# ECSE 210: Circuit Analysis

## Lecture #22:

## Frequency Response

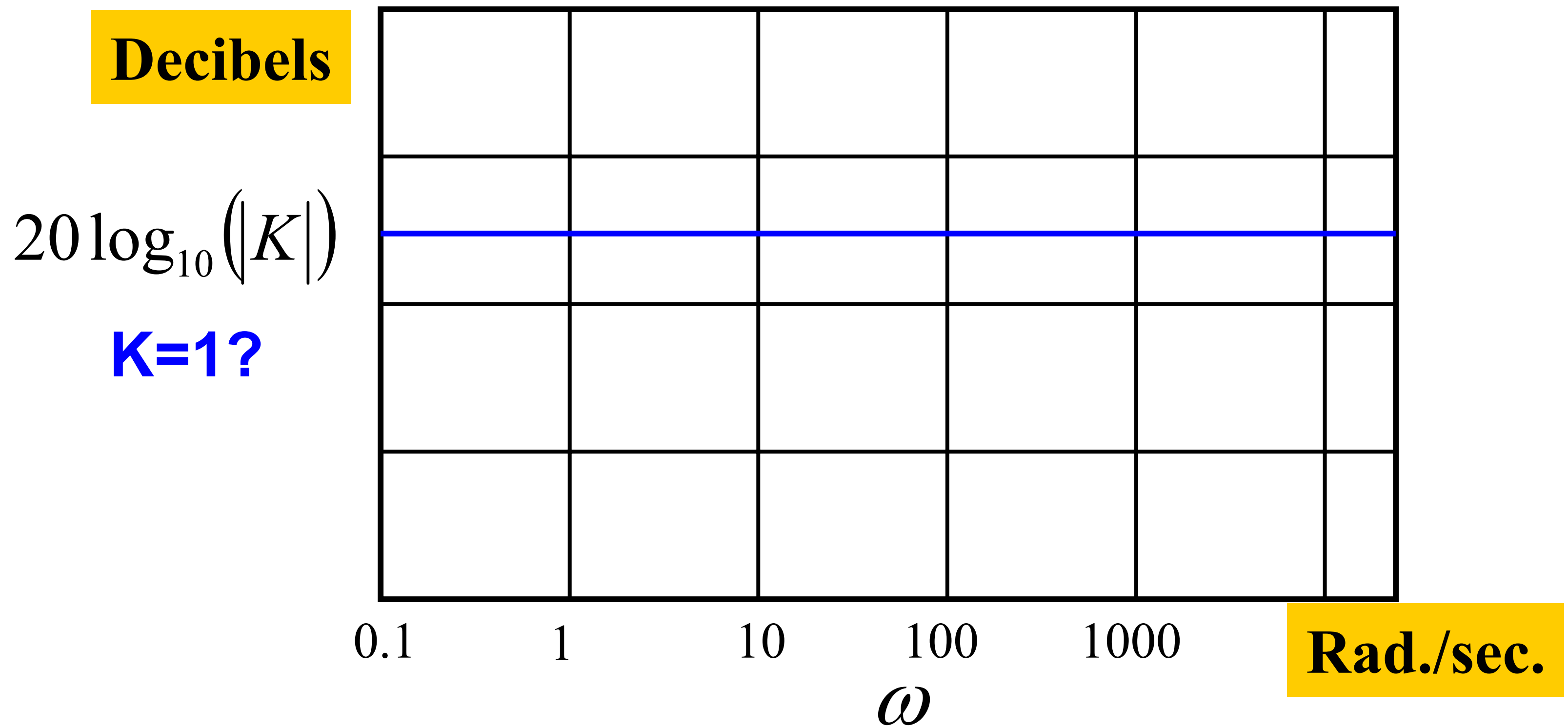
## Bode Plots

# Bode Plots: Constant Factor

**Constant factor:** $\mathbf{H}(\mathbf{s}) = K$

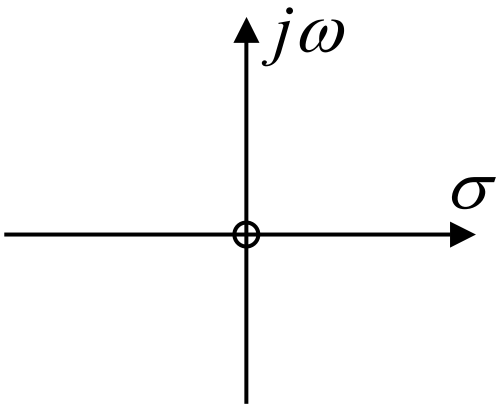
$$A(dB) = 20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = 20\log_{10}\left(\left|K\right|\right) = const.$$

$$\angle\mathbf{H}(\mathbf{s}) = 0^{o}$$



# Bode Plots: Zero at the Origin

## Zeros at the origin




Single zero $\mathbf{H}(\mathbf{s}) = \mathbf{s}$

Two zeros $\mathbf{H}(\mathbf{s}) = \mathbf{s}^2$

Multiple zeros $\mathbf{H}(\mathbf{s}) = \mathbf{s}^N$

(zeros with *multiplicity* N)

$$A(dB) = 20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = 20\log_{10}\left(\left|(j\omega)^N\right|\right) = 20N\log_{10}(\omega)$$

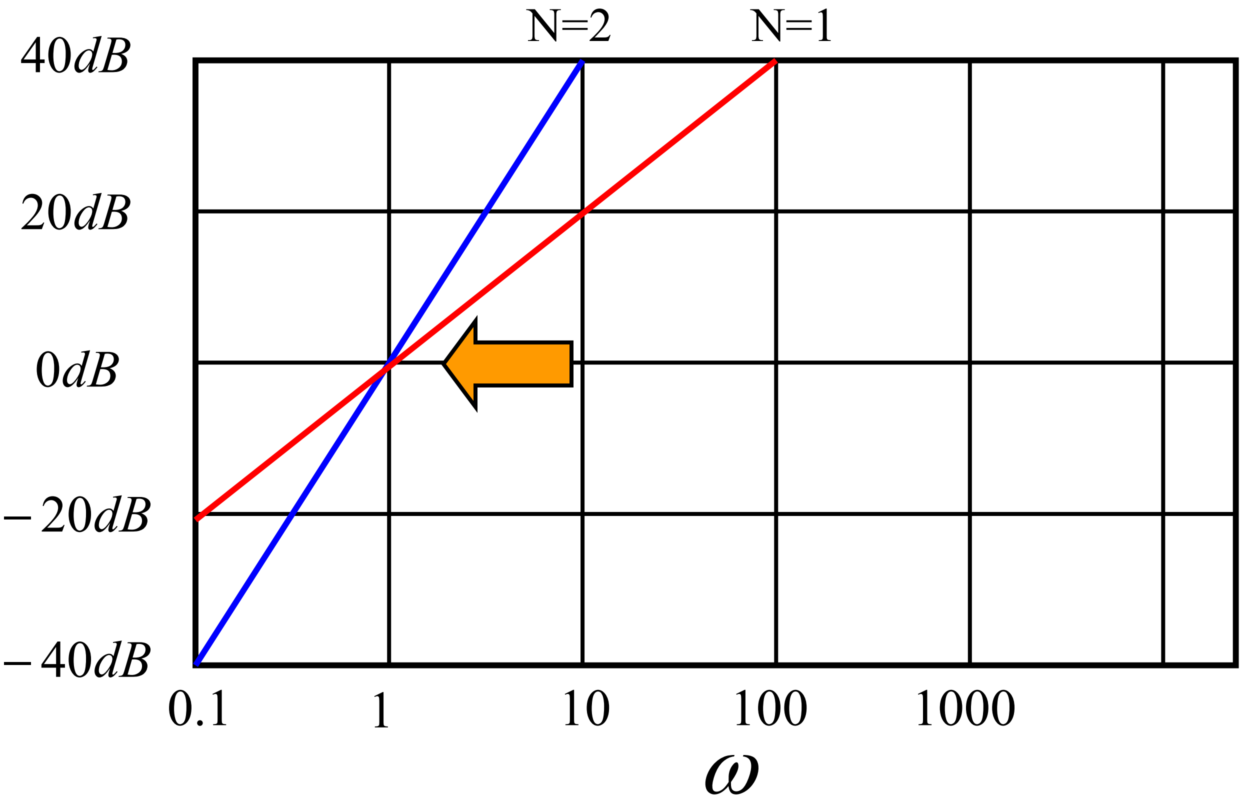
$$\angle\mathbf{H}(j\omega) = N90^o$$

# Bode Plots

$$\mathbf{H(s)} = \mathbf{s}^N$$

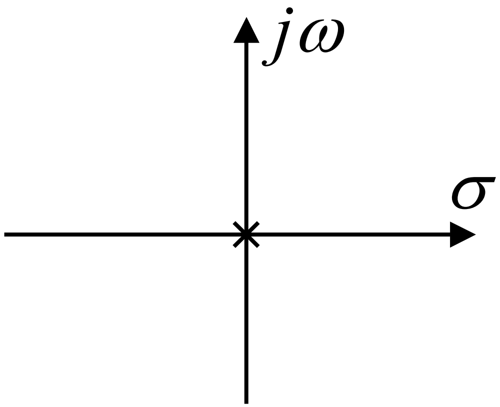
$$20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = 20\log_{10}\left(\left|(j\omega)^N\right|\right) = 20N\log_{10}(\omega)$$

Slope =20N dB/decade



# Bode Plots: Pole at the Origin

**Poles at the origin**




Single pole $\mathbf{H}(\mathbf{s}) = \dfrac{1}{\mathbf{s}}$

Two poles $\mathbf{H}(\mathbf{s}) = \dfrac{1}{\mathbf{s}^2}$

Multiple poles $\mathbf{H}(\mathbf{s}) = \dfrac{1}{\mathbf{s}^N}$

(poles with *multiplicity* N)

$$20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = 20\log_{10}\left(\frac{1}{\left|(j\omega)^N\right|}\right) = -20N\log_{10}(\omega)$$

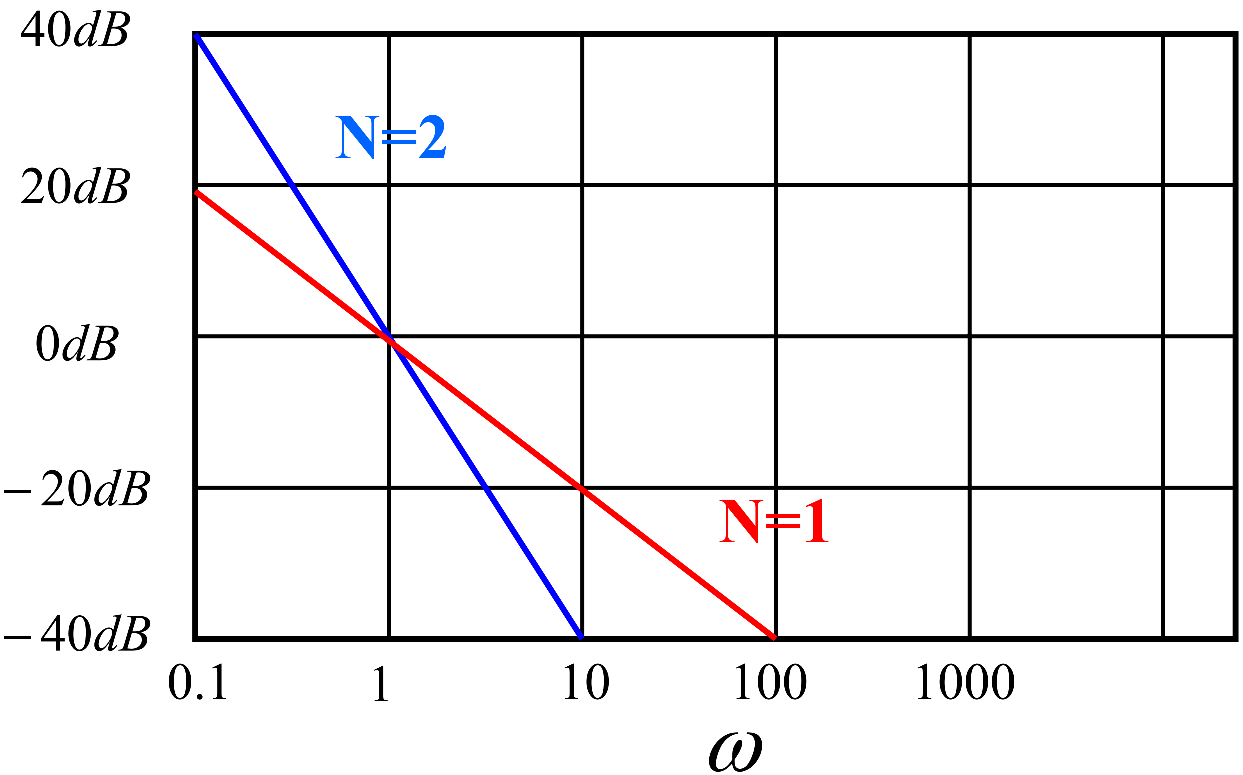
$$\angle\mathbf{H}(j\omega) = -N90^o$$

# Bode Plots

$$\mathbf{H}(\mathbf{s}) = \frac{1}{\mathbf{s}^N}$$

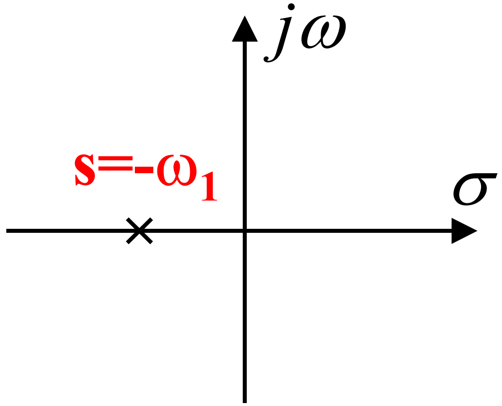
$$20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = 20\log_{10}\left(\frac{1}{\left|(j\omega)^N\right|}\right) = -20N\log_{10}(\omega)$$

Slope =-20N dB/decade



# Bode Plots: Real Pole

**Real poles**




Single pole: $$\mathbf{H}(\mathbf{s}) = \frac{1}{(\mathbf{s}+\omega_1)}$$

Pole of multiplicity N: $$\mathbf{H}(\mathbf{s}) = \frac{1}{(\mathbf{s}+\omega_1)^N}$$

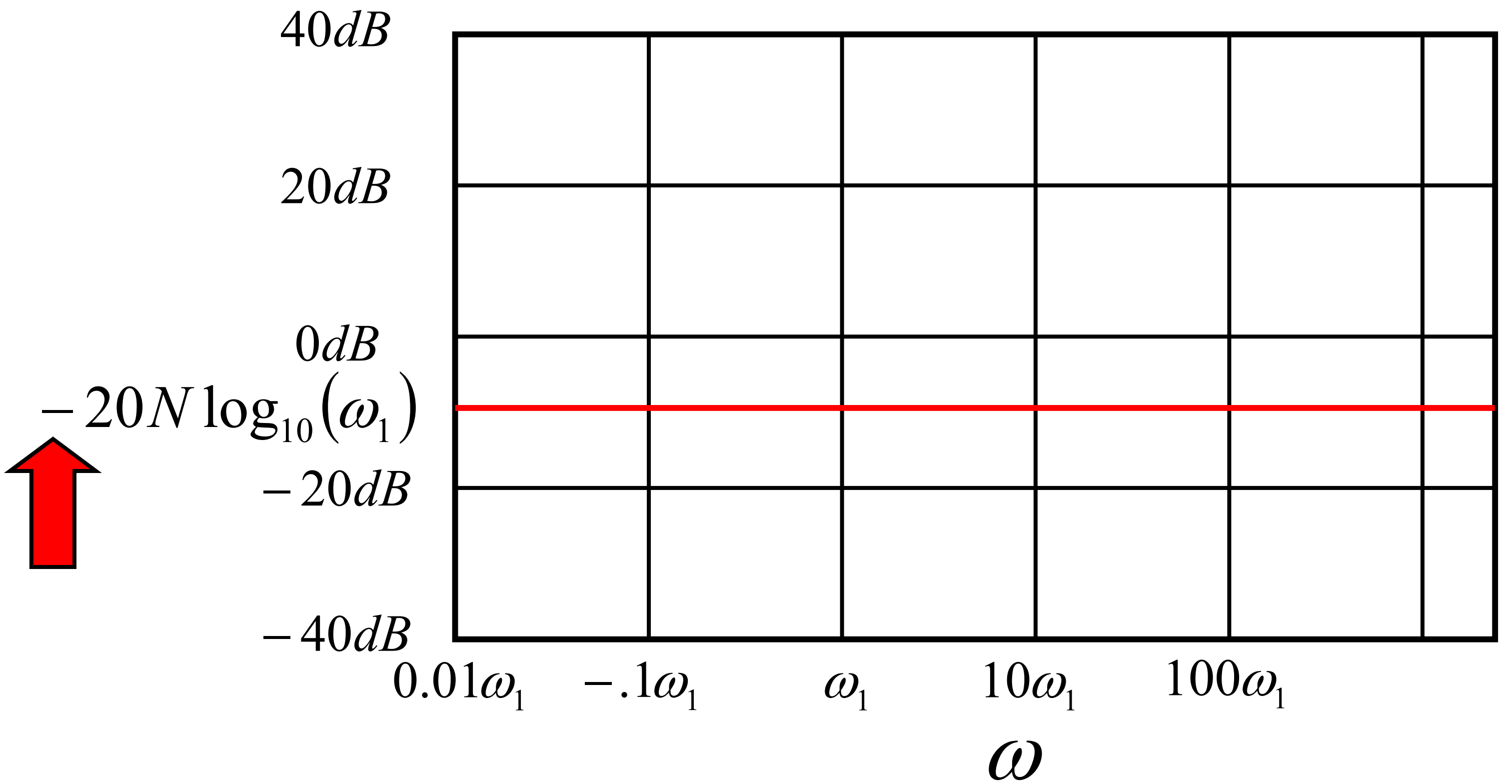
$$20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = 20\log_{10}\left(\frac{1}{\left|(j\omega+\omega_1)^N\right|}\right) = -20N\log_{10}\left(\left|j\omega+\omega_1\right|\right)$$

# Bode Plots: Low Frequency Asymptote

$\omega << \omega_1$ $\quad$ $\angle \mathbf{H(s)} \rightarrow 0$

$$20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = -20N\log_{10}\left(\left|j\omega+\omega_1\right|\right) = -20N\log_{10}(\omega_1)$$

$= constant$

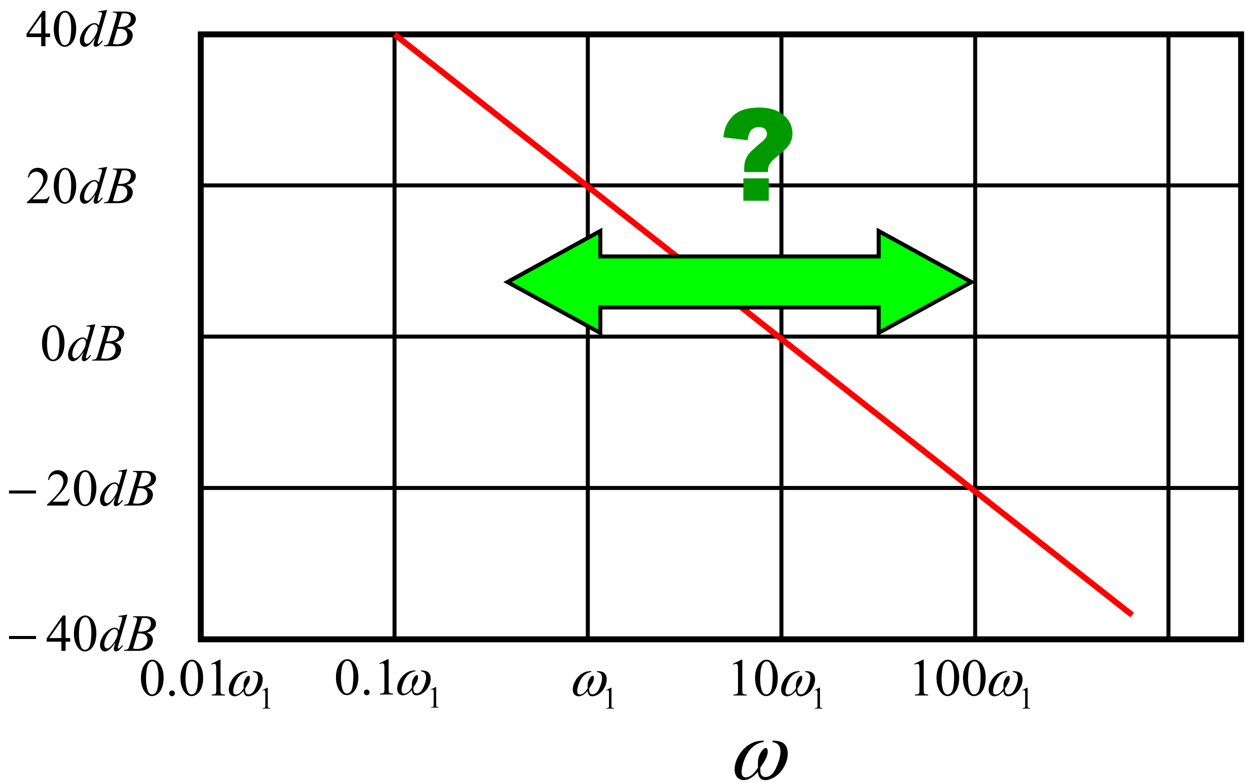


# Bode Plots: High Frequency Asymptote

$$\omega >> \omega_1 \qquad \angle \mathbf{H}(\mathbf{s}) \rightarrow -N90^{o}$$

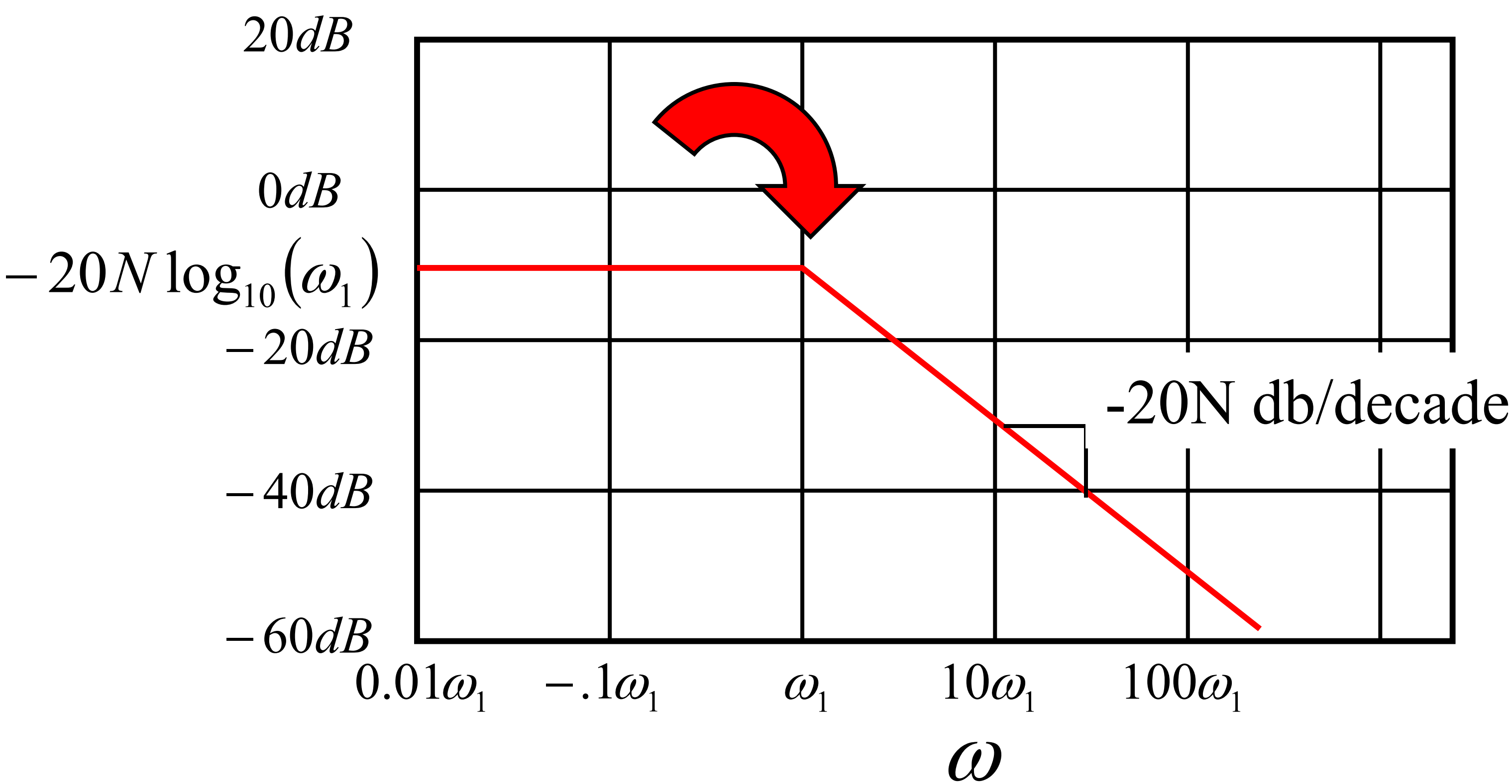
$$20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = -20N\log_{10}\left(\left|j\omega + \omega_1\right|\right) = -20N\log_{10}(\omega)$$

Slope = -20N dB/decade

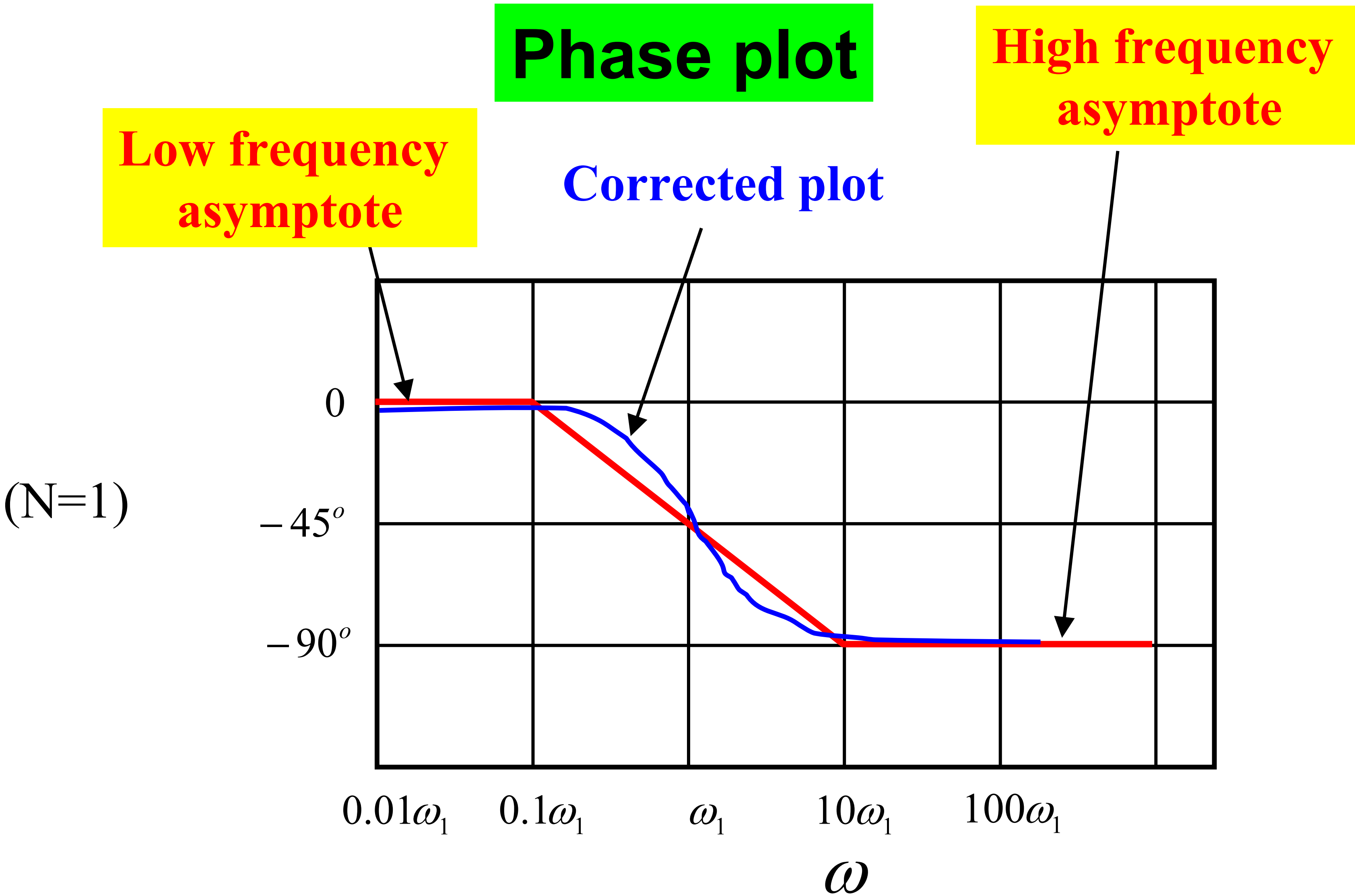


# Bode Plots: Real Pole

Note that the two lines resulting from the **"extreme cases"** intersect at $\omega = \omega_1$.

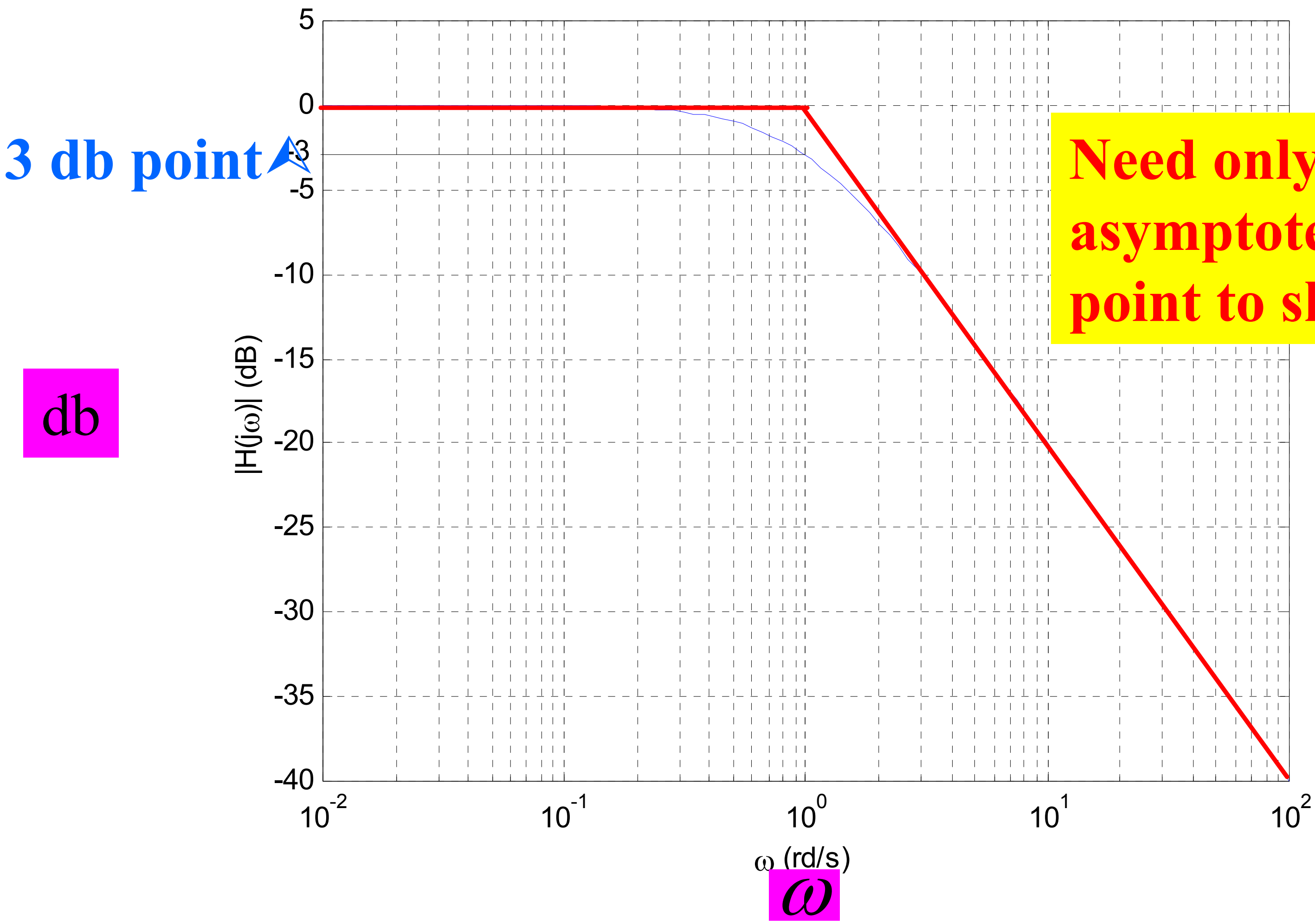


# Bode Plots: Real Pole



# Bode Plots: Magnitude

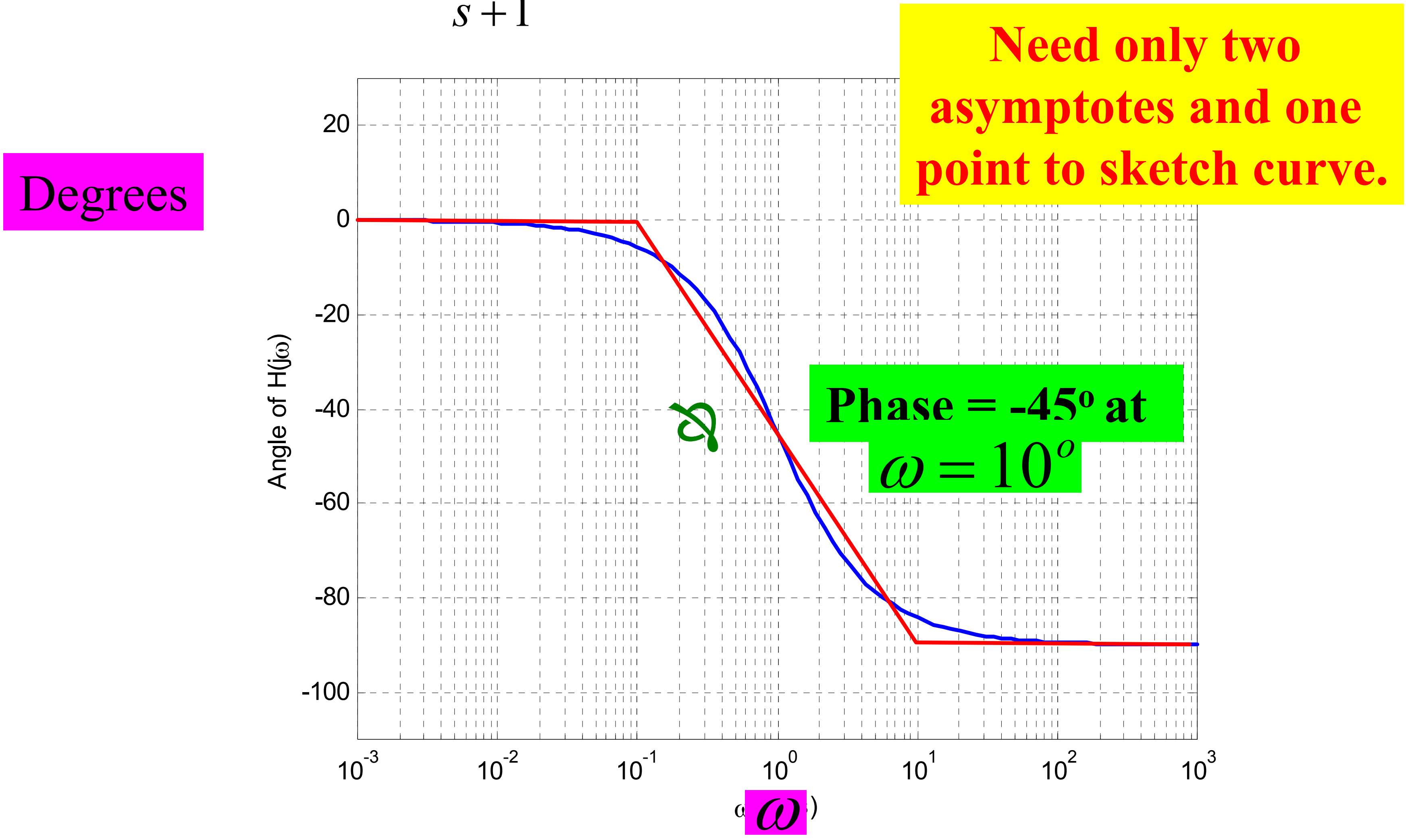
Example: $\mathbf{H}(s) = \dfrac{1}{s+1}$




**Need only two asymptotes and one point to sketch curve.**

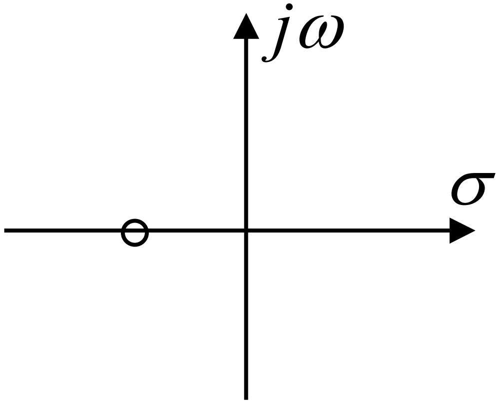
# Bode Plots: Phase

Example: $\mathbf{H}(s) = \dfrac{1}{s+1}$



# Bode Plots: Real Zero

**Real zeros**




Single zero $\mathbf{H}(s) = s + \omega_1$

Zero of multiplicity N $\mathbf{H}(s) = \left(s + \omega_1\right)^N$

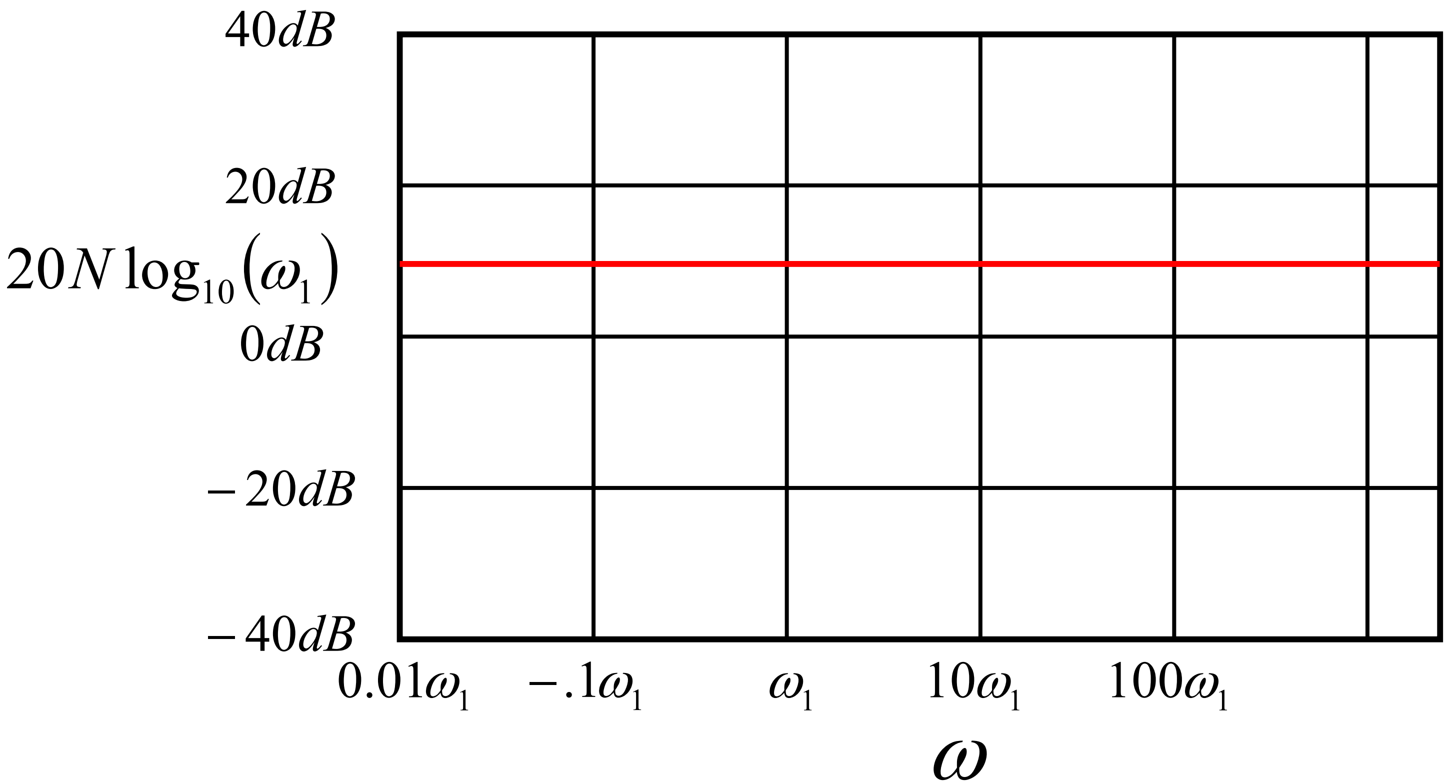
$$20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = 20\log_{10}\left(\left|(j\omega + \omega_1)^N\right|\right) = 20N\log_{10}\left(\left|j\omega + \omega_1\right|\right)$$

# Bode Plots: Low Frequency Asymptote

$$\omega << \omega_1 \qquad \angle \mathbf{H}(s) \to 0$$

$$20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = 20N\log_{10}\left(\left|j\omega + \omega_1\right|\right) = 20N\log_{10}(\omega_1)$$

$$= \textbf{constant}$$

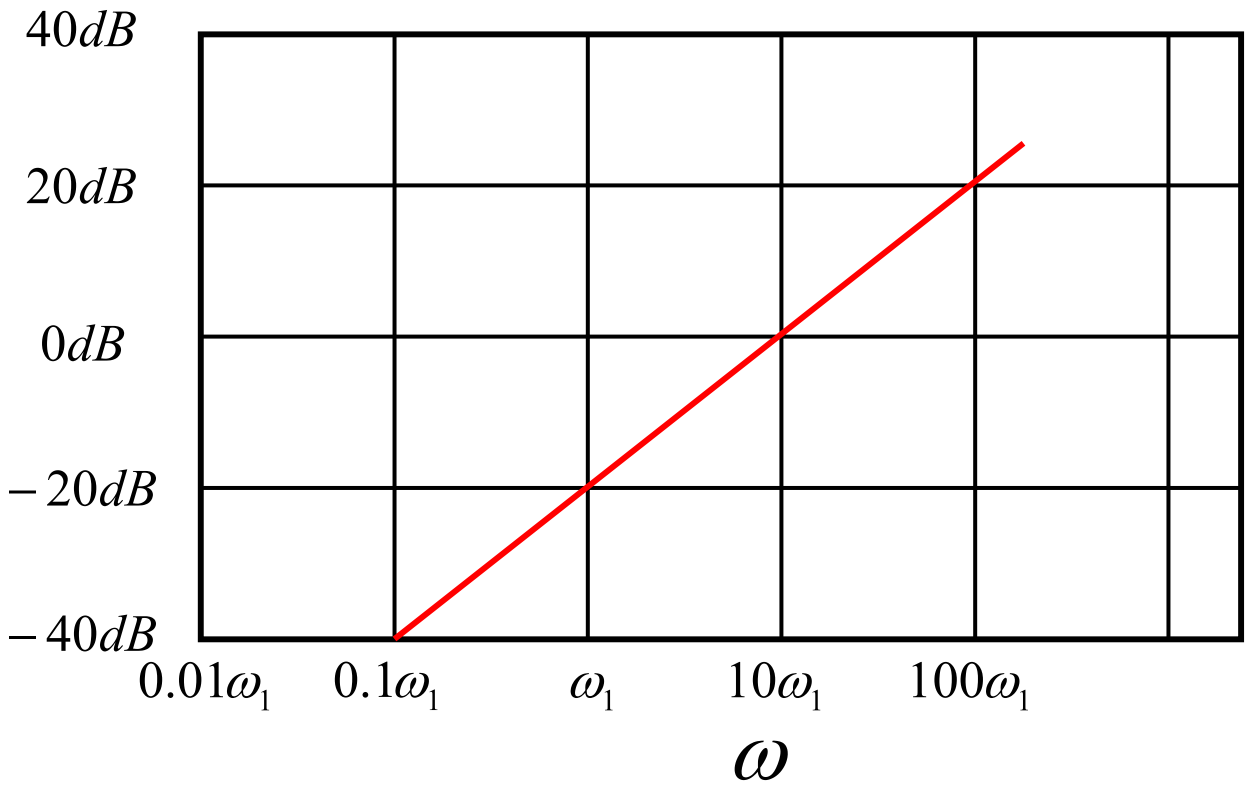


# Bode Plots: High Frequency Asymptote

$$\omega >> \omega_1 \qquad \angle \mathbf{H}(s) \rightarrow +N90^o$$

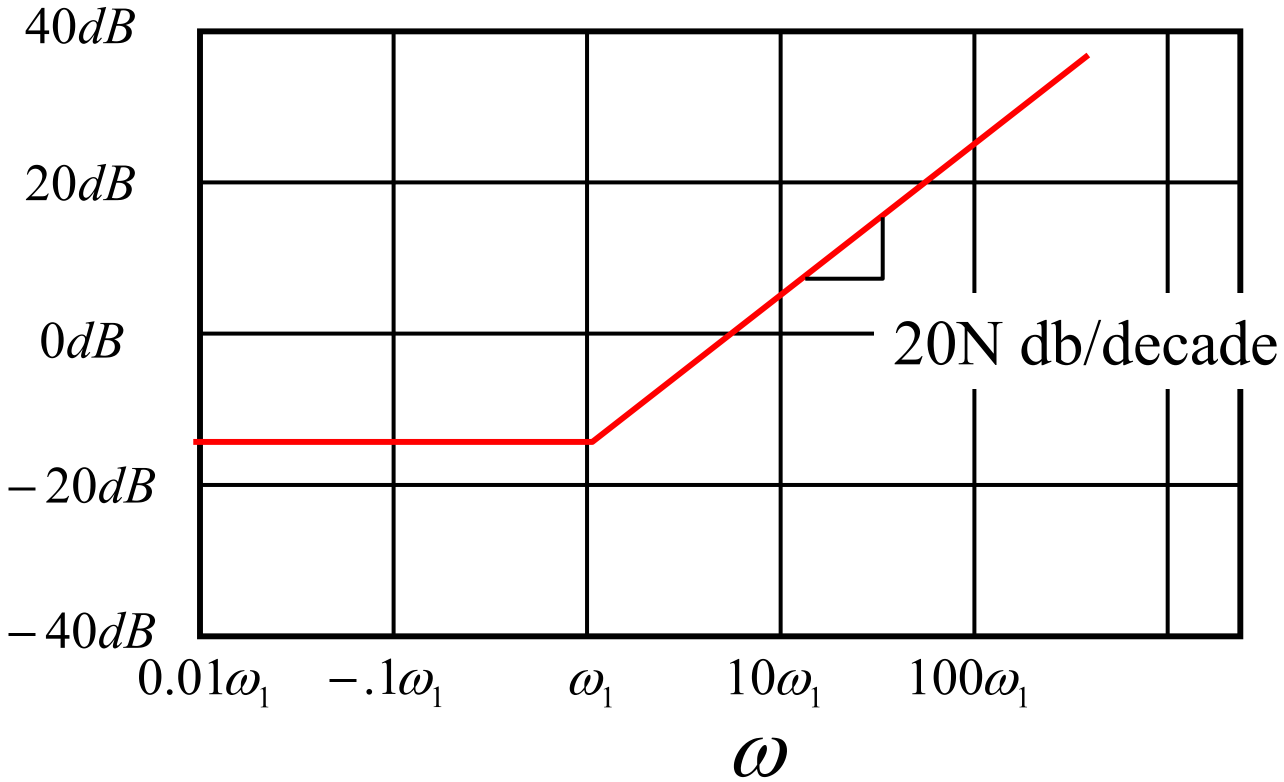
$$20\log_{10}\left(\left|\mathbf{H}(j\omega)\right|\right) = 20N\log_{10}\left(\left|j\omega + \omega_1\right|\right) = 20N\log_{10}(\omega)$$

Slope =20N dB/decade

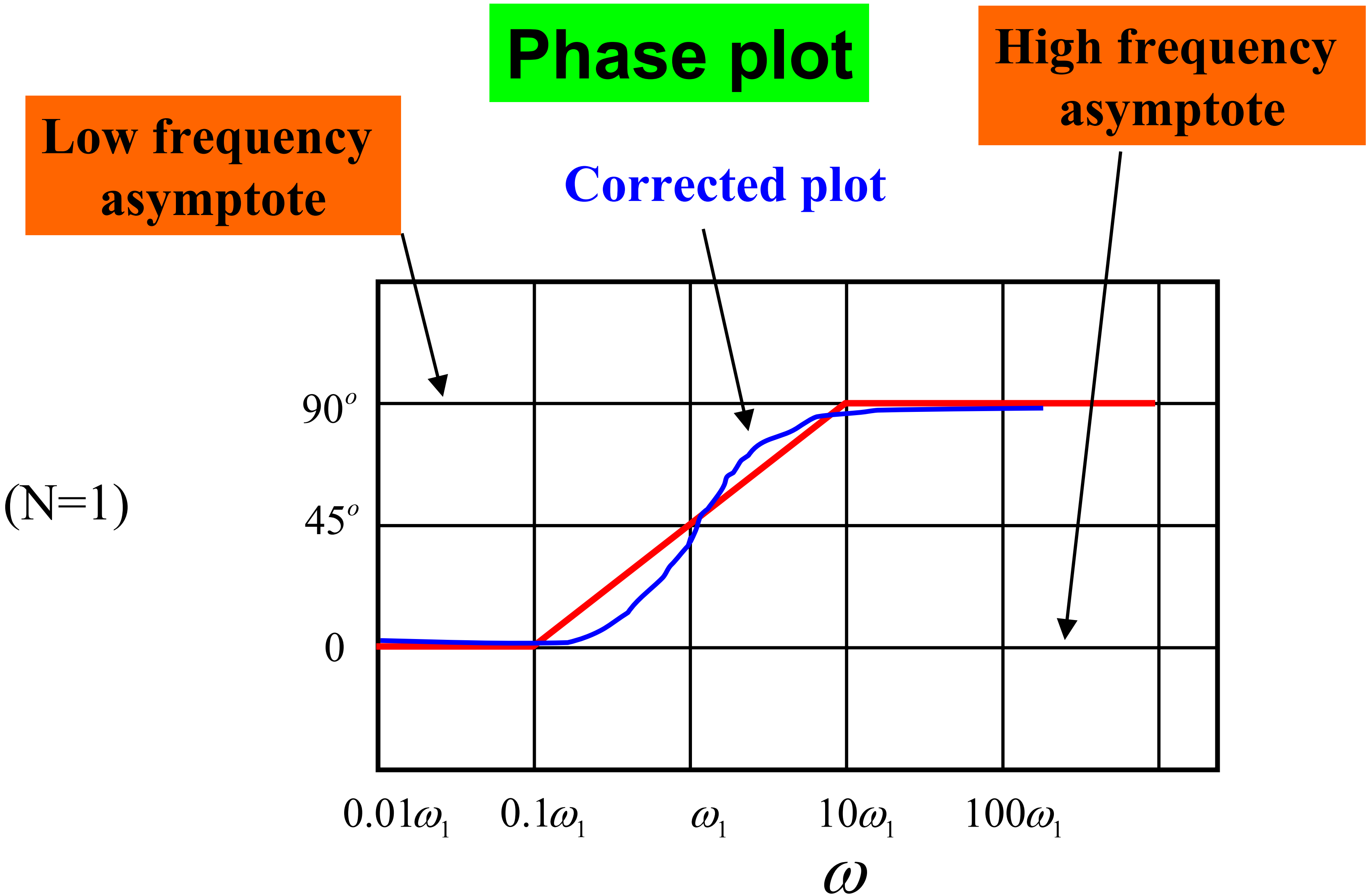


# Bode Plots

Note that the two lines resulting from the **"extreme cases"** intersect at $\omega=\omega_1$.



# Bode Plots

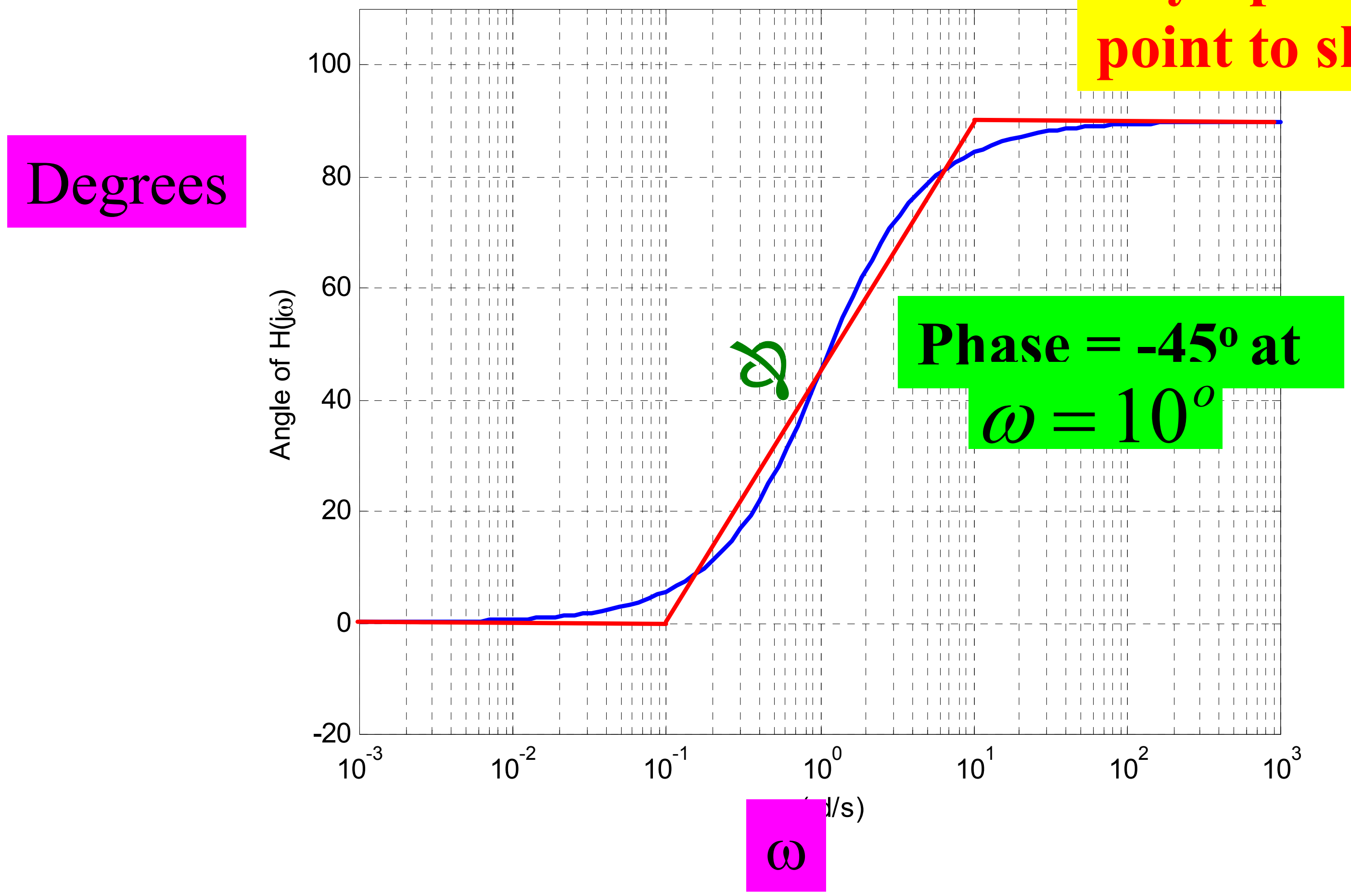


Phase plot
High frequency asymptote
Low frequency asymptote
Corrected plot
(N=1)
$90^o$
$45^o$
0
$0.01\omega_1$
$0.1\omega_1$
$\omega_1$
$10\omega_1$
$100\omega_1$
$\omega$

# Bode Plots: Phase

Example: $\mathbf{H(s)} = \mathbf{s} + 1$

Need only two asymptotes and one point to sketch curve.



# Example 1

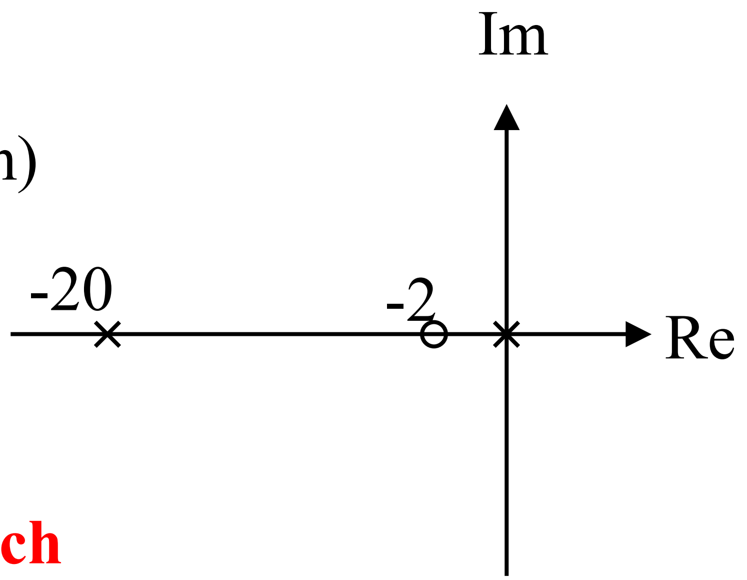
$$\mathbf{H}(\mathbf{s}) = \frac{(-5)(\mathbf{s}+2)}{\mathbf{s}(\mathbf{s}+20)}$$

→ **See textbook example 14.7**

One zero, two poles:

→ Poles at **s**=-20, and **s**=0 (origin)

→ Zero at **s**=-2.



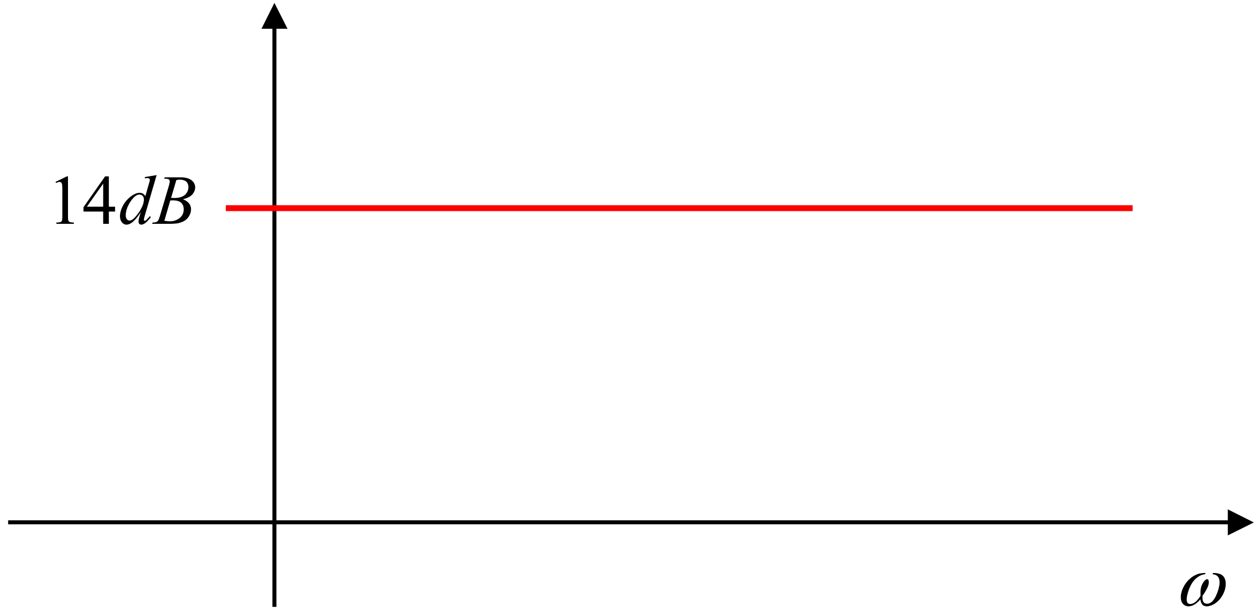

→ ***Method***

**Find the Bode plot due to each component and add the results.**

# Example 1

**Constant term:**

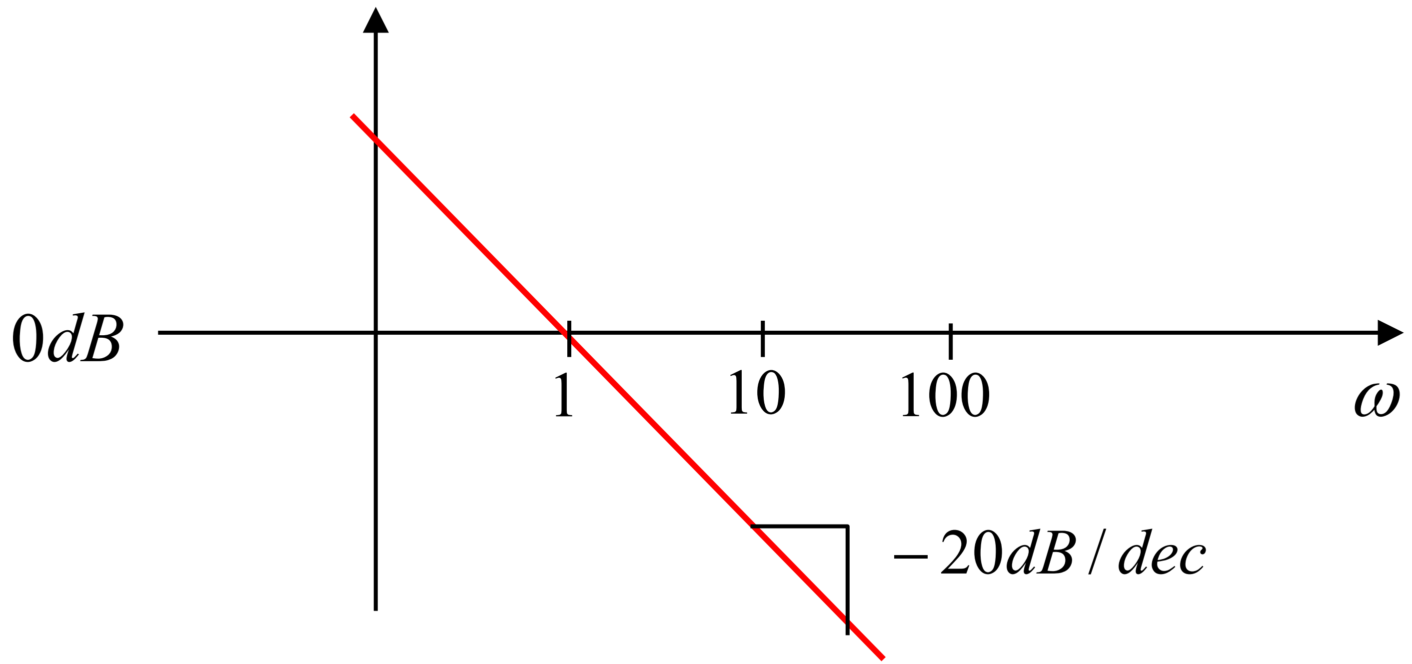
$$20\log(|-5|) = 14dB$$



# Example 1

**Pole at origin** $\mathbf{H(s)} = \dfrac{1}{\mathbf{s}}$

$\mathbf{H}(j1) = 20\log(1) = 0dB$ **at $\omega$=*1* rad./sec.**

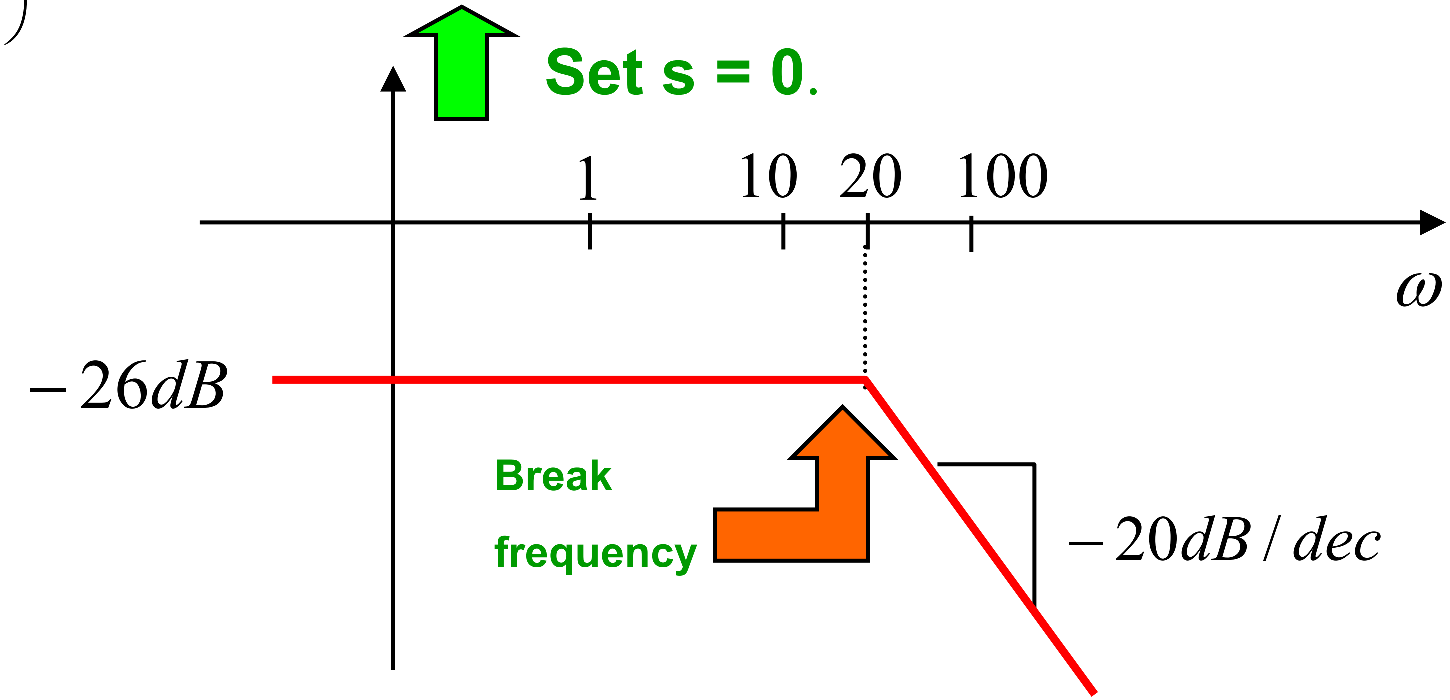


# Example 1

**Pole at s=-20**

$$\mathbf{H(s)} = \frac{1}{\mathbf{s}+20}$$

$$20\log\left(\frac{1}{20}\right) = -26dB \text{ for } \omega << 20 \text{ rad./sec.}$$

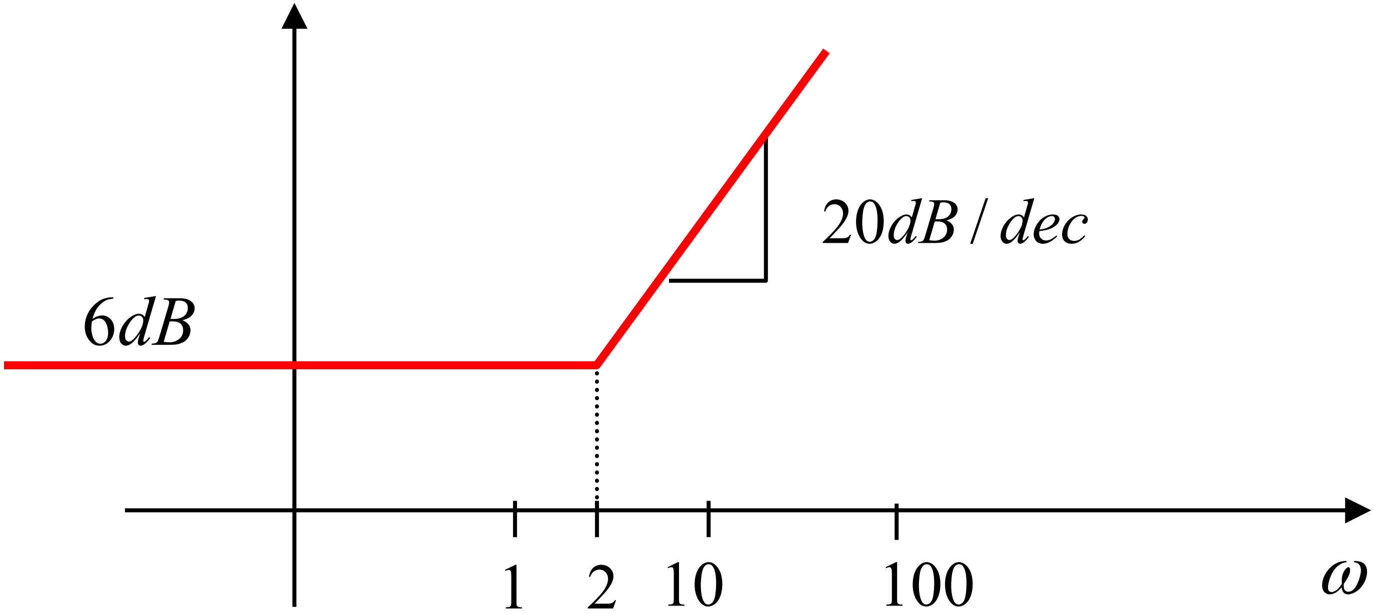


# Example 1

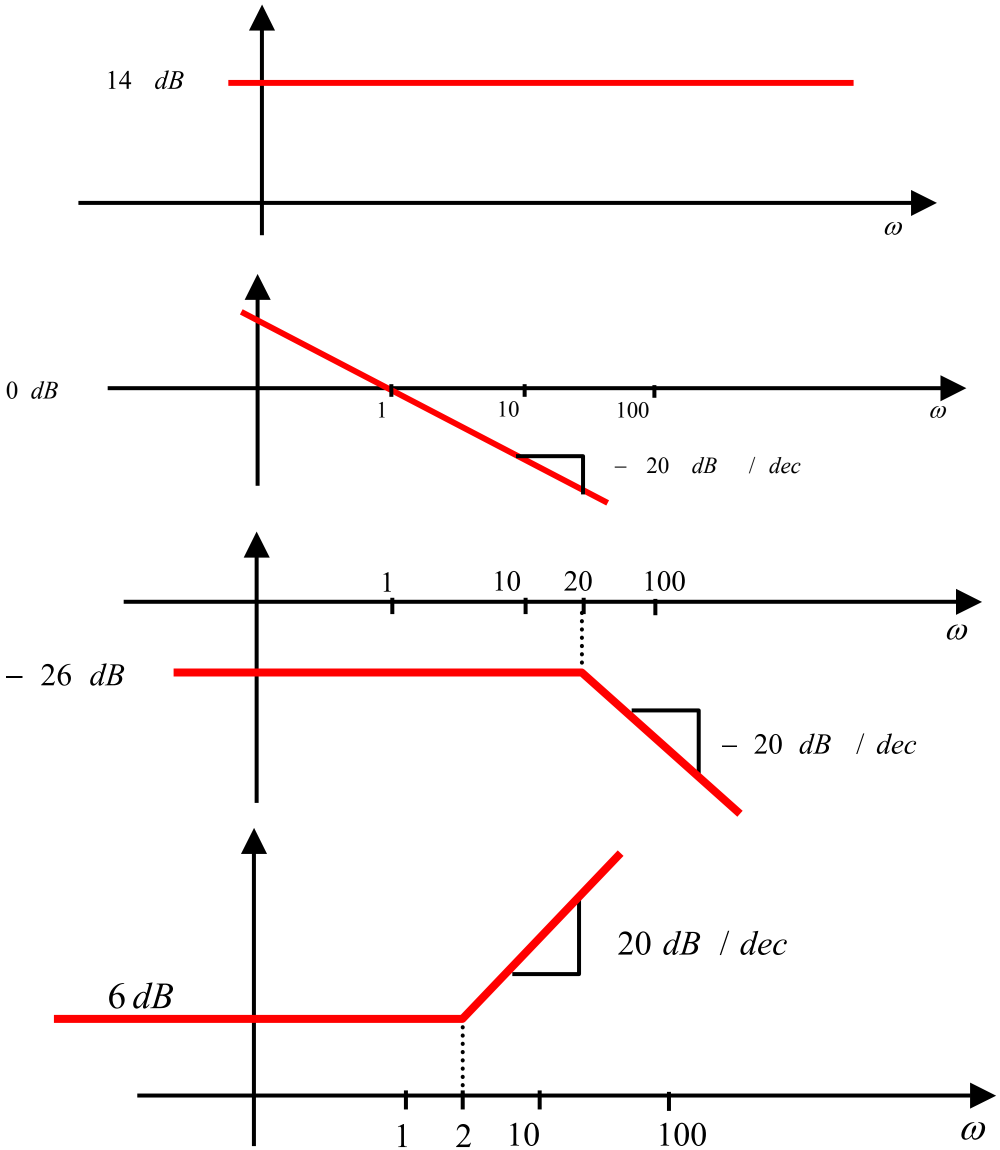
**Zero at s=-2** $\mathbf{H}(\mathbf{s}) = \mathbf{s} + 2$

**Set s = 0.**

$20\log(2) = 6dB$ for $\omega << 2$ rad./sec.



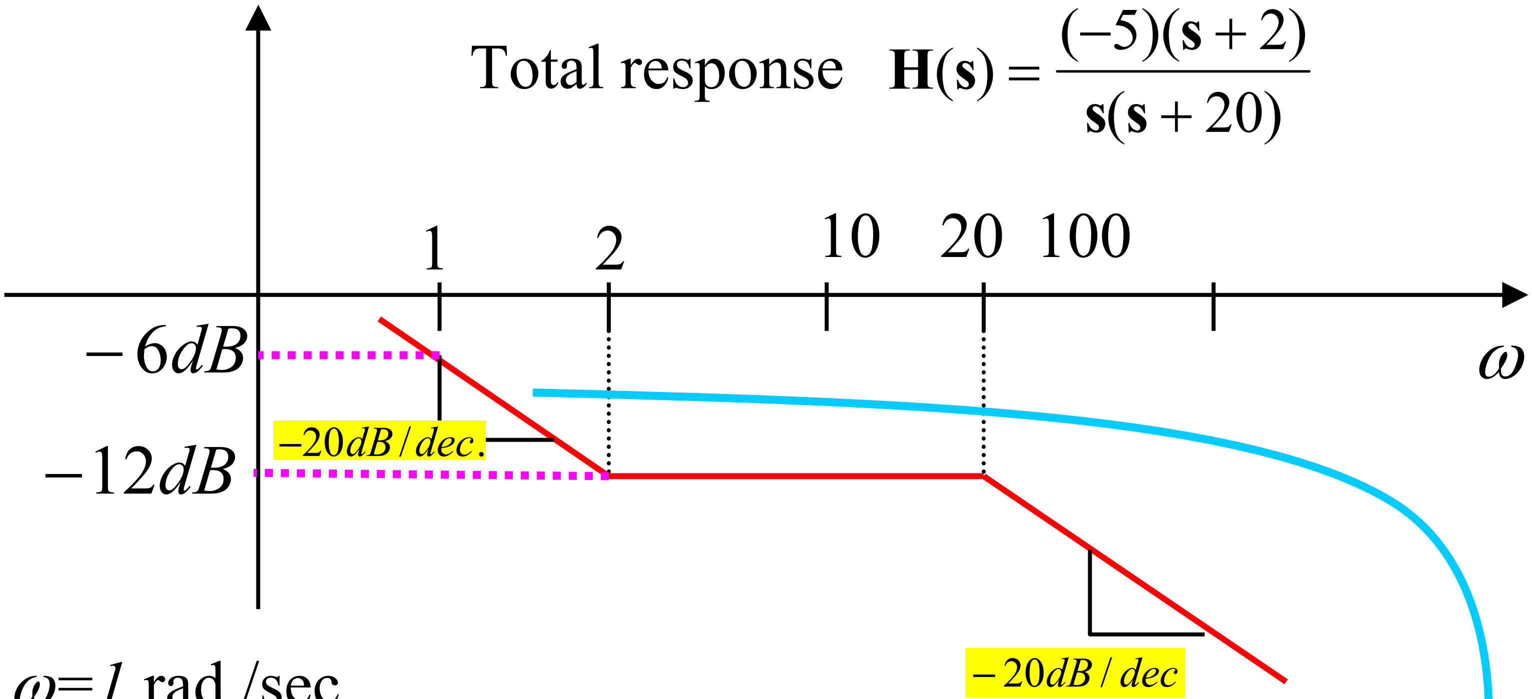
# Example 1



14 dB
ω
0 dB
1
10
100
ω
− 20 dB / dec
1
10
20
100
ω
− 26 dB
− 20 dB / dec
20 dB / dec
6 dB
1
2
10
100
ω

# Example 1

Total response $\mathbf{H(s)} = \dfrac{(-5)(\mathbf{s}+2)}{\mathbf{s}(\mathbf{s}+20)}$




At $\omega = 1$ rad./sec.

$$20\log\left(\left|\mathbf{H(s)}\right|\right) = 14 + 0 + 6 - 26 = -6dB$$

$$Decades = \log_{10}\frac{\omega_2}{\omega_1}$$

At $\omega = 2$ rad./sec.

$$\left|\mathbf{H(s)}\right|_{dB} = -6dB - 20\log\left(\frac{2}{1}\right) = -12dB$$

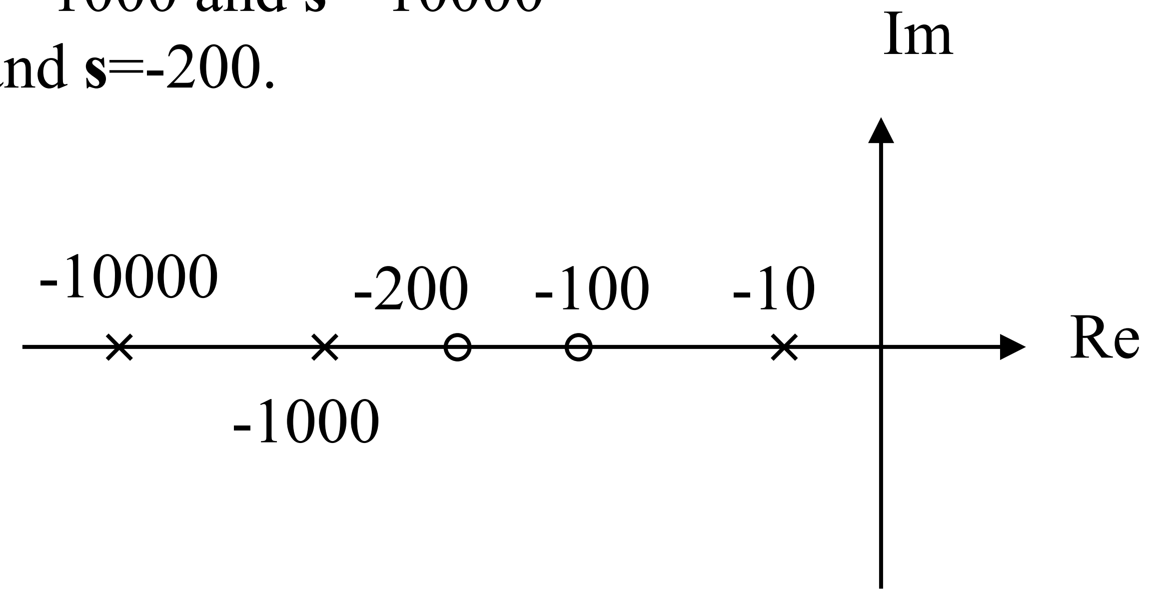
# Example 2

$$\mathbf{H}(\mathbf{s}) = \frac{(\mathbf{s}+100)(\mathbf{s}+200)}{(\mathbf{s}+10)(\mathbf{s}+1000)(\mathbf{s}+10000)}$$

Three poles, two zeros:

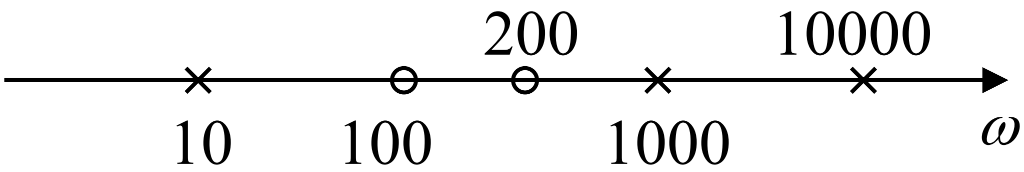
→ Poles at **s**=-10, **s**=-1000 and **s**=-10000

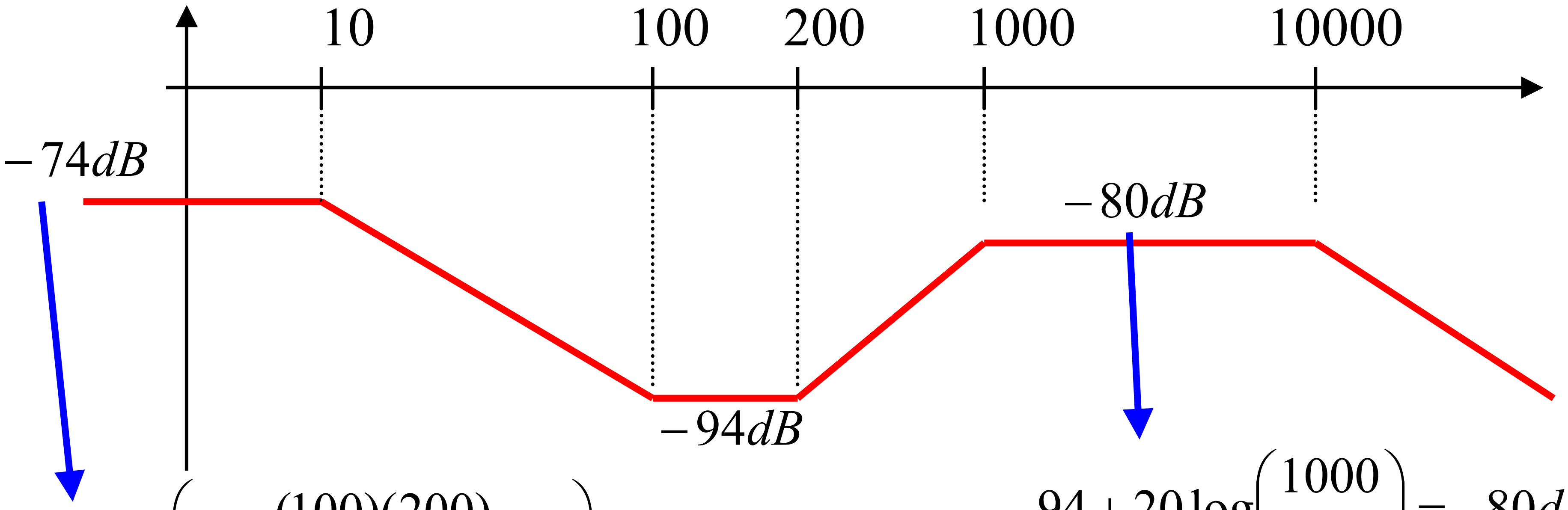
→ Zeros at **s**=-100 and **s**=-200.



# Example 2

$$\mathbf{H(s)} = \frac{(\mathbf{s}+100)(\mathbf{s}+200)}{(\mathbf{s}+10)(\mathbf{s}+1000)(\mathbf{s}+10000)}$$




$$20\log\left(\frac{(100)(200)}{(10)(1000)(10000)}\right) = -74dB$$

$$-94 + 20\log\left(\frac{1000}{200}\right) = -80dB$$