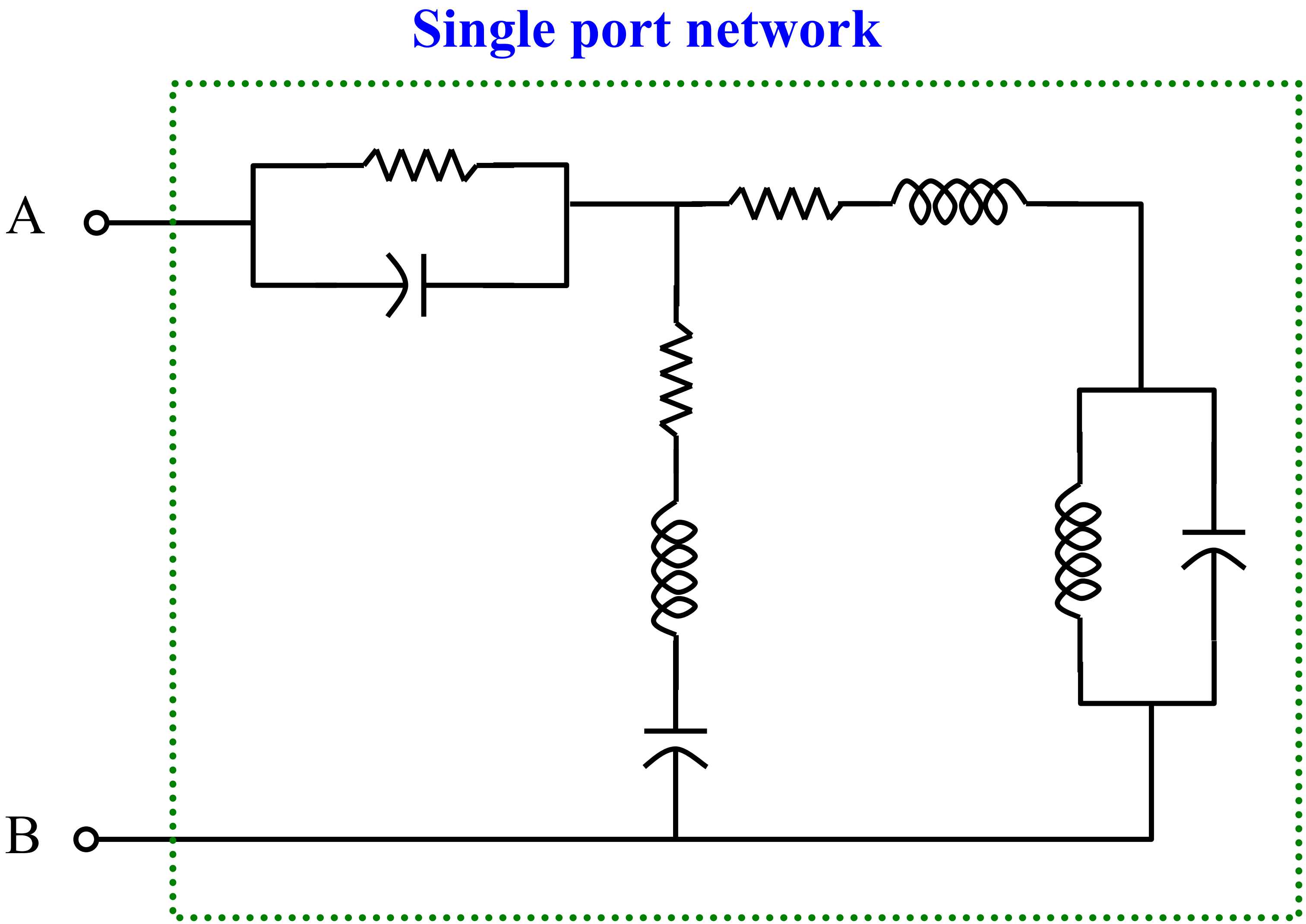
Example: Two Port Network

→ A single-port network may consist of a single circuit element (e.g., R, L or C) or a complex interconnection of such elements.

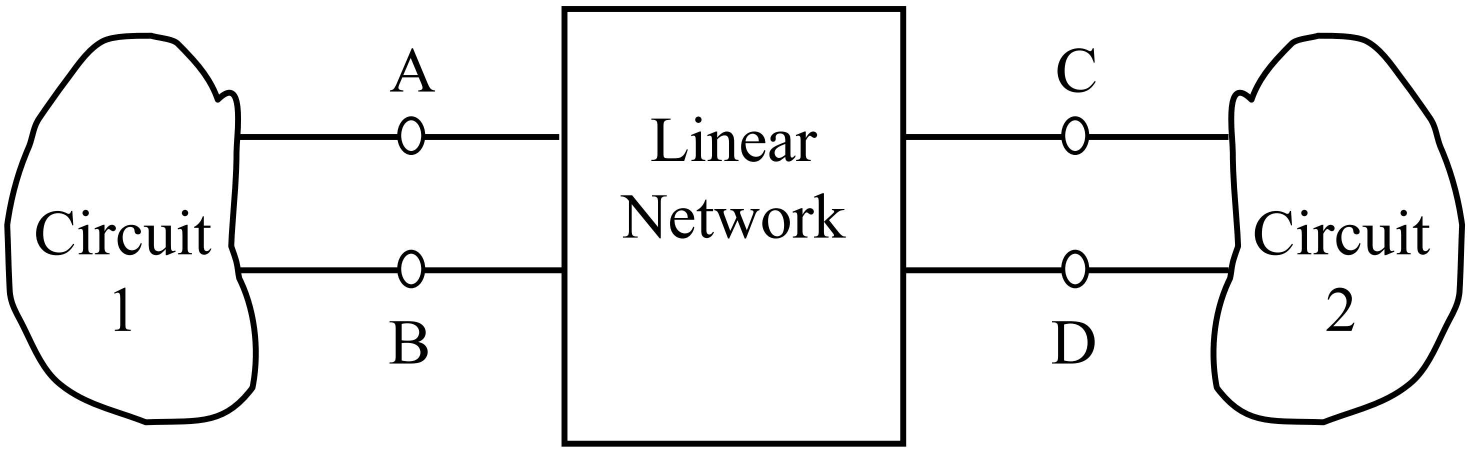


Single port network

Example: Single Port Network



Two-Port Network



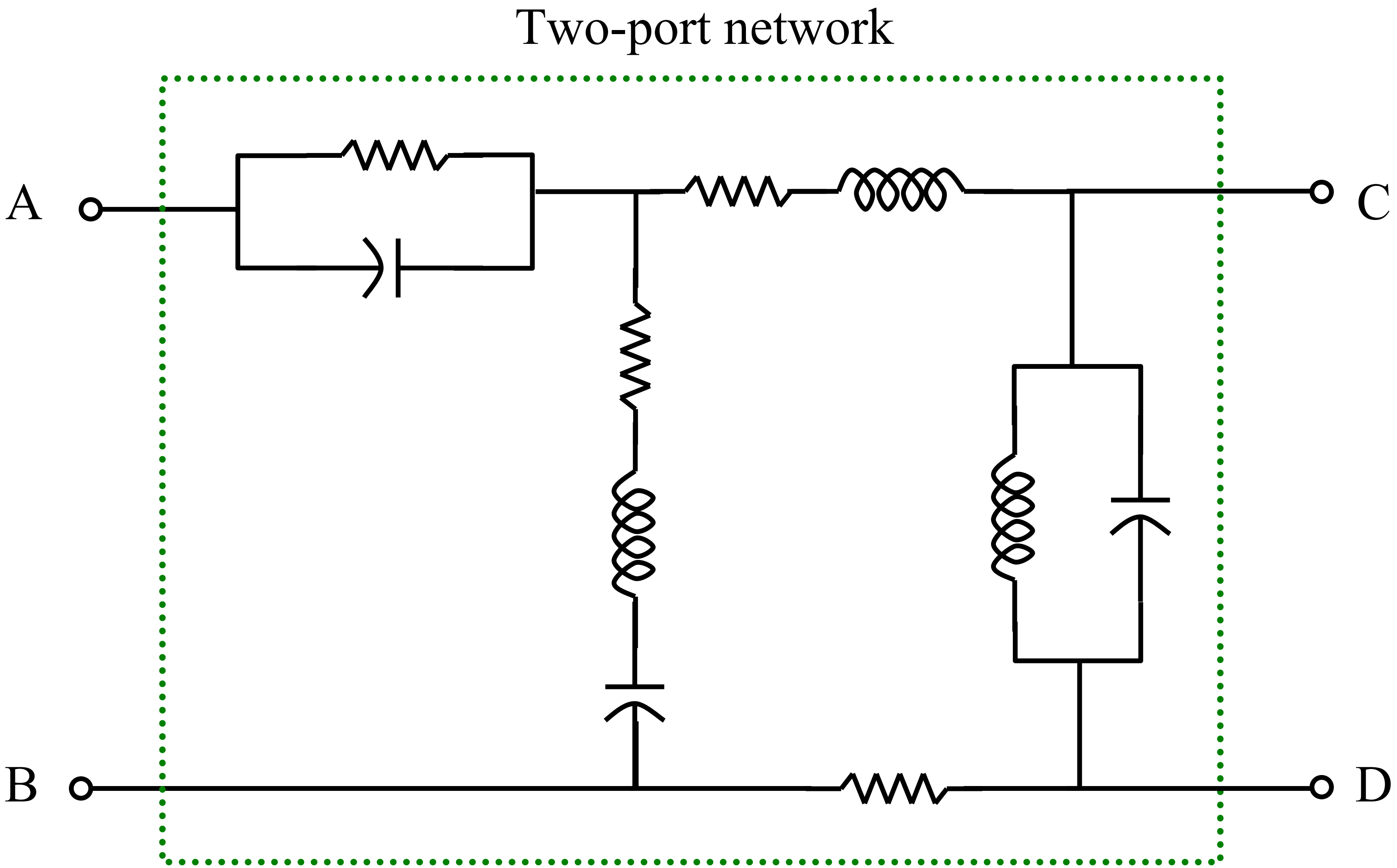
→ This linear network is called a ***two-port*** network.

→ Generally, the terminal pairs or *ports* are identified as:

A-B: ***input*** *port* or the linear network;

C-D: ***output*** *port* of the linear network.

Example: Two-Port Network



Two-Port Networks

→ Most practical circuits and systems have at least two ports (an input and an output).

→ Two-ports are used extensively in the modeling of many electronic devices and system components:

1. Transistors and OpAmps
2. Transformers and transmission lines

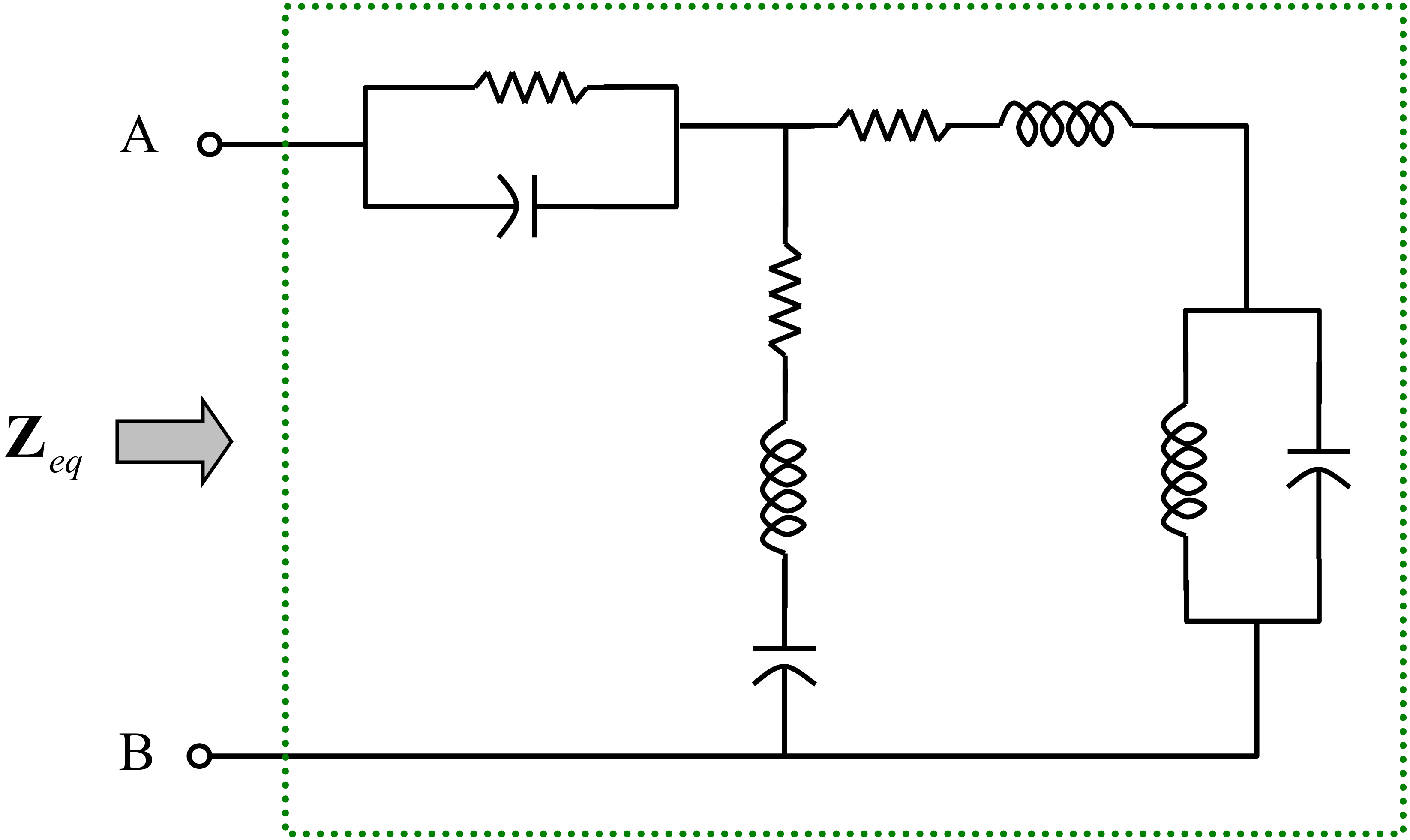
→ In general, a two-port linear network may contain any combination of ***R, L and C circuit elements, op-amps and controlled sources***.

 * **However, *independent* sources are excluded.**

→ The operation of a two-port network is fully described by the voltage-current relationships at the two ports, called the **two port-parameters**.

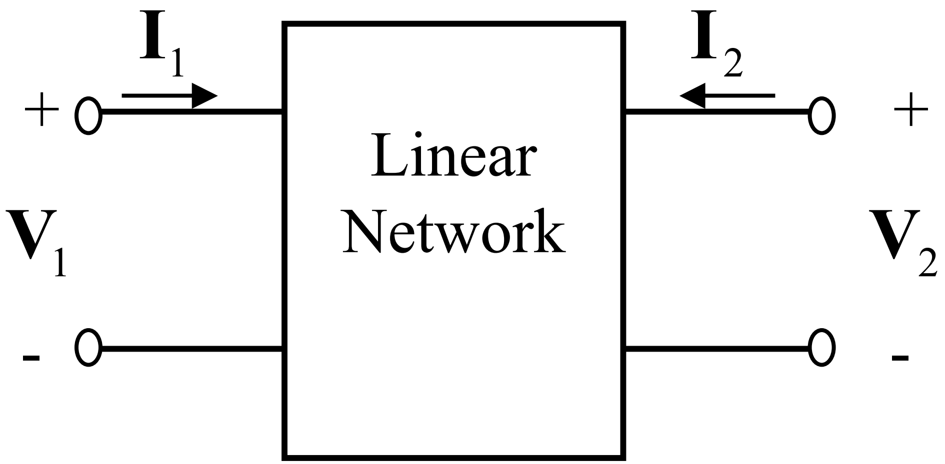
Two-Port vs. Single-Port Parameters

→ The **driving point impedance or admittance** (input impedance or admittance) completely describe the circuit.



Two-Port Admittance Parameters

→ Commonly referred to as the ***Y-parameters***.



→ Note the voltage polarities, and current directions.

→ Using the principle of superposition we get:

$$\mathbf{I}_1 = y_{11}\mathbf{V}_1 + y_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = y_{21}\mathbf{V}_1 + y_{22}\mathbf{V}_2$$

Y- Parameters

$$\mathbf{I}_1 = y_{11}\mathbf{V}_1 + y_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = y_{21}\mathbf{V}_1 + y_{22}\mathbf{V}_2$$

$$\Rightarrow \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

- → y_{ij} are complex constants of proportionality with units of siemens (S).
- → Two equations describe the two-port operation.
- → Once y_{11} , y_{12} , y_{21} , y_{22} are known, the input/output operation of the two-port network is ***completely*** defined.
- → y_{ij} are the **admittance parameters** or Y-parameters.
- → The Y-parameter matrix is:

$$\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

Y- Parameters

$$\mathbf{I}_1 = y_{11}\mathbf{V}_1 + y_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = y_{21}\mathbf{V}_1 + y_{22}\mathbf{V}_2$$

$$\Rightarrow \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\mathbf{V}_2 = 0 \quad \rightarrow \quad \mathbf{I}_1 = y_{11}\mathbf{V}_1 \quad \rightarrow \quad y_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$$

$$\mathbf{V}_1 = 0 \quad \rightarrow \quad \mathbf{I}_1 = y_{12}\mathbf{V}_2 \quad \rightarrow \quad y_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{V}_2 = 0 \quad \rightarrow \quad \mathbf{I}_2 = y_{21}\mathbf{V}_1 \quad \rightarrow \quad y_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$$

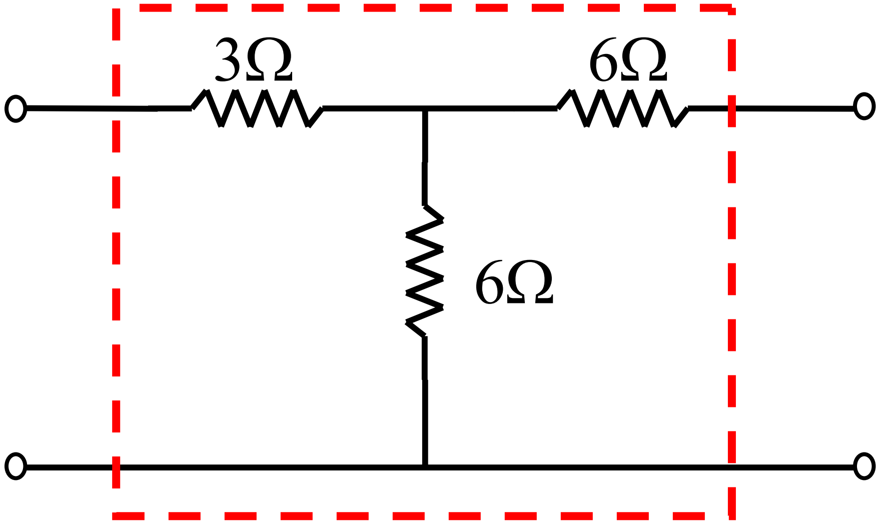
$$\mathbf{V}_1 = 0 \quad \rightarrow \quad \mathbf{I}_2 = y_{22}\mathbf{V}_2 \quad \rightarrow \quad y_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

Example

→ **Find the Y-parameters for the two-port below.**

$$\mathbf{I}_1 = y_{11}\mathbf{V}_1 + y_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = y_{21}\mathbf{V}_1 + y_{22}\mathbf{V}_2$$

$$\Rightarrow \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

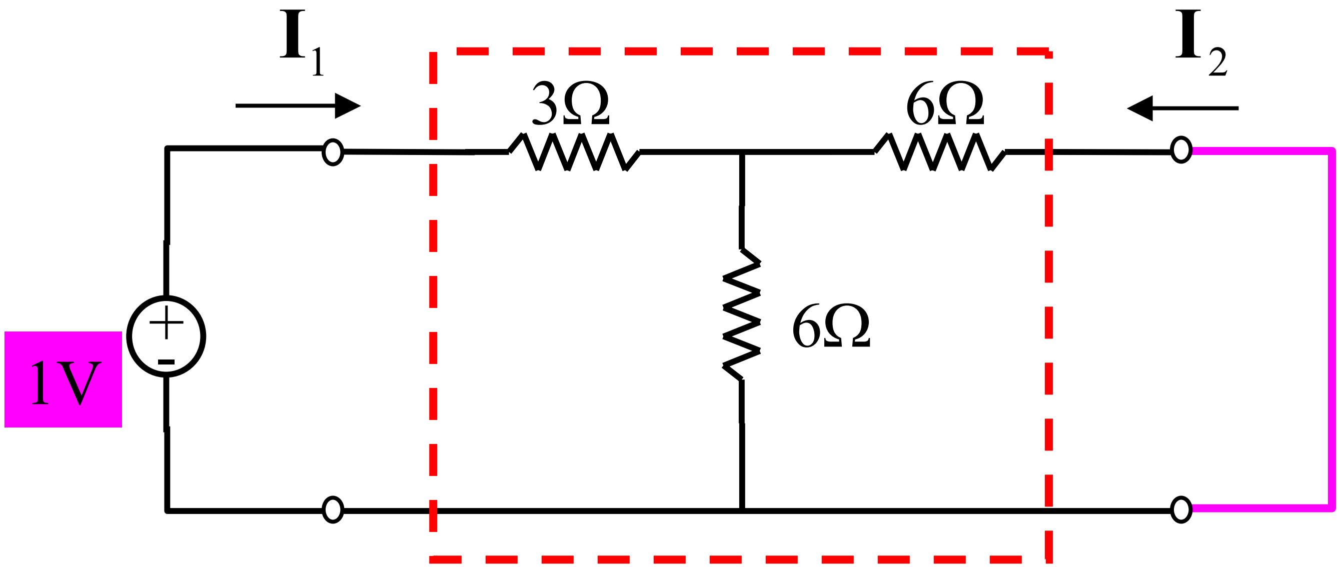


Example

→ **Y-parameters can be found "experimentally"**

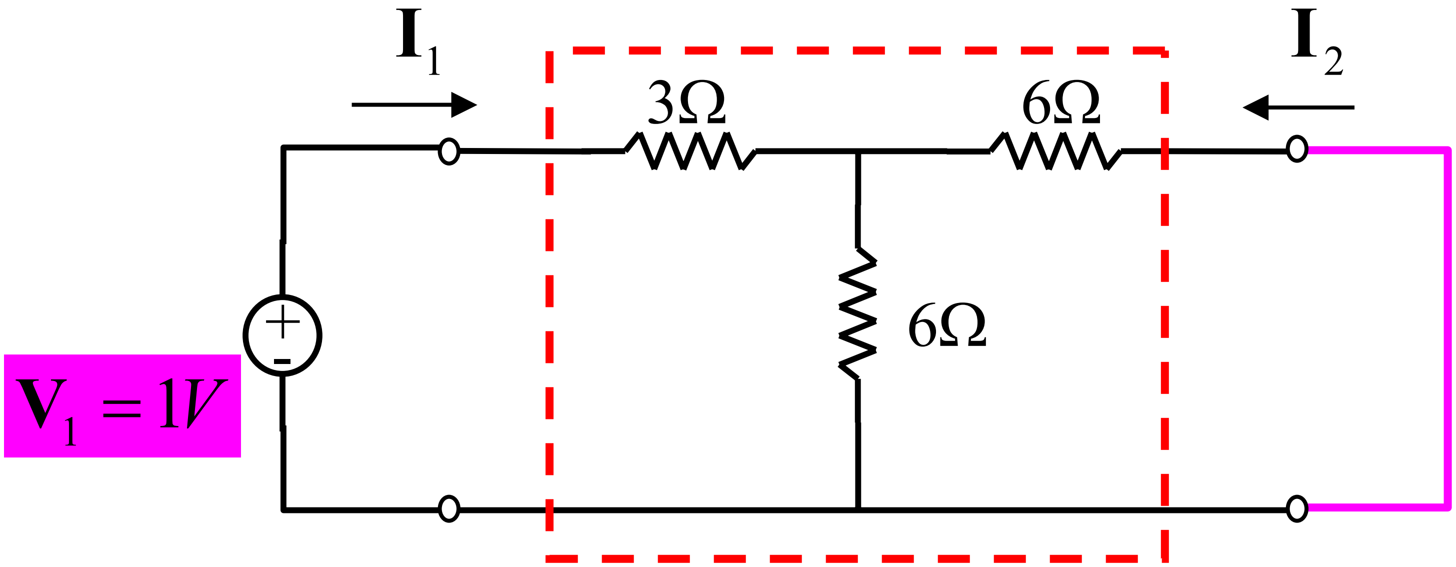
$$\mathbf{I}_1 = y_{11}\mathbf{V}_1 + y_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = y_{21}\mathbf{V}_1 + y_{22}\mathbf{V}_2$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$



From the above equations: $\mathbf{I}_1 = y_{11}(1V)$ $\mathbf{I}_2 = y_{21}(1V)$

Example



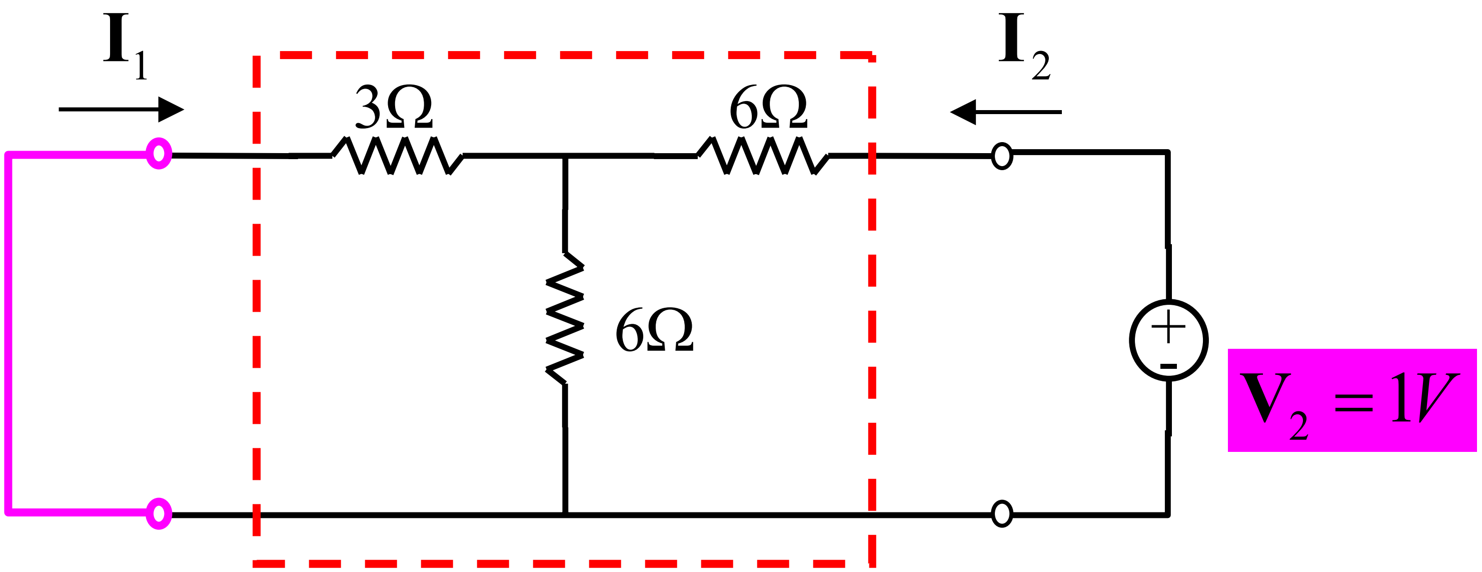
$$\mathbf{V}_1 = (3 + 6 \,||\, 6)\mathbf{I}_1 = 6\mathbf{I}_1$$

$$y_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = \frac{1}{6} S$$

$$\mathbf{I}_2 = -\frac{6}{6+6}\mathbf{I}_1 = -\frac{1}{12}\mathbf{V}_1$$

$$y_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = -\frac{1}{12} S$$

Example



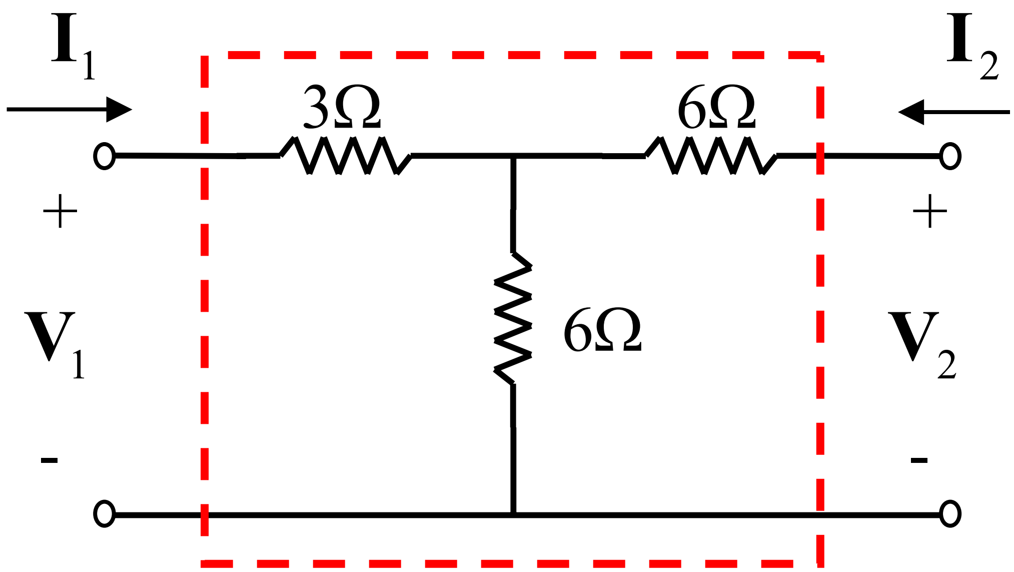
$$\mathbf{V}_2 = (6 + 3 \,||\, 6)\mathbf{I}_2 = 8\mathbf{I}_2$$

$$y_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = \frac{1}{8} S$$

$$\mathbf{I}_1 = -\frac{6}{6+3}\mathbf{I}_2 = -\frac{1}{12}\mathbf{V}_2$$

$$y_{21} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = -\frac{1}{12} S$$

Example



$$I_1 = \frac{1}{6}V_1 - \frac{1}{12}V_2$$

$$I_2 = -\frac{1}{12}V_1 + \frac{1}{8}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{1}{8} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Example 2

π-network