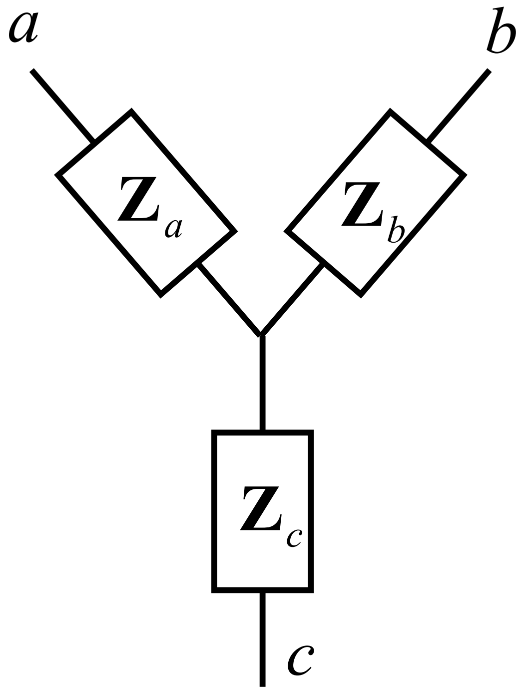
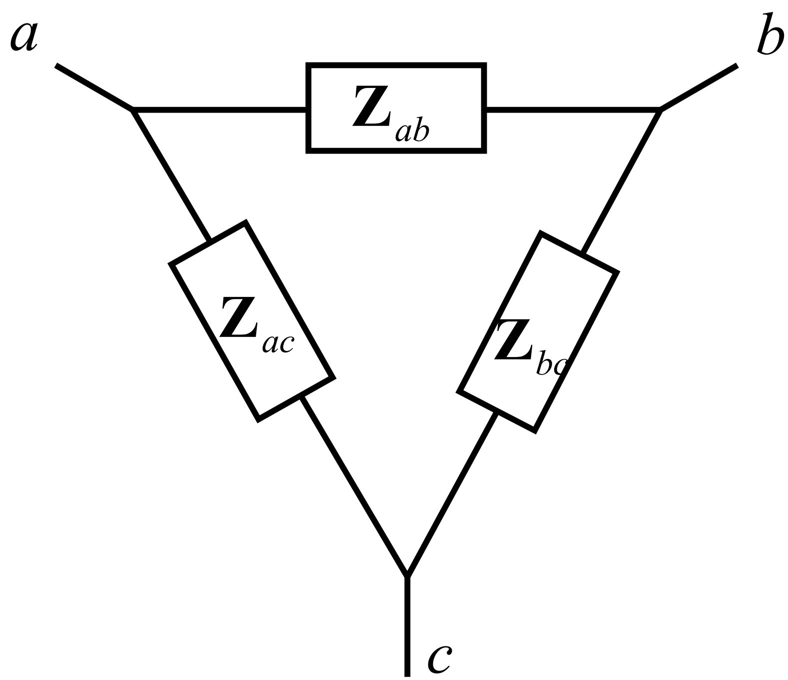
ECSE 210: Circuit Analysis

Lecture#19:

Three-Phase Circuits

Wye-Delta Transformation - Review





$$\mathbf{Z}_a = \frac{\mathbf{Z}_{ab}\mathbf{Z}_{ac}}{\mathbf{Z}_{ab} + \mathbf{Z}_{ac} + \mathbf{Z}_{bc}}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_{ab}\mathbf{Z}_{bc}}{\mathbf{Z}_{ab} + \mathbf{Z}_{ac} + \mathbf{Z}_{bc}}$$

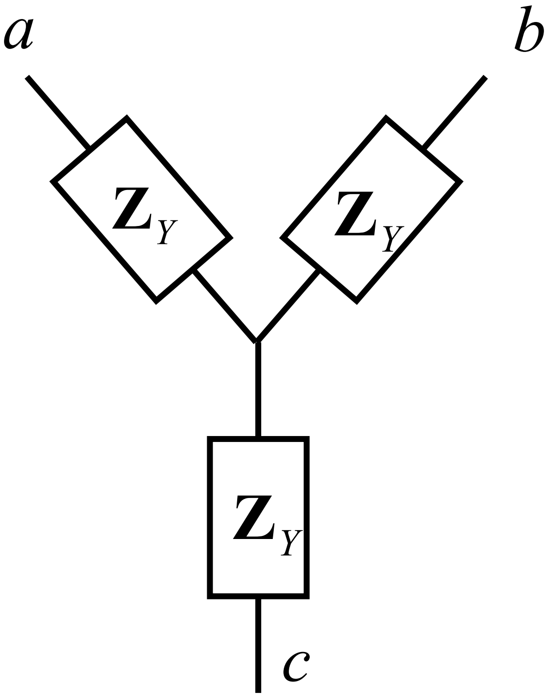
$$\mathbf{Z}_c = \frac{\mathbf{Z}_{ac}\mathbf{Z}_{bc}}{\mathbf{Z}_{ab} + \mathbf{Z}_{ac} + \mathbf{Z}_{bc}}$$

$$\mathbf{Z}_{ab} = \frac{\mathbf{Z}_a\mathbf{Z}_b + \mathbf{Z}_b\mathbf{Z}_c + \mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_c}$$

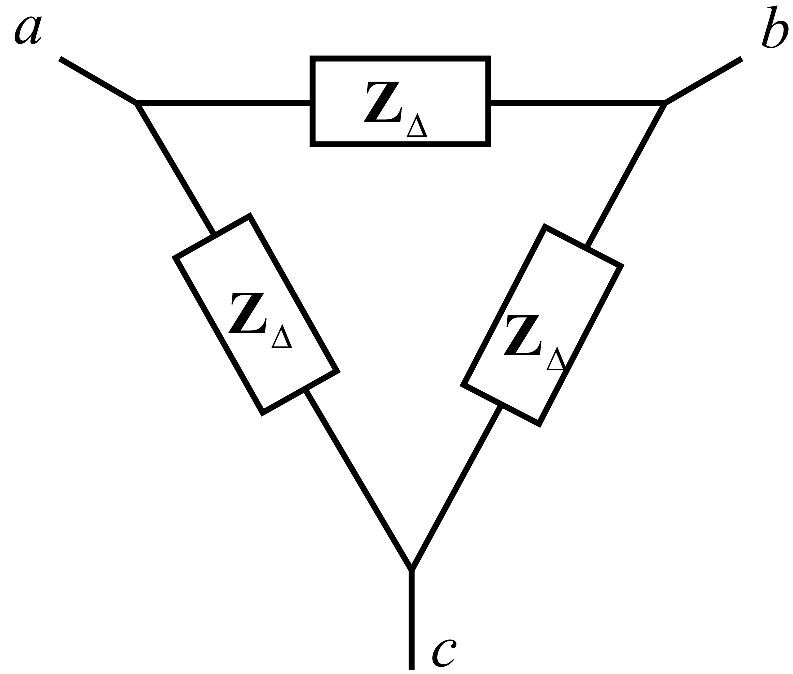
$$\mathbf{Z}_{ac} = \frac{\mathbf{Z}_a\mathbf{Z}_b + \mathbf{Z}_b\mathbf{Z}_c + \mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_b}$$

$$\mathbf{Z}_{bc} = \frac{\mathbf{Z}_a\mathbf{Z}_b + \mathbf{Z}_b\mathbf{Z}_c + \mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a}$$

Wye-Delta for Balanced Loads

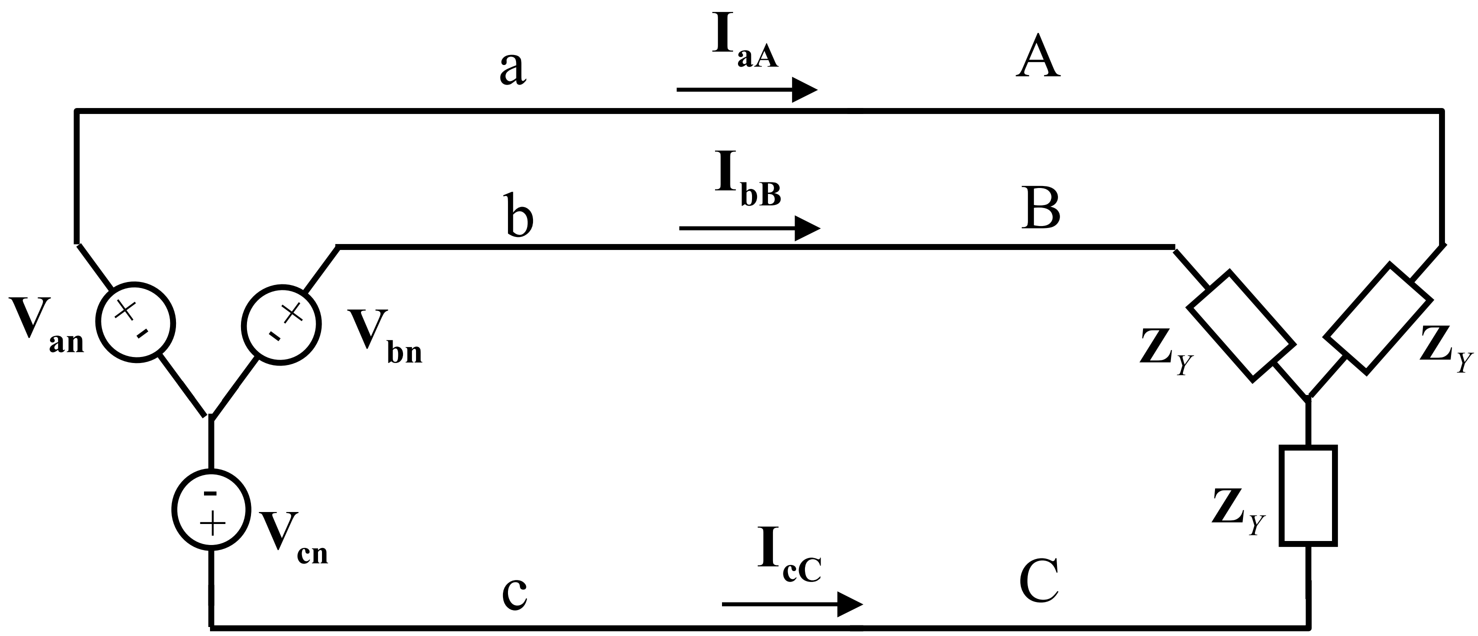


$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$



$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y$$

Balanced Wye-to-Wye Connection



Balanced Wye-to-Wye Connection

Phase voltages:

$$\mathbf{V_a} = V_p \angle 0^o$$

$$\mathbf{V_b} = V_p \angle -120^o$$

$$\mathbf{V_c} = V_p \angle +120^o$$

Line-to-line voltages (or line voltages):

$$\mathbf{V_{ab}} = \mathbf{V_L} = \sqrt{3} V_p \angle 30^o \qquad \mathbf{L \equiv line}$$

$$\mathbf{V_{bc}} = \sqrt{3} V_p \angle -90^o$$

$$\mathbf{V_{ca}} = \sqrt{3} V_p \angle -210^o = \sqrt{3} V_p \angle +150^o$$

Balanced Wye-to-Wye Connection

Currents:

$$\mathbf{I_L} = \mathbf{I_P}$$ (Line current = Phase current)

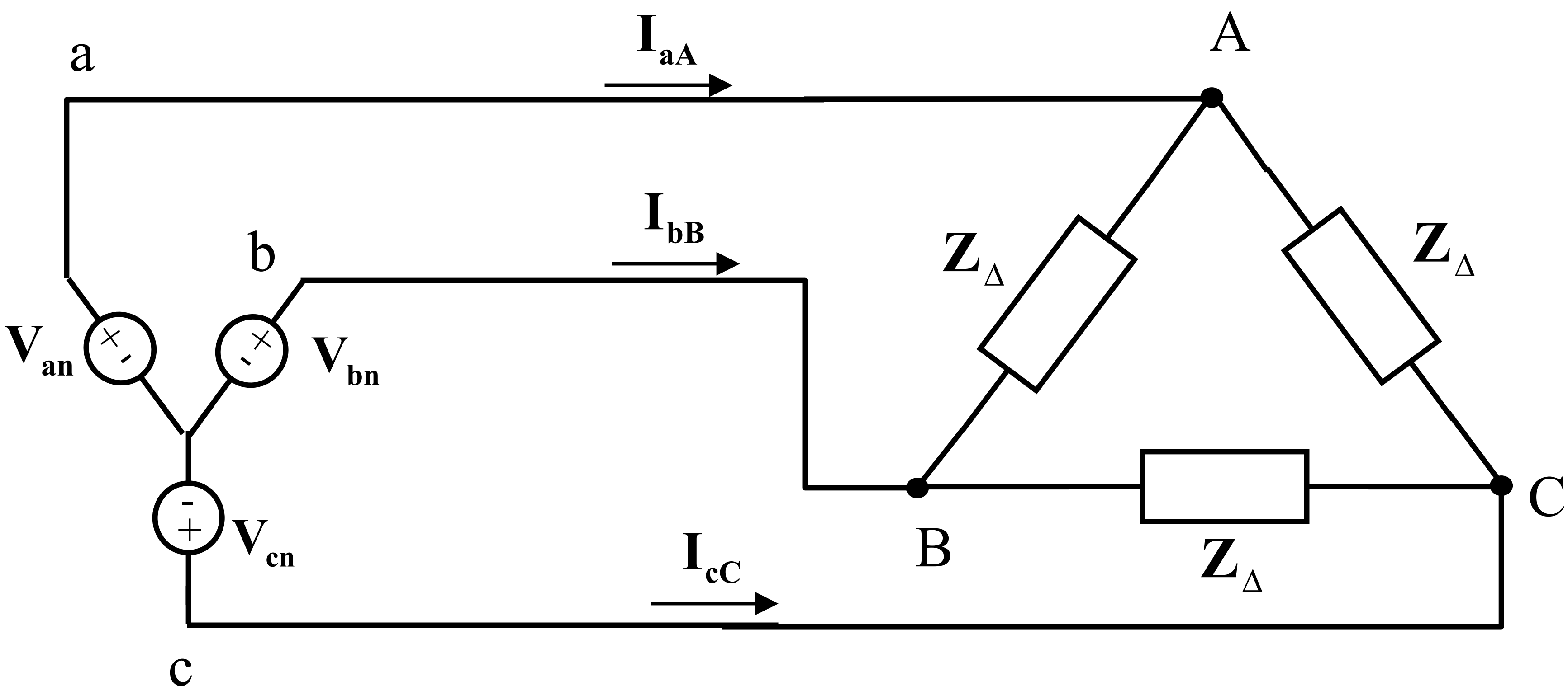
$$\mathbf{I_{aA}} = \frac{\mathbf{V_a}}{\mathbf{Z}_Y} = \frac{V_p\angle 0}{|\mathbf{Z}_Y|\angle\phi} = |\mathbf{I_L}|\angle-\phi = I_p\angle-\phi$$

$$\mathbf{I_{bB}} = |\mathbf{I_L}|\angle(-\phi-120) = I_p\angle(-\phi-120)$$

$$\mathbf{I_{cC}} = |\mathbf{I_L}|\angle(-\phi-240) = I_p\angle(-\phi-240)$$

Phase voltage at the source is across the Y load impedance.

Balanced Wye-to-Delta Connection



Phase voltages: $\mathbf{V_{AB}} = V_{AB}\angle 0^o = V_p \angle 0^o$

Line-to-line voltages (line voltages)

At the Δ load impedance.

Balanced Wye-to-Delta Connection

Note

Phase voltages: $\mathbf{V_{AB}} = V_{AB}\angle 0^o = V_p\angle 0^o$

$\mathbf{V_{BC}} = V_p\angle -120^o$

$\mathbf{V_{CA}} = V_p\angle -240^o$

At the Δ load impedance.

Line-to-line voltages (line voltages):

$\mathbf{V_{AB}} = \mathbf{V_L}$ (Line voltage = Phase voltage)

Phase Currents:

$$\mathbf{I_{AB}} = \frac{\mathbf{V_{AB}}}{\mathbf{Z_\Delta}} = I_p\angle -\phi$$

$$\mathbf{Z_\Delta} = Z_\Delta\angle\phi$$

$$\mathbf{I_{BC}} = \frac{\mathbf{V_{BC}}}{\mathbf{Z_\Delta}} = I_p\angle -120-\phi$$

Note

$$\mathbf{I_{CA}} = \frac{\mathbf{V_{CA}}}{\mathbf{Z_\Delta}} = I_p\angle -240-\phi$$

Balanced Wye-to-Delta Connection

Line Currents:

$$\mathbf{I_{aA}} = \sqrt{3} I_p \angle -\phi - 30^o$$

$$\mathbf{I_{bB}} = \sqrt{3} I_p \angle -\phi - 150^o$$

$$\mathbf{I_{cC}} = \sqrt{3} I_p \angle -\phi + 90^o$$

Example 1

Assume that an abc-sequence three-phase voltage source connected as a **wye** supplies power to a balanced **delta**-connected load. Given the load current $\mathbf{I_{AB}}$, find the line currents.

$$\mathbf{I_{AB}} = 4\angle 20^0 \; A \; rms$$

$$\mathbf{I_{aA}} = 4\sqrt{3}\angle -10^0 \; A \; rms$$

$$\mathbf{I_{bB}} = 4\sqrt{3}\angle -130^0 \; A \; rms$$

$$\mathbf{I_{cC}} = 4\sqrt{3}\angle +110^0 \; A \; rms$$

Line currents

Example 2

Assume a 60Hz abc-sequence three-phase voltage source connected in a balanced **wye** supplies power to a balanced **delta**-connected load. If $\mathbf{V}_{\mathbf{an}} = 120\angle 30^{o}$ V rms and the load contains a 10Ω resistance in series with a 20mH inductance in all three phases, what are the line and phase currents?

The load impedance per phase is:

$$\mathbf{Z}_{\Delta} = 10 + j(2\pi 60)(20mH) = 10 + j7.54\Omega$$

Since there is no line impedance:

$$\mathbf{V}_{\mathbf{AB}} = \mathbf{V}_{\mathbf{ab}} = 120\sqrt{3}\angle 60^{o}\ V\ rms$$

$$\mathbf{I}_{\mathbf{AB}} = \frac{120\sqrt{3}\angle 60^{o}}{10 + j7.54} = 16.6\angle 22.98^{o}\ A\ rms$$

Example 2

$$\mathbf{I}_{\mathbf{AB}} = \frac{120\sqrt{3}\angle 60^o}{10 + j7.54} = 16.6\angle 22.98^o \; A\, rms$$

Load (phase) current

$$\mathbf{I}_{\mathbf{aA}} = 16.6\sqrt{3}\angle -7.02^o \; A\, rms$$

Line current

Using the abc-sequence, the remaining phase and line currents are:

$$\mathbf{I}_{\mathbf{BC}} = 16.6\angle -97.02^o \; A\, rms$$

$$\mathbf{I}_{\mathbf{CA}} = 16.6\angle 142.98^o \; A\, rms$$

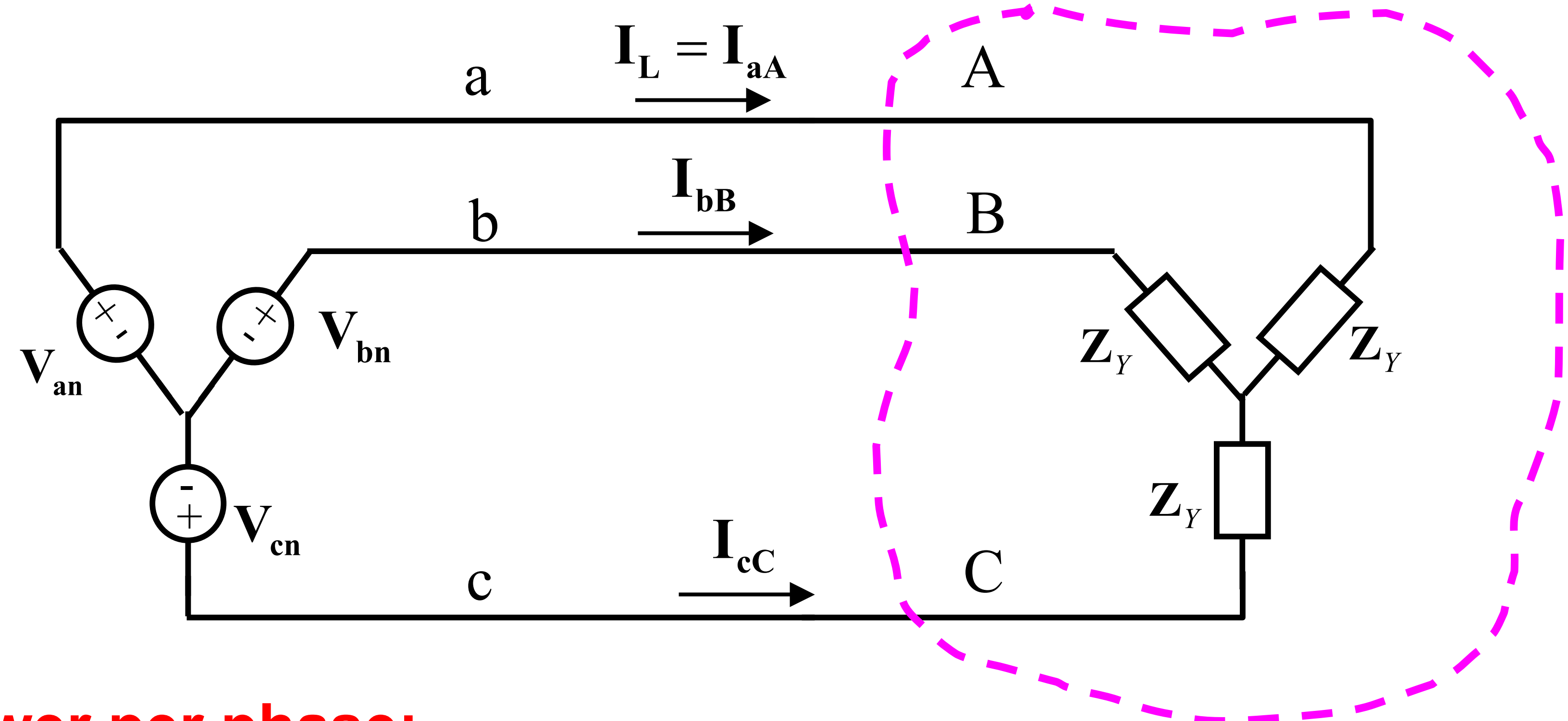
Phase currents

$$\mathbf{I}_{\mathbf{bB}} = 16.6\sqrt{3}\angle -127.02^o \; A\, rms$$

$$\mathbf{I}_{\mathbf{cC}} = 16.6\sqrt{3}\angle 112.98^o \; A\, rms$$

Line currents

Three-Phase Power



Power per phase:

$$P_p = |\mathbf{V_{an}}||\mathbf{I_L}|\overbrace{\cos(\theta)}^{\text{Power factor}} \longrightarrow \text{Average power}$$

$$Q_p = |\mathbf{V_{an}}||\mathbf{I_L}|\sin(\theta) \longrightarrow \text{Reactive power}$$

Example 3

Assume a balanced three-phase **wye-delta** system has a line voltage of 208V rms. If the ***total*** real power (avg. power) absorbed by the load is **1200**W, and the load power factor angle is 20° lagging, determine the magnitude of the **line current** and the value of the **load impedance** per phase in the delta.

Solution

Amplitude of **line-to neutral-voltage**: $|\mathbf{V_{an}}| = \dfrac{208}{\sqrt{3}} V\ rms$

Per phase power = **400**W $= \dfrac{208|\mathbf{I_L}|}{\sqrt{3}}\cos(20^o)$

$$|\mathbf{I_L}| = 3.5\,A\ rms$$ **Line current**

Example 3

$$|\mathbf{I}_L| = 3.5\,A\,rms$$

For a delta-connected load → The phase current is:

$$|\mathbf{I}_V| = \frac{|\mathbf{I}_L|}{\sqrt{3}} = 2.04\,A\,rms$$

The magnitude of the impedance in each phase is:

$$|\mathbf{Z}_\Delta| = \frac{|\mathbf{V}_L|}{|\mathbf{I}_\Delta|} = \frac{208}{2.04} = 101.77\,\Omega$$

Since the power factor angle is 20^o lagging (means current is lagging):

$$\mathbf{Z}_\Delta = 101.77\angle 20^o\,\Omega$$

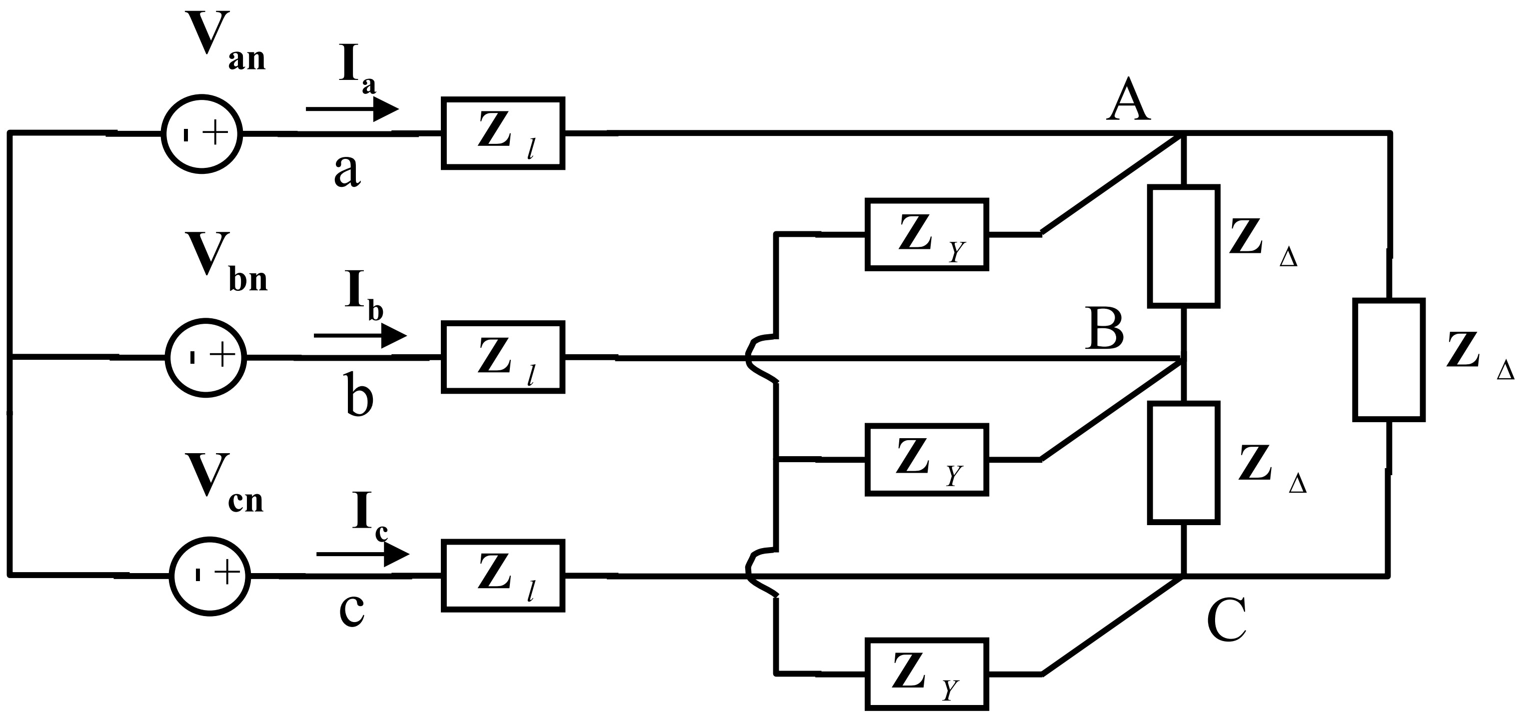
Example 4

Consider a balanced three-phase circuit consisting of:

- → A wye-connected abc-sequence voltage **source** with $\mathbf{V}_{ab} = 208\angle 30^{o}$ V rms;
- → A **wye-connected load**, with per phase impedance 10+6jΩ, in parallel with a **delta-connected** load with per phase impedance 24+9jΩ;
- → A line impedance with per phase impedance 1+0.5j Ω.

Determine the line currents and the load phase voltages when the load is converted into **an equivalent wye**.

Example 4



$$\mathbf{Z}_l = 1 + 0.5j\Omega$$

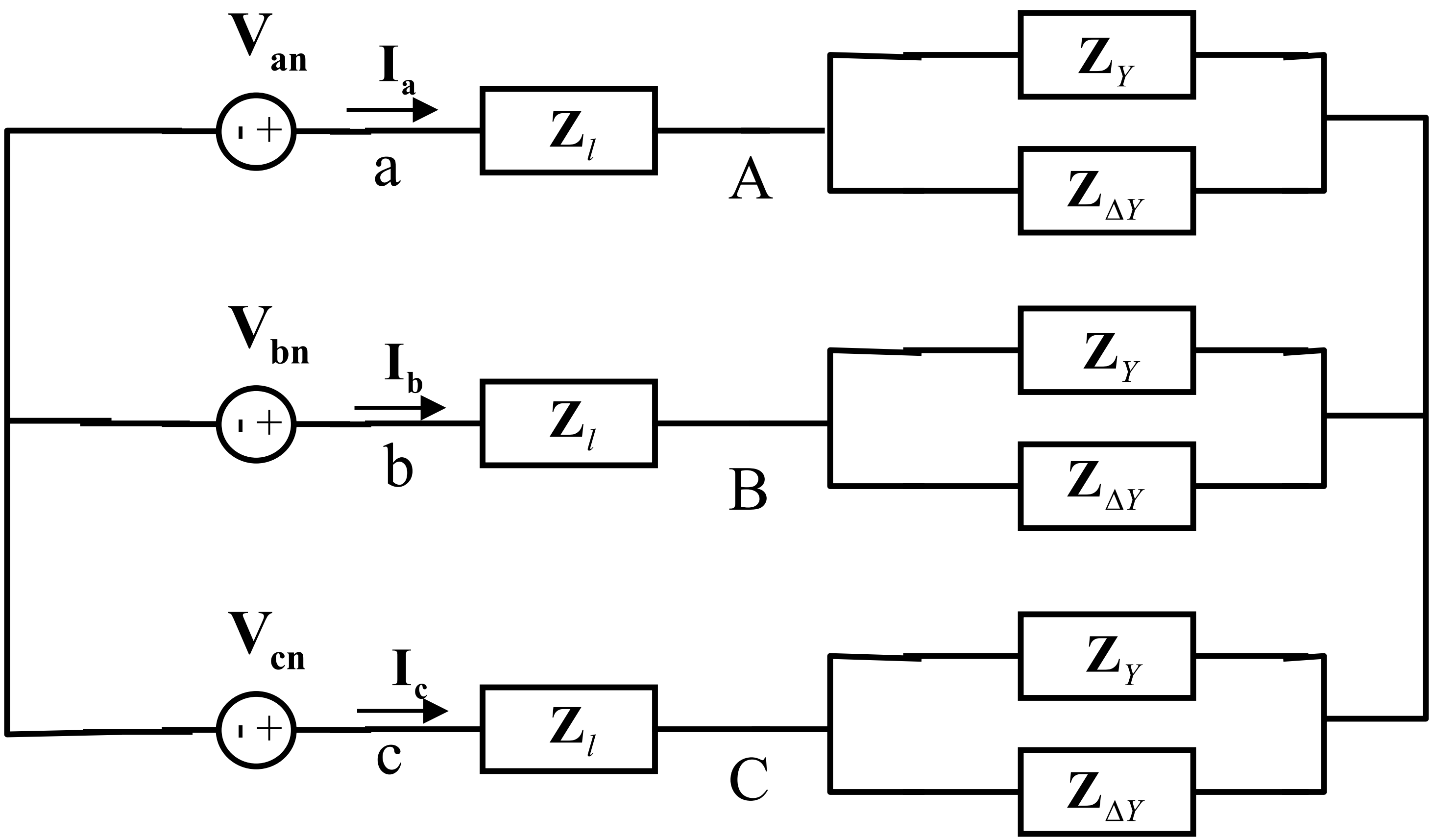
$$\mathbf{Z}_Y = 10 + 6j\Omega$$

$$\mathbf{Z}_\Delta = 24 + 9j\Omega$$

Example 4

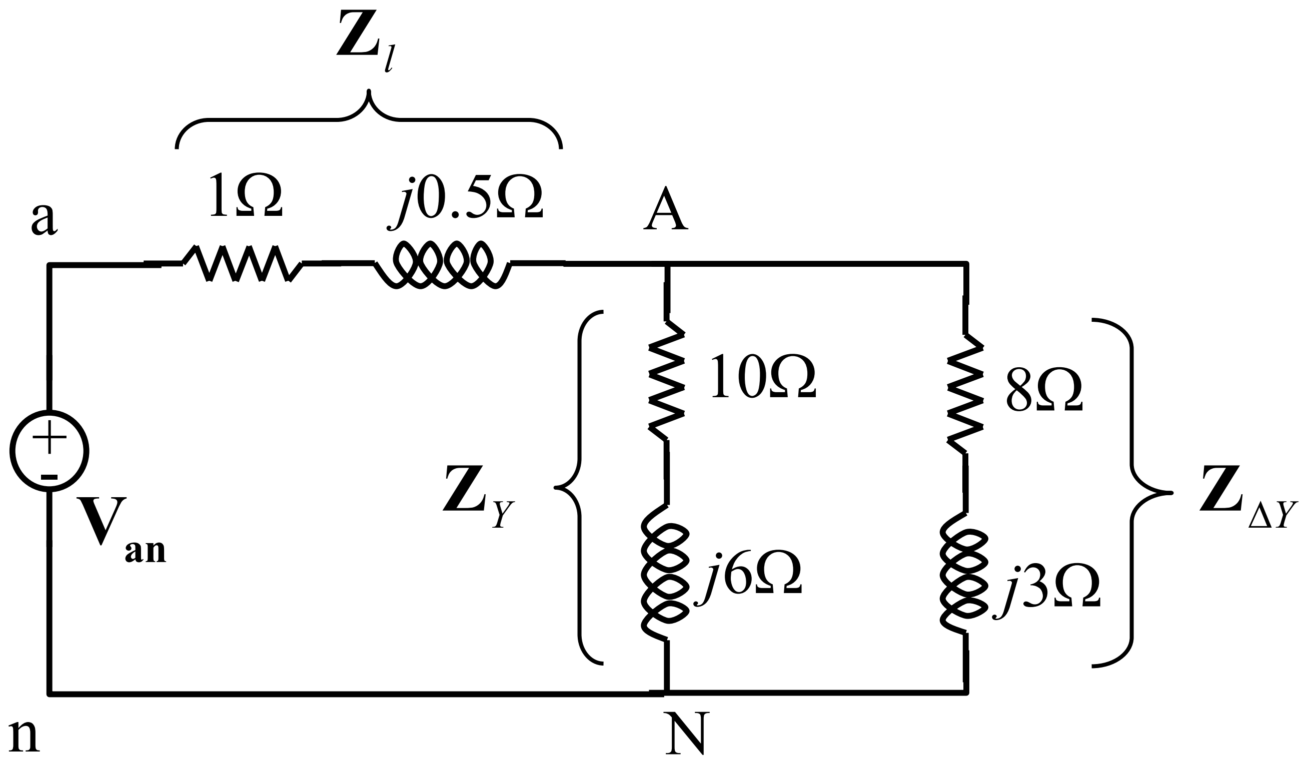
Delta to wye transformation:

$$\mathbf{Z}_{\Delta Y} = \frac{\mathbf{Z}_{\Delta}}{3} = 8 + 3j\Omega$$



Example 4

Per-phase equivalent circuit:



Example 4

Line-voltage at the source is $\mathbf{V_{ab}} = 208\angle 30^o$ V rms **Given**

→ The **source phase voltage** is:

$$\mathbf{V_{an}} = \frac{208}{\sqrt{3}}\angle 0^o\, V\, rms = 120\angle 0^o\, V\, rms$$

→ The line current is: $\mathbf{I_{aA}} = \dfrac{\mathbf{V_{an}}}{\mathbf{Z}_{total}}$

where $\mathbf{Z}_{total} = \mathbf{Z}_l + \mathbf{Z}_Y \,||\, \mathbf{Z}_{\Delta Y} = 5.49 + 2.59\, j\Omega$

→ $\mathbf{I_{aA}} = 19.77\angle -25.26^o\, A\, rms$

→ The **phase voltage at the load** is: $\mathbf{V_{AN}} = \mathbf{I_{aA}}\mathbf{Z}_{load}$

where $\mathbf{Z}_{load} = \mathbf{Z}_Y \,||\, \mathbf{Z}_{\Delta Y} = 4.49 + 2.09\, j\Omega$

→ $\mathbf{V_{AN}} = 97.86\angle -0.31\, V\ rms$

Example 4

Therefore, the balanced three-phase **line currents** and **load phase voltages** are given by:

$$\mathbf{I_{aA}} = 19.77\angle -25.26^o\ A\,rms$$

$$\mathbf{I_{bB}} = 19.77\angle -145.26^o\ A\,rms$$

$$\mathbf{I_{cC}} = 19.77\angle 94.74^o\ A\,rms$$

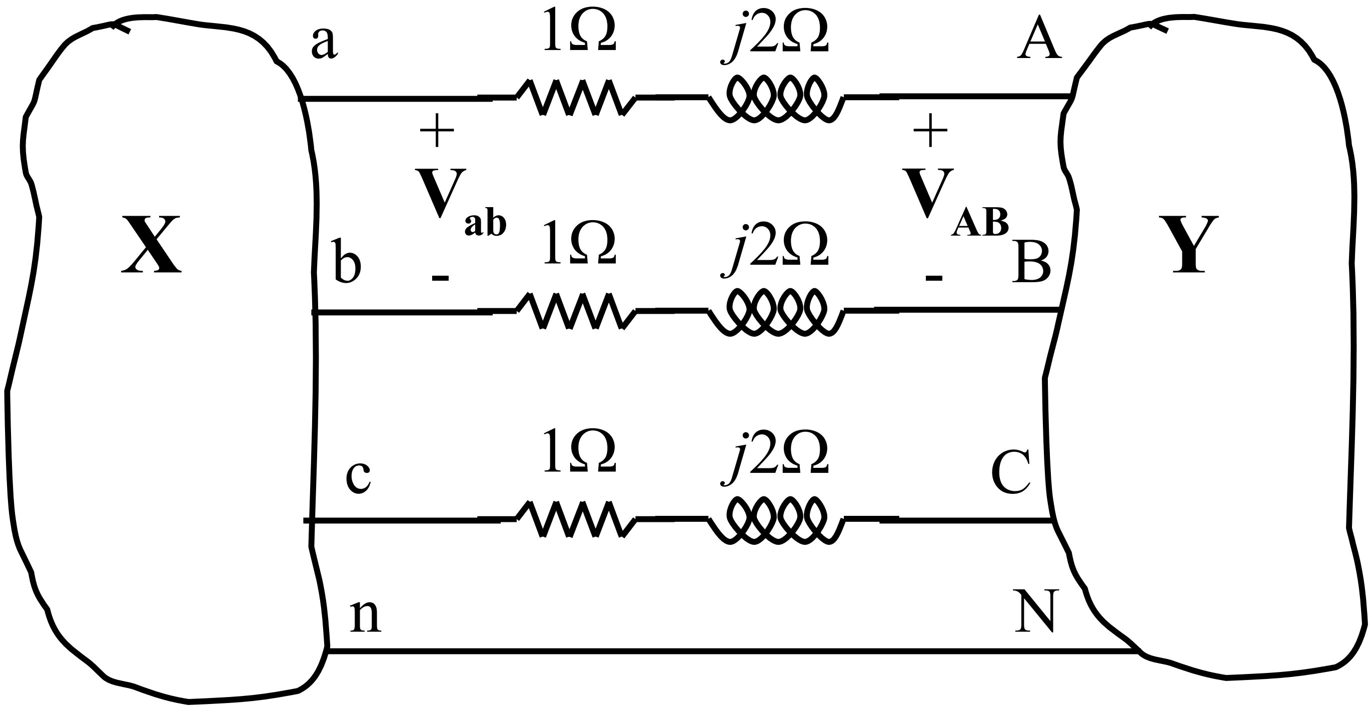
$$\mathbf{V_{AN}} = 97.86\angle -0.31^o\ V\ rms$$

$$\mathbf{V_{BN}} = 97.86\angle -120.31^o\ V\ rms$$

$$\mathbf{V_{BN}} = 97.86\angle 119.69^o\ V\ rms$$

Example 5

Consider the balanced circuit below:



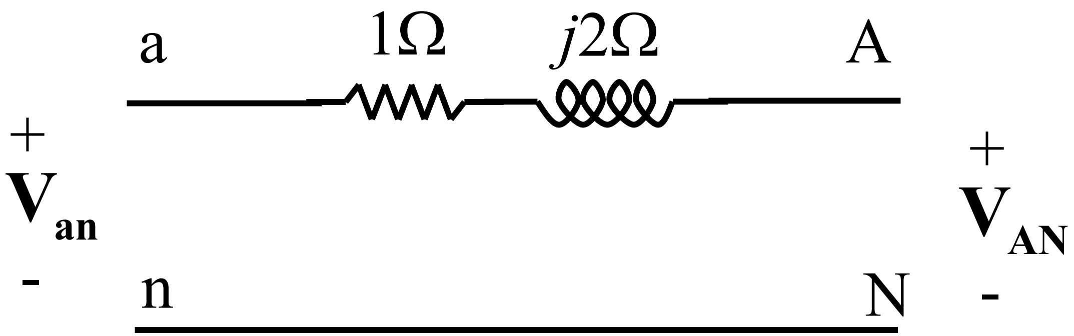
Determine which is the load and which is the source.

Given: $\mathbf{V}_{ab} = 12\angle 0^{o}\ kV\ rms$ $\mathbf{V}_{AB} = 12\angle 5^{o}\ kV\ rms$

$\mathbf{Z}_{line} = 1 + 2j\Omega$

Example 5

The **per-phase** equivalent circuit is:



$$\mathbf{V_{an}} = \frac{12}{\sqrt{3}}\angle -30^o\ kV\ rms \qquad \mathbf{V_{AN}} = \frac{12}{\sqrt{3}}\angle -25^o\ kV\ rms$$

The line current is given by:

$$\mathbf{I_{aA}} = \frac{\mathbf{V_{an}} - \mathbf{V_{AN}}}{\mathbf{Z}_{line}} = 270.3\angle -180.93^o\ A\ rms$$

Example 5

➔ The per-phase power supplied to system **Y** is:

$$P_{Yp} = |\mathbf{V}_{\mathbf{AN}}||\mathbf{I}_{\mathbf{aA}}|\cos(-25+180.93) = -1.71MW$$

The total power supplied to system **Y** is:

$$P_Y = 3P_{Yp} = -5.13MW$$

Thus system Y is not the load, it *supplies* 5.13MW.

➔ The per-phase power supplied to system **X** is:

$$P_{Xp} = -|\mathbf{V}_{\mathbf{an}}||\mathbf{I}_{\mathbf{aA}}|\cos(-30+180.93) = 1.64MW$$

The total power supplied to system **X** is:

$$P_Y = 3P_{Yp} = 4.91MW$$

➔ Is power being conserved?