ECSE 210: Circuit Analysis

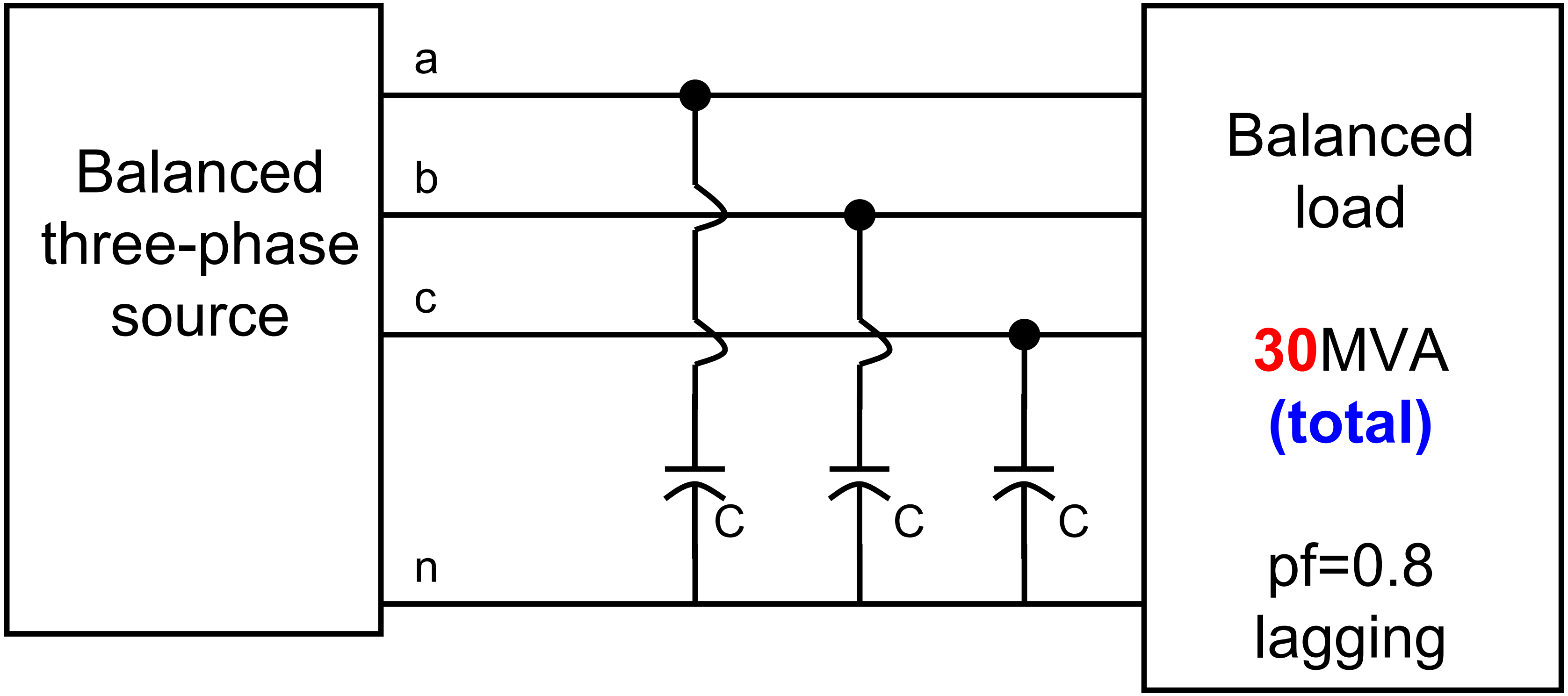
Lecture #20:

Three-Phase Circuits

Frequency Response

Power Factor Correction

→ Line voltage is 40kV rms at 60Hz.
→ Choose capacitors such that pf=0.9 lagging.

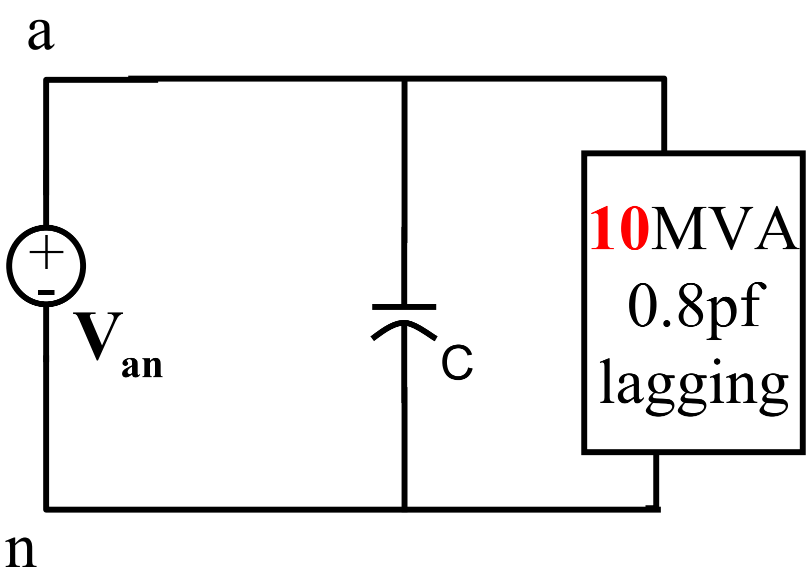


Power Factor Correction

→ The line to neutral voltage is given by:

$$\left|\mathbf{V_{an}}\right| = \frac{40kV}{\sqrt{3}} = 23.09kV\ rms$$

→ The per-phase equivalent circuit is therefore:



$$\mathbf{S_{old}} = 10\angle \arccos(0.8)\ MVA$$

$$= 10\angle 36.87^{o}\ MVA$$

↓

$$\theta_V - \theta_I > 0$$

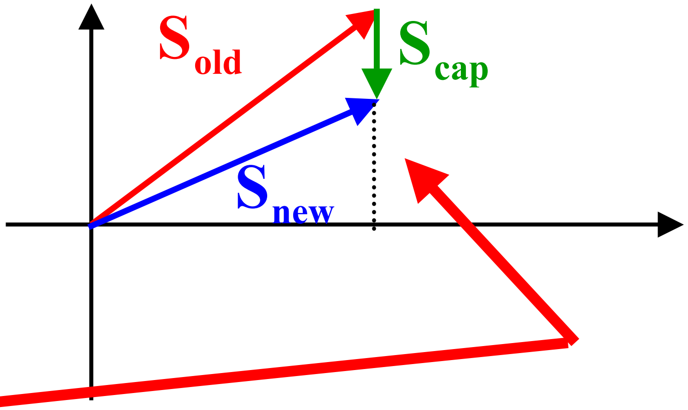
For the old pf:

$$\mathbf{S_{old}} = 8 + 6j\ MVA$$

Power Factor Correction

$$\mathbf{S}_{\mathbf{old}} = 10\angle 36.87^o \; MVA = 8 + 6j \; MVA$$

$$\theta_{old} = 36.87^o$$



$$\theta_{new} = \arccos(0.9) = 25.84^o$$

$$\text{lagging pf} \rightarrow \; \theta_{new} > 0$$

$$\mathbf{S}_{\mathbf{new}} = 8 + j8\tan(25.84^o) = 8 + j3.88 \; MVA$$

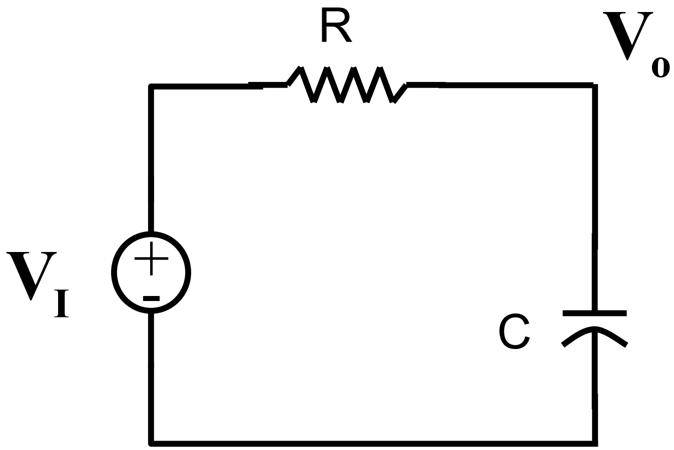
$$\mathbf{S}_{\mathbf{new}} = \mathbf{S}_{\mathbf{cap}} + \mathbf{S}_{\mathbf{old}} \qquad \mathbf{S}_{\mathbf{cap}} = \mathbf{S}_{\mathbf{new}} - \mathbf{S}_{\mathbf{old}} = -2.12j \; MVA$$

$$\mathbf{S}_{\mathbf{cap}} = \frac{V_{an}^2}{\mathbf{Z}^*} = -j\omega C V_{an}^2$$

$$C = \frac{2.12 \times 10^6}{377(23.09 \times 10^3)^2} = 10.6 \mu F$$

Frequency Characteristics

→ What is $\mathbf{V_o}$ as a function of $\mathbf{V_I}$?



$$\mathbf{Z}_C = \frac{1}{sC}$$

$$\mathbf{Z}_R = R$$

$$s = j\omega$$

→ Voltage divider:

$$\mathbf{V_o} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}}\mathbf{V_I} = \frac{1}{1 + sRC}\mathbf{V_I} = \frac{1}{1 + j\omega RC}\mathbf{V_I}$$

Frequency Characteristics

$$\mathbf{V_o} = \frac{1}{1+sRC}\mathbf{V_I}$$

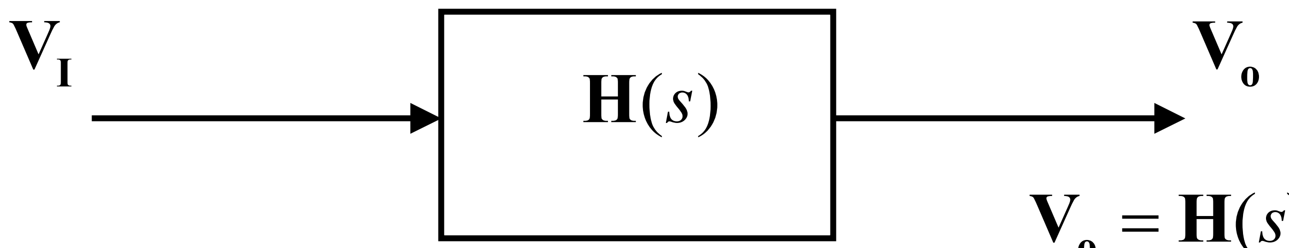
→ Input output relation is a function of frequency.

→ The input and output have the same frequency, but a different magnitude and phase.

(This is a fundamental property of linear circuits.)

→ We define the transfer function:

$$\mathbf{H}(s) = \frac{\mathbf{V_o}}{\mathbf{V_I}} = \frac{1}{1+sRC}$$



$$\mathbf{V_o} = \mathbf{H}(s)\mathbf{V_I}$$

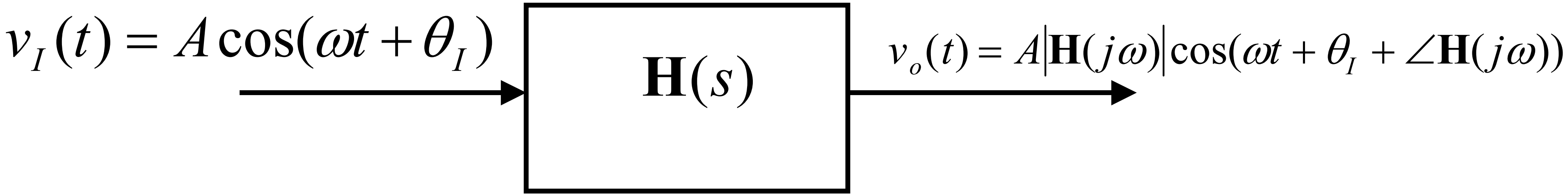
Frequency Response

→ At frequency ω:

$$\mathbf{V_o} = \mathbf{H}(j\omega)\mathbf{V_I}$$

$$|\mathbf{V_o}|\angle\theta_o = \left(|\mathbf{H}(j\omega)|\angle\theta_h\right)\left(|\mathbf{V_I}|\angle\theta_I\right)$$

$$|\mathbf{V_o}|\angle\theta_o = |\mathbf{H}(j\omega)||\mathbf{V_I}|\angle\theta_I + \theta_h$$



→ Same frequency ω.

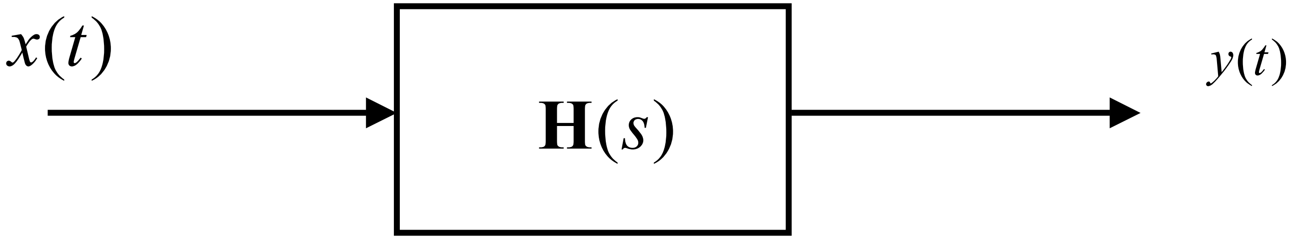
→ Amplitude is multiplied by |H(jω)|.

→ Phase is shifted by the angle of |H(jω)|.

→ **H**(s) is not a phasor!

Network Function

→ In general:



→ *x(t)* may be an input **voltage** *or* **current**.

→ y(t) may be a **voltage** at the "output" node *or* a **branch current**.

→ This leads to four types of network transfer functions.

Network Function

(1) $x(t)=i(t);\ \ y(t)=v(t)$ → $\mathbf{Y}(s) = \mathbf{H}(s)\mathbf{X}(s);\ \ \mathbf{V}(s) = \mathbf{H}(s)\mathbf{I}(s)$

$$\mathbf{H}(s) = \mathbf{Z}(s)$$

Note: in this case **Z**(s) is a **transfer** impedance,
not a **driving point** impedance.

(2) $x(t)=v(t);\ \ y(t)=i(t)$ → $\mathbf{Y}(s) = \mathbf{H}(s)\mathbf{X}(s);\ \ \mathbf{I}(s) = \mathbf{H}(s)\mathbf{V}(s)$

$\mathbf{H}(s)$ is a **transfer** admittance.

(3) $x(t)=v_i(t);\ \ y(t)=v_o(t)$ → $\mathbf{Y}(s) = \mathbf{H}(s)\mathbf{X}(s); \mathbf{V}_o(s) = \mathbf{H}(s)\mathbf{V}_I(s)$

$$\mathbf{H}(s) = \mathbf{G}_V(s)$$

Note: $\mathbf{G}_V$(s) is a **voltage gain**.

Network Function

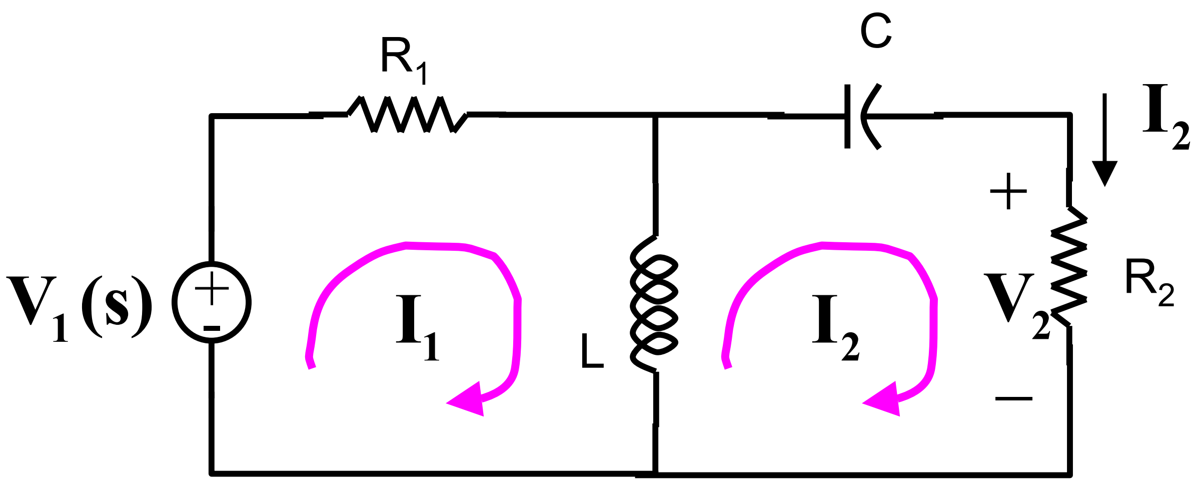
(4) $x(t)=i_i(t)$; $y(t)=i_o(t)$ → $\mathbf{Y}(s)=\mathbf{H}(s)\mathbf{X}(s)$; $\mathbf{I}_o(s)=\mathbf{H}(s)\mathbf{I}_I(s)$

$$\mathbf{H}(s)=\mathbf{G}_I(s)$$

Note: $\mathbf{G}_I(s)$ is a **current gain**.

Example 1

Find the transfer admittance $\mathbf{I_2(s)/V_1(s)}$ and the voltage gain $\mathbf{V_2(s)/V_1(s)}$ for the network provided below.



Mesh equations:

$$R_1\mathbf{I}_1(\mathbf{s}) + \mathbf{s}L\left(\mathbf{I}_1(\mathbf{s}) - \mathbf{I}_2(\mathbf{s})\right) = \mathbf{V}_1(\mathbf{s})$$

$$\mathbf{s}L\left(\mathbf{I}_2(\mathbf{s}) - \mathbf{I}_1(\mathbf{s})\right) + \frac{\mathbf{I}_2(\mathbf{s})}{\mathbf{s}C} + R_2\mathbf{I}_2(\mathbf{s}) = 0$$

Example 1

$$\begin{cases} (R_1 + \mathbf{s}L)\mathbf{I_1}(\mathbf{s}) - \mathbf{s}L\mathbf{I_2}(\mathbf{s}) = \mathbf{V_1}(\mathbf{s}) & ☆ \\ -\mathbf{s}L\mathbf{I_1}(\mathbf{s}) + \left(R_2 + \mathbf{s}L + \dfrac{1}{\mathbf{s}C}\right)\mathbf{I_2}(\mathbf{s}) = 0 & 🕐 \end{cases}$$

→ Solve for $\mathbf{I_2}$:

🕐 → $$\mathbf{I_1}(\mathbf{s}) = \frac{1}{\mathbf{s}L}\left(R_2 + \mathbf{s}L + \frac{1}{\mathbf{s}C}\right)\mathbf{I_2}(\mathbf{s})$$

Substituting into ☆ →

$$\frac{(R_1 + \mathbf{s}L)}{\mathbf{s}L}\left(R_2 + \mathbf{s}L + \frac{1}{\mathbf{s}C}\right)\mathbf{I_2}(\mathbf{s}) - \mathbf{s}L\mathbf{I_2}(\mathbf{s}) = \mathbf{V_1}(\mathbf{s})$$

Example 1

$$\frac{(R_1 + \mathbf{s}L)}{\mathbf{s}L}\left(R_2 + \mathbf{s}L + \frac{1}{\mathbf{s}C}\right)\mathbf{I_2(s)} - \mathbf{s}L\mathbf{I_2(s)} = \mathbf{V_1(s)}$$

$$\mathbf{I_2(s)} = \frac{\mathbf{s}L\mathbf{V_1(s)}}{(R_1 + \mathbf{s}L)\left(R_2 + \mathbf{s}L + \frac{1}{\mathbf{s}C}\right) - \mathbf{s}^2 L^2}$$

$$\mathbf{I_2(s)} = \frac{LC\mathbf{s}^2\mathbf{V_1(s)}}{(R_1 + R_2)LC\mathbf{s}^2 + (L + R_1 R_2 C)\mathbf{s} + R_1}$$

$$\mathbf{V_2(s)} = \frac{R_2 LC\mathbf{s}^2\mathbf{V_1(s)}}{(R_1 + R_2)LC\mathbf{s}^2 + (L + R_1 R_2 C)\mathbf{s} + R_1}$$

Example 1

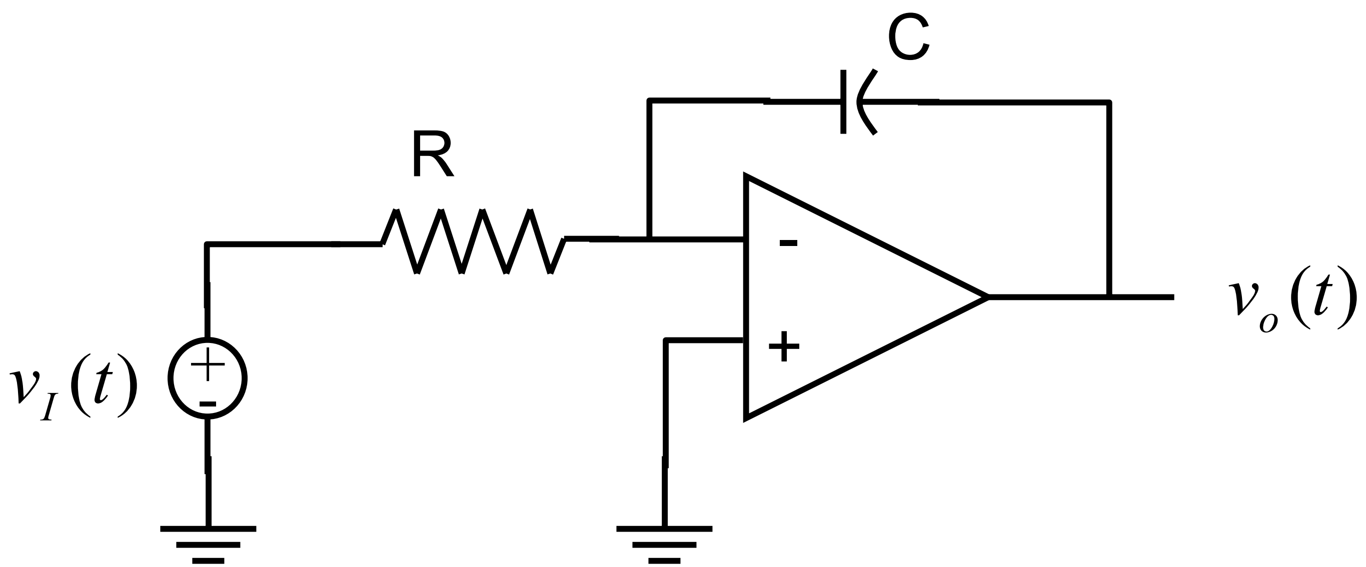
→ Transfer admittance:

$$\mathbf{Y}(\mathbf{s}) = \frac{\mathbf{I}_2(\mathbf{s})}{\mathbf{V}_1(\mathbf{s})} = \frac{LC\mathbf{s}^2}{(R_1 + R_2)LC\mathbf{s}^2 + (L + R_1R_2C)\mathbf{s} + R_1}$$

→ Voltage gain:

$$\mathbf{G}_v(\mathbf{s}) = \frac{\mathbf{V}_2(\mathbf{s})}{\mathbf{V}_1(\mathbf{s})} = \frac{R_2LC\mathbf{s}^2}{(R_1 + R_2)LC\mathbf{s}^2 + (L + R_1R_2C)\mathbf{s} + R_1}$$

Example 2: Integrator



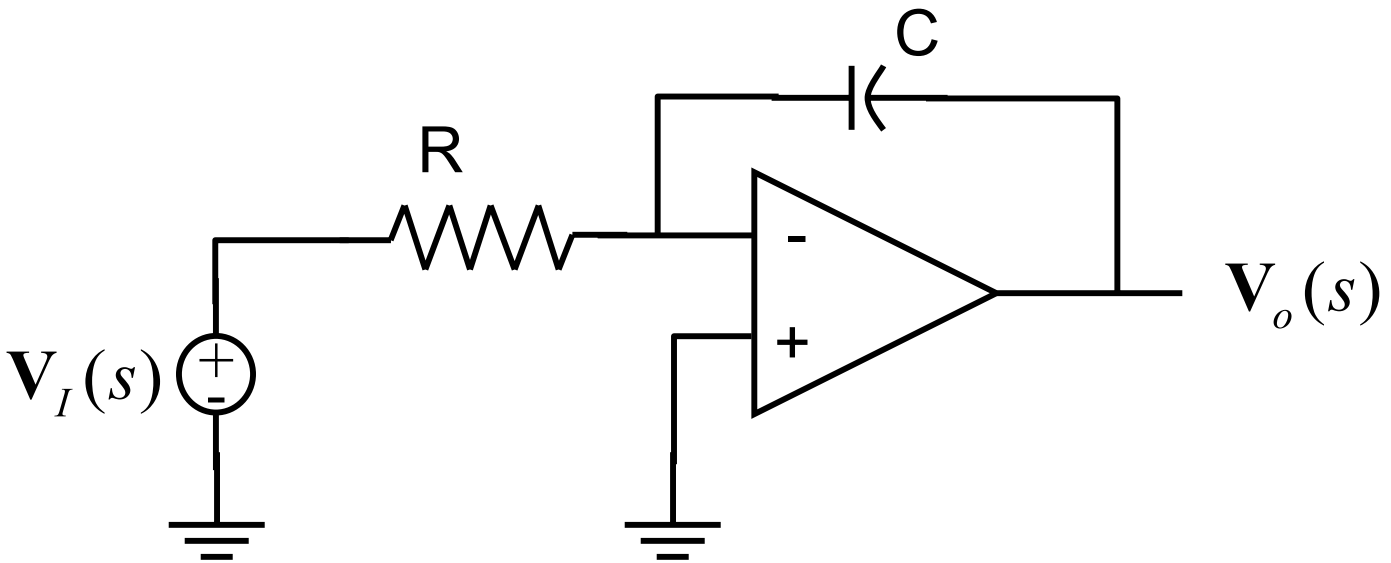
KCL in time domain

$$\frac{v_I(t)}{R} + C\frac{dv_o(t)}{dt} = 0 \quad \rightarrow \quad \frac{dv_o(t)}{dt} = -\frac{1}{RC}v_I(t)$$

$$v_o(t) = -\frac{1}{RC}\int_{0+}^{t} v_I(t)dt + v_o(0+) = -\frac{1}{RC}\int_{0+}^{t} v_I(t)dt$$

Assuming zero initial conditions.

Example 2: Integrator



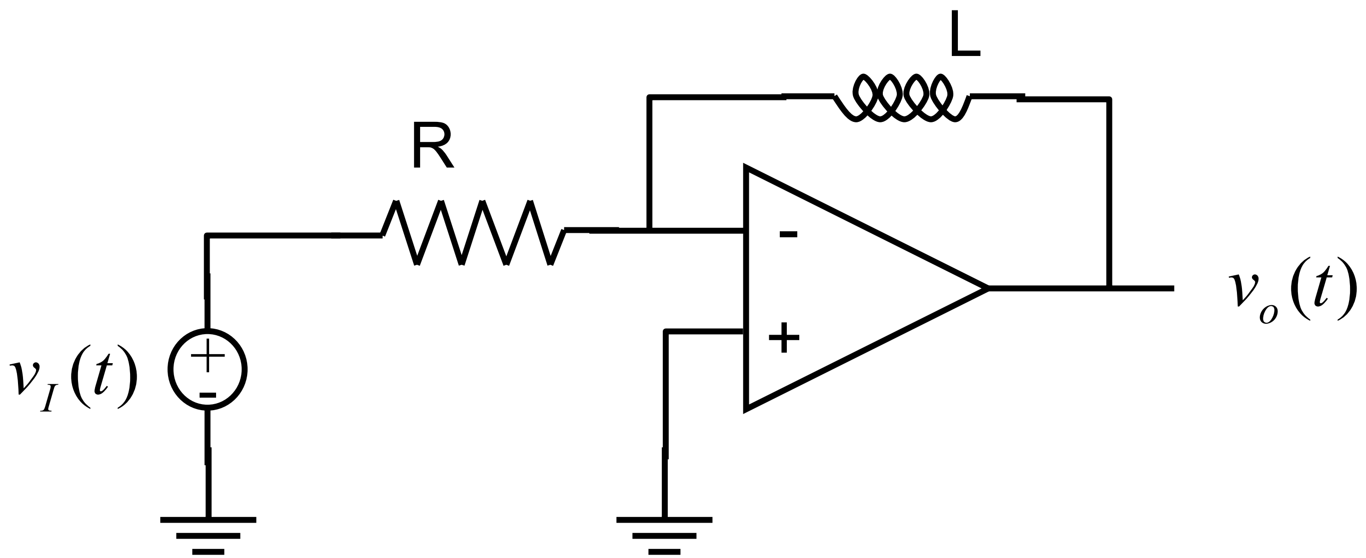
KCL in frequency domain

$$\frac{\mathbf{V_I(s)}}{R} + \mathbf{s}C\mathbf{V_o(s)} = 0 \quad \rightarrow \quad \mathbf{V_o(s)} = -\frac{1}{\mathbf{s}RC}\mathbf{V_I(s)}$$

→ Division by **s** is equivalent to integration

(Assuming zero initial conditions)

Example 3: Differentiator



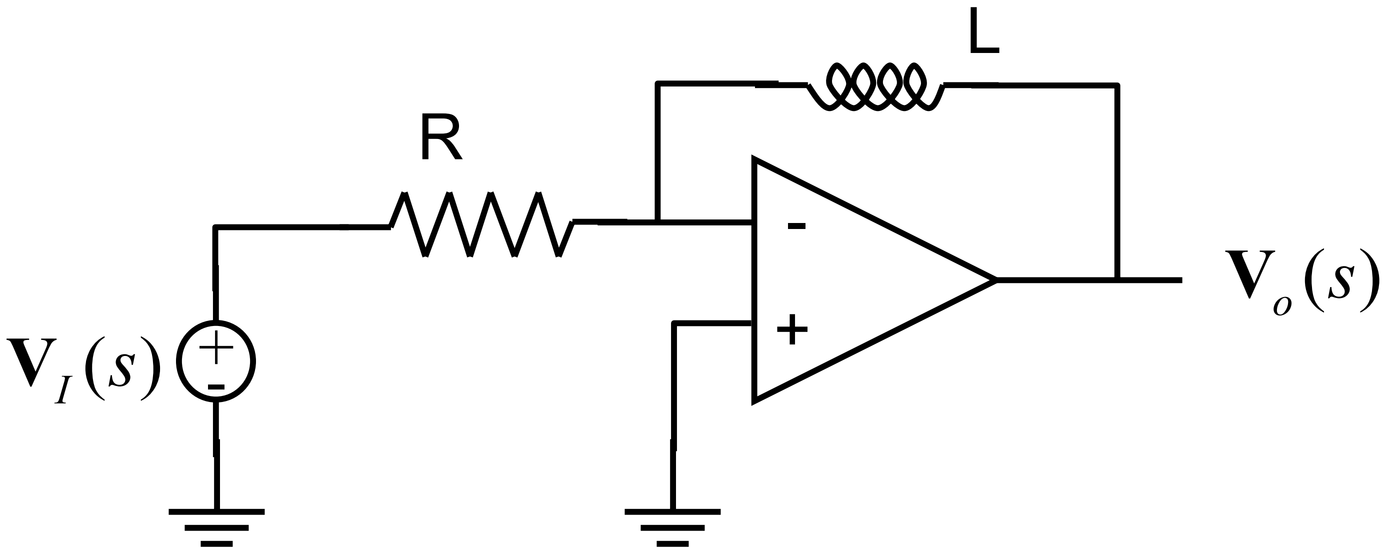
KCL in time domain

$$\frac{v_I(t)}{R} + \frac{1}{L}\int_{0+}^{t} v_o(t)dt + i_L(0+) = 0 \rightarrow \int_{0+}^{t} v_o(t)dt = -\frac{L}{R}v_I(t)$$

Assuming zero initial condition.

$$v_o(t) = -\frac{L}{R}\frac{dv_I(t)}{dt}$$

Example 3: Differentiator



KCL in frequency domain

$$\frac{\mathbf{V_I(s)}}{R} + \frac{\mathbf{V_o(s)}}{\mathbf{s}L} = 0 \quad \rightarrow \quad \mathbf{V_o(s)} = -\frac{\mathbf{s}L}{R}\mathbf{V_I(s)}$$

→ Multiplication by **s** is equivalent to differentiation

(Assuming zero initial conditions)