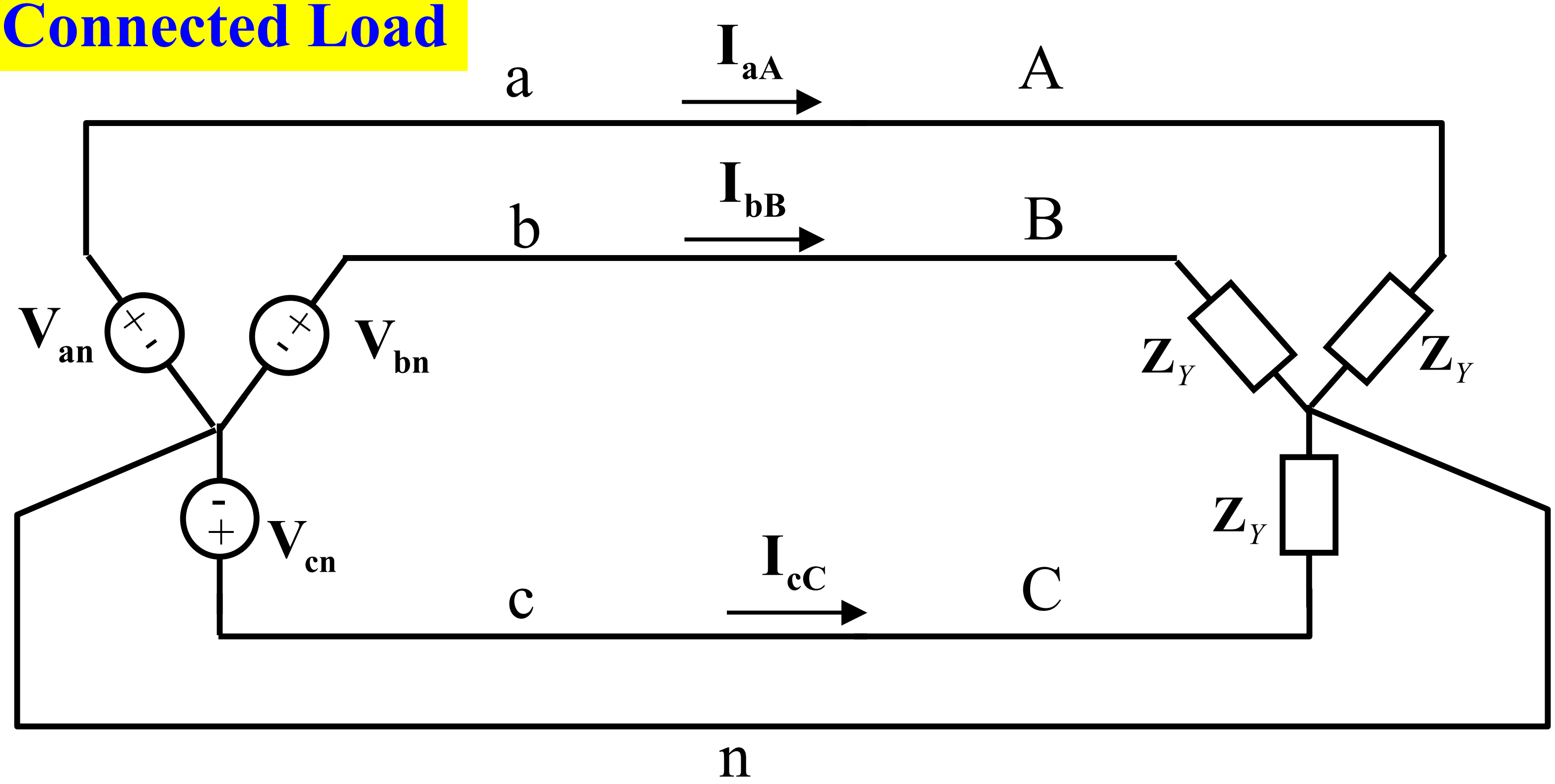
ECSE 210: Circuit Analysis

Lecture #18:

Three-Phase Circuits

Balanced Load Connections

Wye-Connected Load



The above is also called the Wye-Wye configuration (Wye-connected source, Wye connected load).

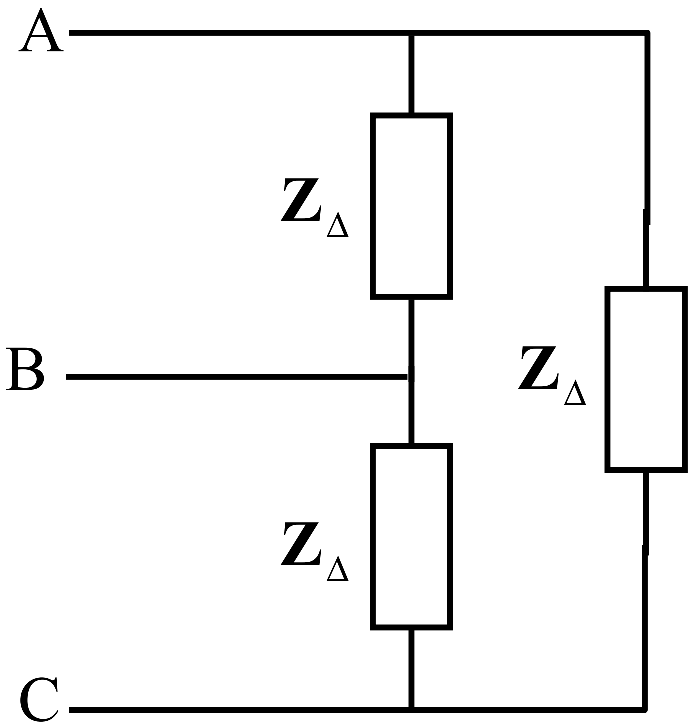
→ **For balanced circuit, current in neutral is zero.**

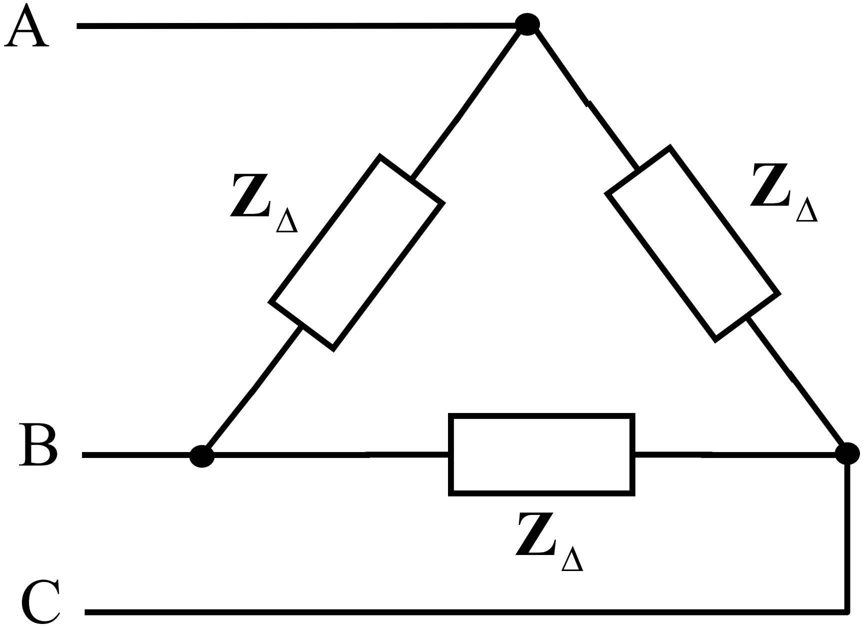
Definitions

1. The ***Phase Voltage*** is the voltage that appears across the load.
2. The ***Line Voltage*** is the voltage difference from line to line.
3. The ***Line Current*** is the current through the transmission line.
4. The ***Phase Current*** is the current through the load.

Balanced Load Connections

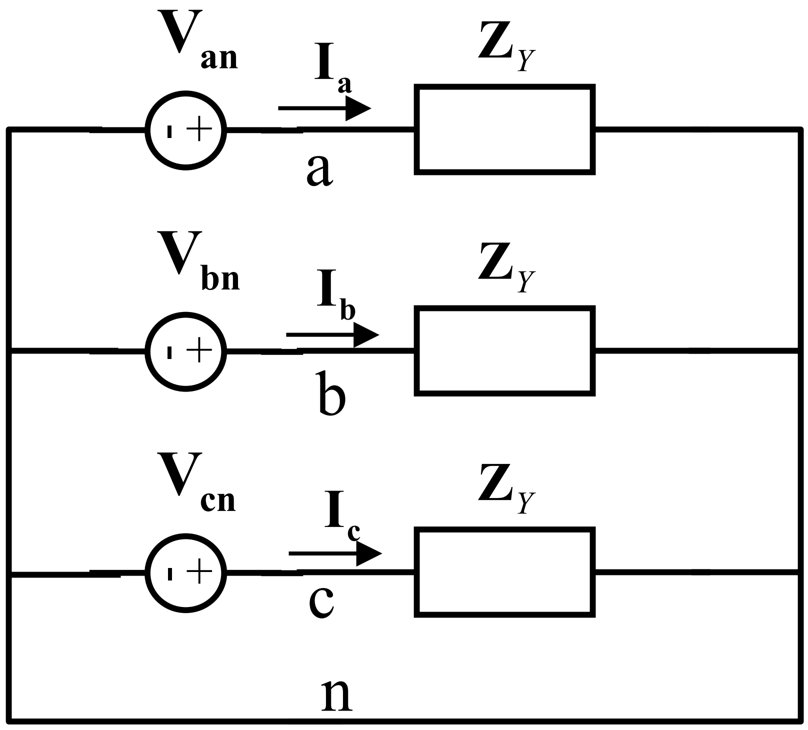
Delta-Connected Load





Note: Delta-connected load has no neutral line.

Balanced Load Connections (Y-Y)



The phase voltages are:

$$\mathbf{V_{an}} = V_p \angle 0^o$$

$$\mathbf{V_{bn}} = V_p \angle -120^o$$

$$\mathbf{V_{cn}} = V_p \angle +120^o$$

The line voltages are:

$$\mathbf{V_{ab}} = \mathbf{V_{an}} - \mathbf{V_{bn}} = \sqrt{3} V_p \angle 30^o$$

$$\mathbf{V_{bc}} = \mathbf{V_{bn}} - \mathbf{V_{cn}} = \sqrt{3} V_p \angle -90^o$$

$$\mathbf{V}_{ca} = \mathbf{V_{cn}} - \mathbf{V_{an}} = \sqrt{3} V_p \angle -210^o = \sqrt{3} V_p \angle +150^o$$

Positive Phase Sequence

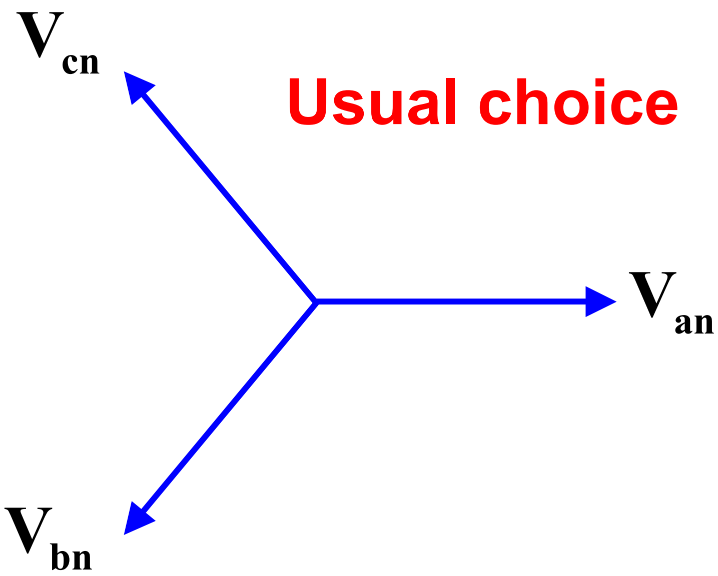
By Convention

<u>Positive Phase Sequence (abc)</u>

$$\mathbf{V_{an}} = V_p \angle 0^o$$

$$\mathbf{V_{bn}} = V_p \angle -120^o$$

$$\mathbf{V_{cn}} = V_p \angle +120^o$$

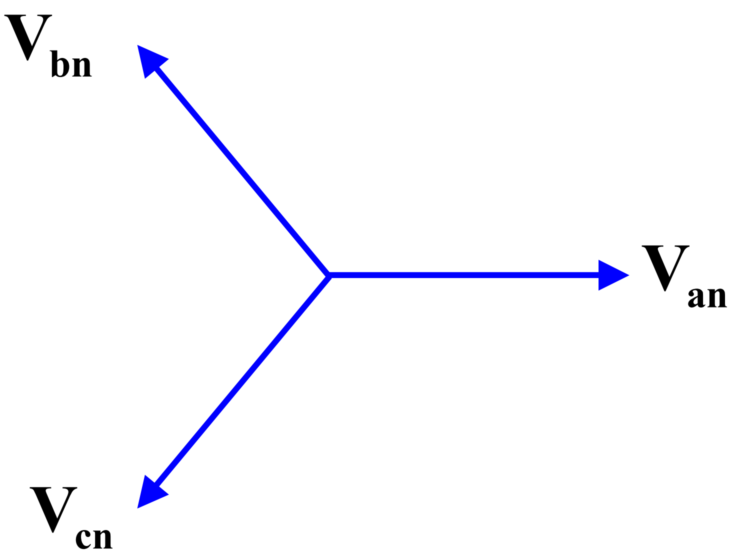


<u>Negative Phase Sequence (acb)</u>

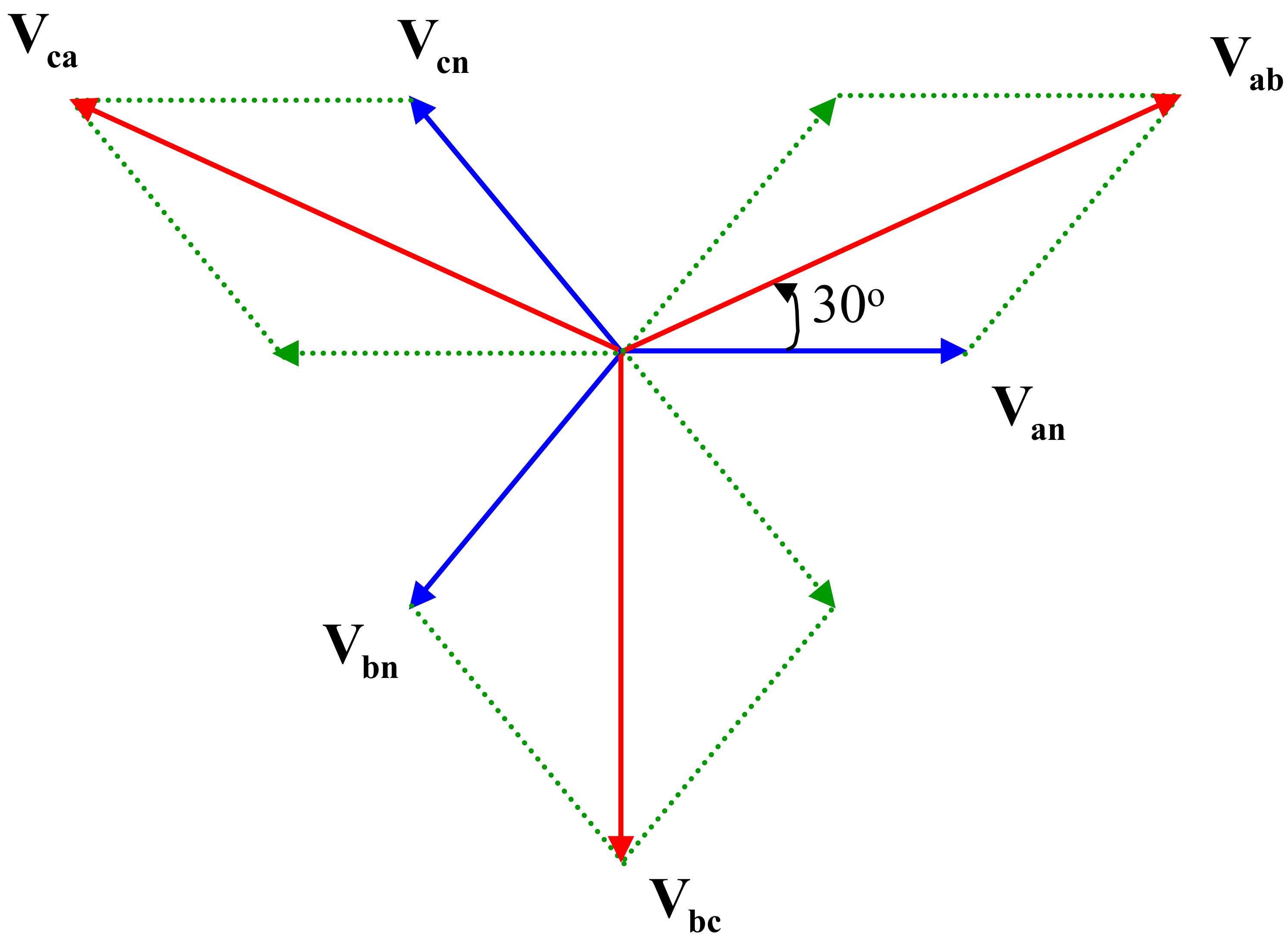
$$\mathbf{V_{an}} = V_p \angle 0^o$$

$$\mathbf{V_{bn}} = V_p \angle +120^o$$

$$\mathbf{V_{cn}} = V_p \angle -120^o$$



Phase and Line Voltages of Wye Load



Balanced Currents

Line currents

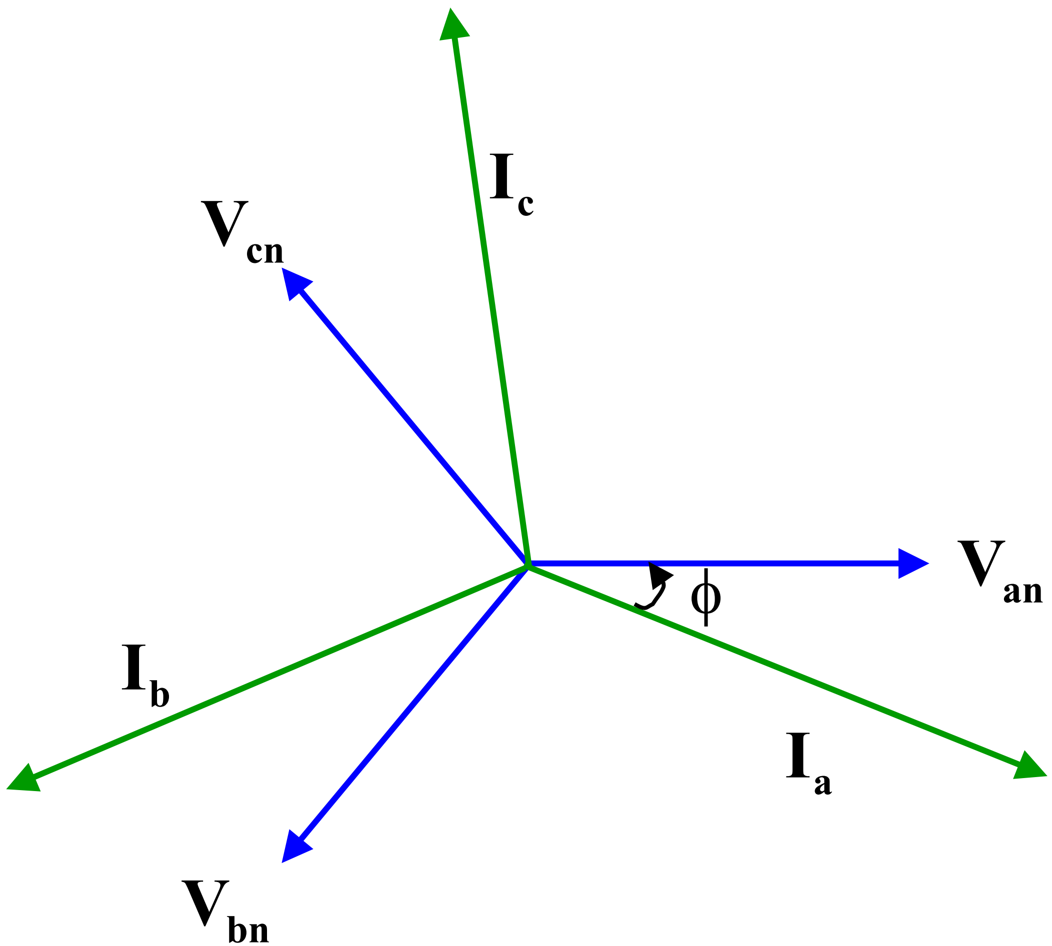
$$\mathbf{I_a} = \frac{\mathbf{V_{an}}}{\mathbf{Z}_Y} = \frac{V_p\angle 0}{|\mathbf{Z}_Y|\angle\phi} = |\mathbf{I_L}|\angle-\phi$$

$$\mathbf{I_b} = \frac{\mathbf{V_{bn}}}{\mathbf{Z}_Y} = \frac{V_p\angle-120}{|\mathbf{Z}_Y|\angle\phi} = |\mathbf{I_L}|\angle(-\phi-120)$$

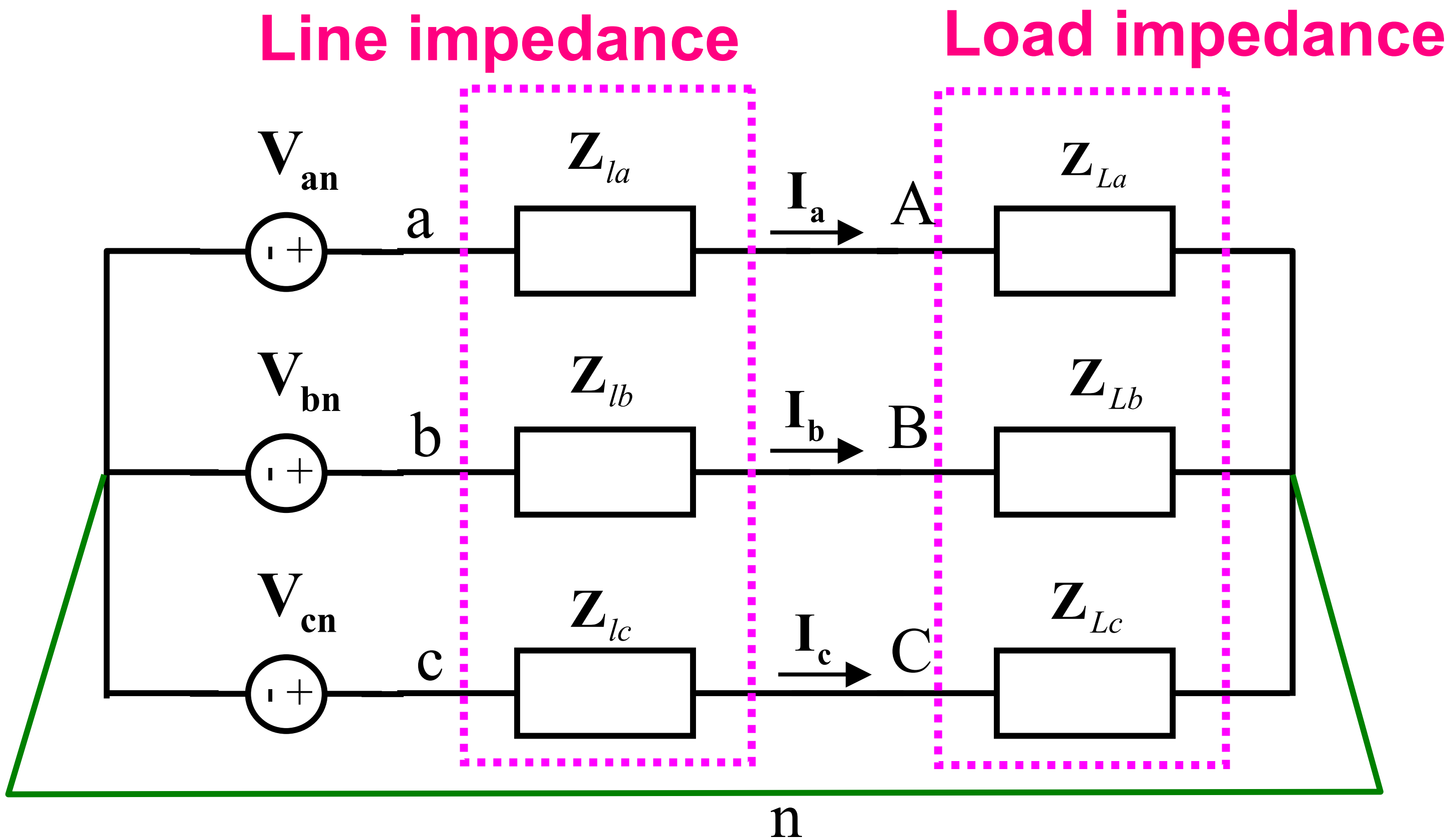
$$\mathbf{I_c} = \frac{\mathbf{V_{cn}}}{\mathbf{Z}_Y} = \frac{V_p\angle+120}{|\mathbf{Z}_Y|\angle\phi} = |\mathbf{I_L}|\angle(-\phi+120)$$

→ Same amplitude
→ Phases are 120 degrees apart
→ Current in neutral is zero
→ The neutral line can be removed from the circuit

Phase Voltages & Line Currents of Wye Load



Wye-Wye Connection with Line Impedance



Balanced case: $|\mathbf{V}_{\mathbf{an}}| = |\mathbf{V}_{\mathbf{bn}}| = |\mathbf{V}_{\mathbf{cn}}|$

$$\angle\mathbf{V}_{\mathbf{an}} = \theta_a; \quad \angle\mathbf{V}_{\mathbf{bn}} = \theta_a - 120; \quad \angle\mathbf{V}_{\mathbf{cn}} = \theta_a - 240$$

$$\mathbf{Z}_{la} = \mathbf{Z}_{lb} = \mathbf{Z}_{lc} \qquad \mathbf{Z}_{La} = \mathbf{Z}_{Lb} = \mathbf{Z}_{Lc}$$

Example 1

Assume that an abc-sequence three-phase voltage source connected in a balanced wye has a line voltage:

$$\mathbf{V_{ab}} = 208\angle -30^o V \ rms$$

Find the three phase voltages:

$$|\mathbf{V_{an}}| = |\mathbf{V_{bn}}| = |\mathbf{V_{cn}}| = V_p = \frac{|\mathbf{V_{ab}}|}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120V \ rms$$

Recall graphical phase relation between phase and line voltages:

$$\mathbf{V_{an}} = 120\angle -60^o V \ rms$$

$$\mathbf{V_{bn}} = 120\angle -180^o V \ rms$$

$$\mathbf{V_{cn}} = 120\angle 60^o V \ rms$$

Example 2

Assume a wye-connected load is supplied by an abc-sequence balanced three-phase wye-connected source with a phase voltage of 120 V rms. Given the line impedance and load impedance per phase are 1+*j*Ω and 20+10*j*Ω, respectively, determine the line current and load voltages.

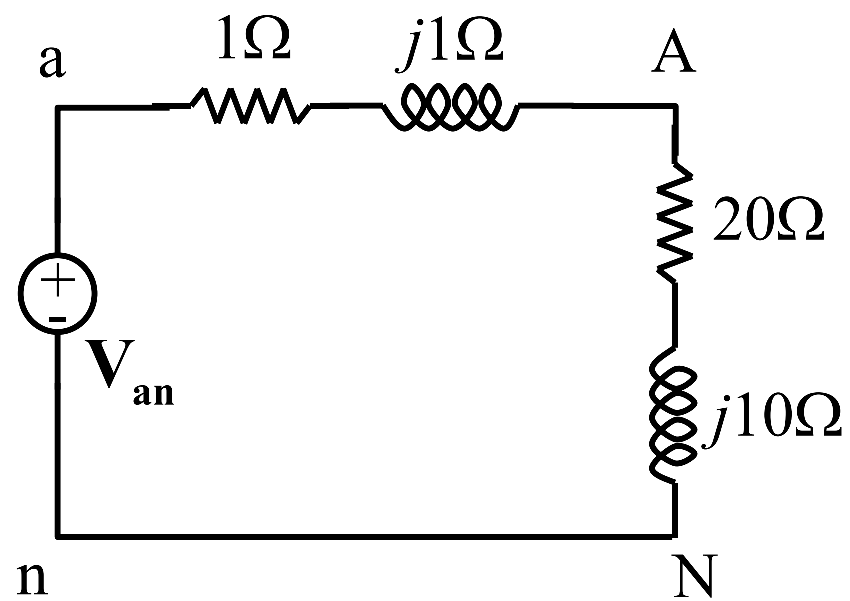
Balanced circuit → ***Analyze one phase only!***

Phase voltages:

$$\mathbf{V_{an}} = 120\angle 0 V \ rms$$

$$\mathbf{V_{bn}} = 120\angle -120^o V \ rms$$

$$\mathbf{V_{cn}} = 120\angle +120^o V \ rms$$



Example 2

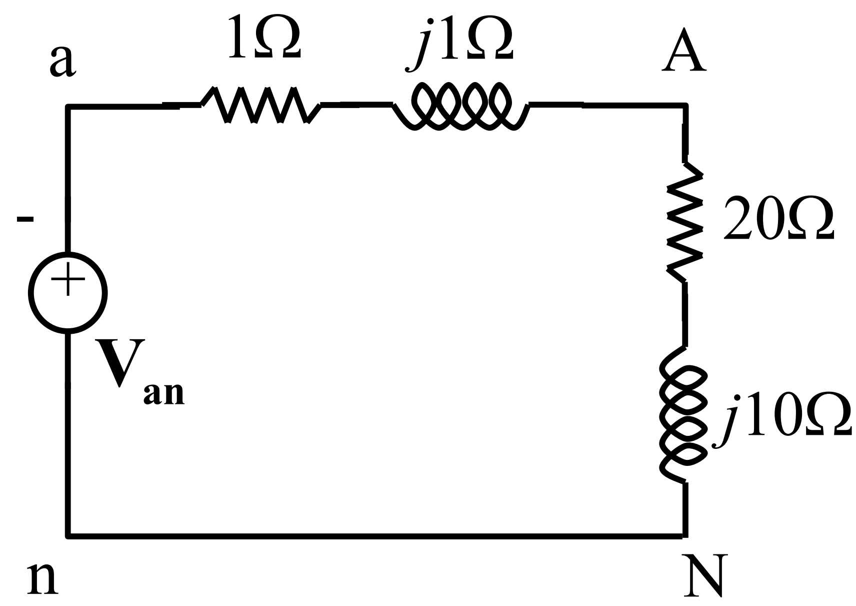
Balanced circuit → ***Analyze one phase only!***

Phase voltages:

$$\mathbf{V_{an}} = 120\angle 0\,V\ \ rms$$

$$\mathbf{V_{bn}} = 120\angle -120^{o}\,V\ \ rms$$

$$\mathbf{V_{cn}} = 120\angle +120^{o}\,V\ \ rms$$



Line current (phase a): $\mathbf{I_{aA}} = \dfrac{120\angle 0}{21 + j11} = 5.06\angle -27.65^{o}\,A\ \ \ rms$

Load voltage (phase a): $\mathbf{V_{AN}} = (5.06\angle -27.65^{o})(20 + j10)$

$$= 113.15\angle -1.08^{o}\,V\ \ \ rms$$

Example 2

Using the abc-sequence we can find phases b and c:

Line Currents:

$$\mathbf{I_{bB}} = 5.06\angle -147.65^o \, A \; rms$$

$$\mathbf{I_{cC}} = 5.06\angle -267.65^o = 5.06\angle 92.35^o \, A \; rms$$

Load Voltages

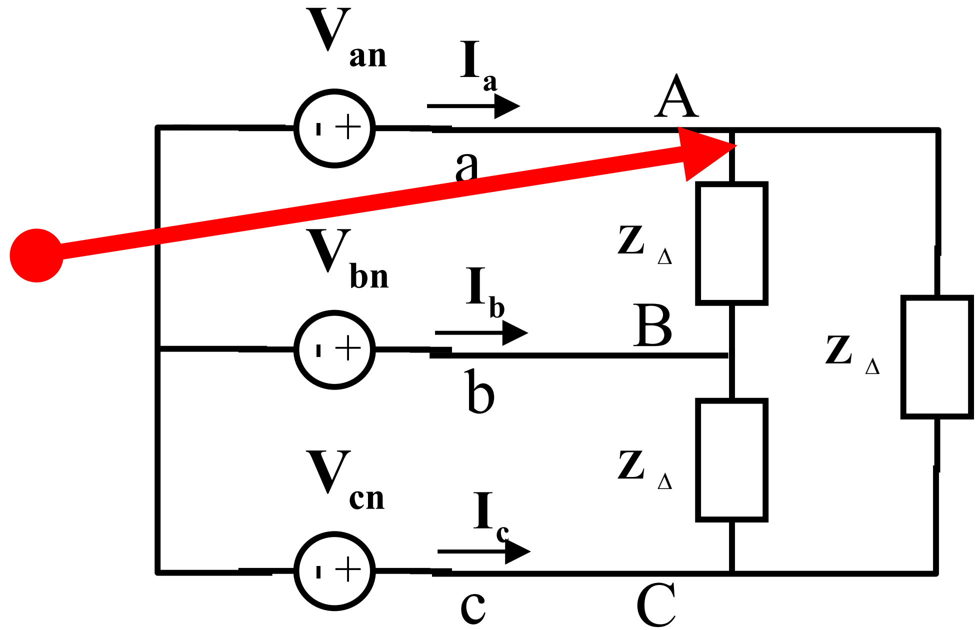
$$\mathbf{V_B} = 113.15\angle -121.08^o \, V \; rms$$

$$\mathbf{V_C} = 113.15\angle -241.08^o = 113.15\angle 118.92^o \, V \; rms$$

Note: For the wye-wye connection, line currents and phase currents are the same.

Balanced Wye-Delta Connection

Note: In this case, line voltages and phase voltages are the same!



For an abc-connected **source**, the **line-to-neutral** voltages are:

$$\mathbf{V_{an}} = V_p \angle 0 V \ rms$$

$$\mathbf{V_{bn}} = V_p \angle -120^o V \ rms$$

$$\mathbf{V_{cn}} = V_p \angle +120^o V \ rms$$

Balanced Wye-Delta Connection

The ***phase*** or ***line*** voltages **(across the load)** are given by:

$$\mathbf{V_{ab}} = \sqrt{3}V_p \angle 30 V\ rms$$

$$\mathbf{V_{bc}} = \sqrt{3}V_p \angle -90 V\ rms$$

$$\mathbf{V_{ca}} = \sqrt{3}V_p \angle -210 = \sqrt{3}V_p \angle 150 V\ rms$$

For a load $\mathbf{Z}_\Delta = |\mathbf{Z}_\Delta| \angle \phi$ The ***phase*** currents at the load are:

$$\mathbf{I_{AB}} = \frac{\mathbf{V_{AB}}}{\mathbf{Z}_\Delta} = I_\Delta \angle 30 - \phi$$

$$\mathbf{I_{BC}} = \frac{\mathbf{V_{BC}}}{\mathbf{Z}_\Delta} = I_\Delta \angle -90 - \phi$$

$$\mathbf{I_{CA}} = \frac{\mathbf{V_{CA}}}{\mathbf{Z}_\Delta} = I_\Delta \angle 150 - \phi$$

Balanced Wye-Delta Connection

The ***line currents*** can be found from the ***phase currents*** using KCL:

$$\mathbf{I_{aA}} = \mathbf{I_{AB}} - \mathbf{I_{CA}}$$

$$\mathbf{I_{aA}} = \sqrt{3} I_\Delta \angle -\phi$$

$$\mathbf{I_{bB}} = \sqrt{3} I_\Delta \angle -120 - \phi$$

$$\mathbf{I_{cC}} = \sqrt{3} I_\Delta \angle -240 - \phi$$