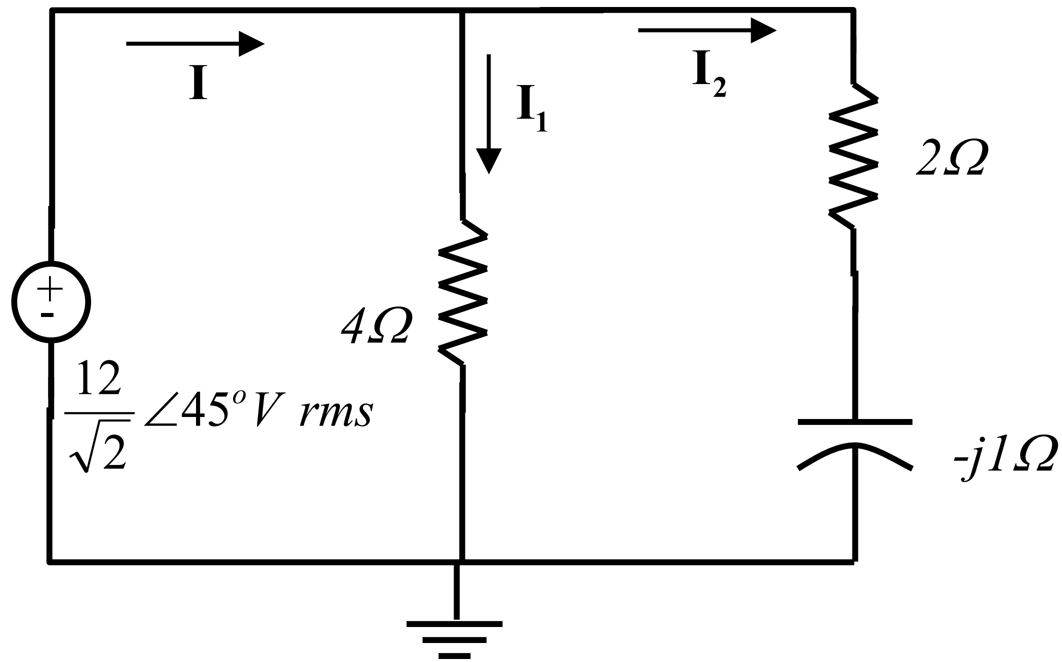
ECSE 210: Circuit Analysis

Lecture #17:

Steady State Power Analysis

Three-Phase Circuits

Conservation of Power



$$\mathbf{I}_1 = \frac{\frac{12}{\sqrt{2}}\angle 45^o}{4} = \frac{3}{\sqrt{2}}\angle 45^o\, A\, rms$$

$$\mathbf{I}_2 = \frac{\frac{12}{\sqrt{2}}\angle 45^o}{2-j} = \frac{5.37}{\sqrt{2}}\angle 71.57^o\, A\, rms$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \frac{8.16}{\sqrt{2}}\angle 62.08^o\, A\, rms$$

Average power absorbed by 4Ω resistance:

$$P_4 = \frac{1}{2}(3)(12) = 18W$$

Average power absorbed by 2Ω resistance:

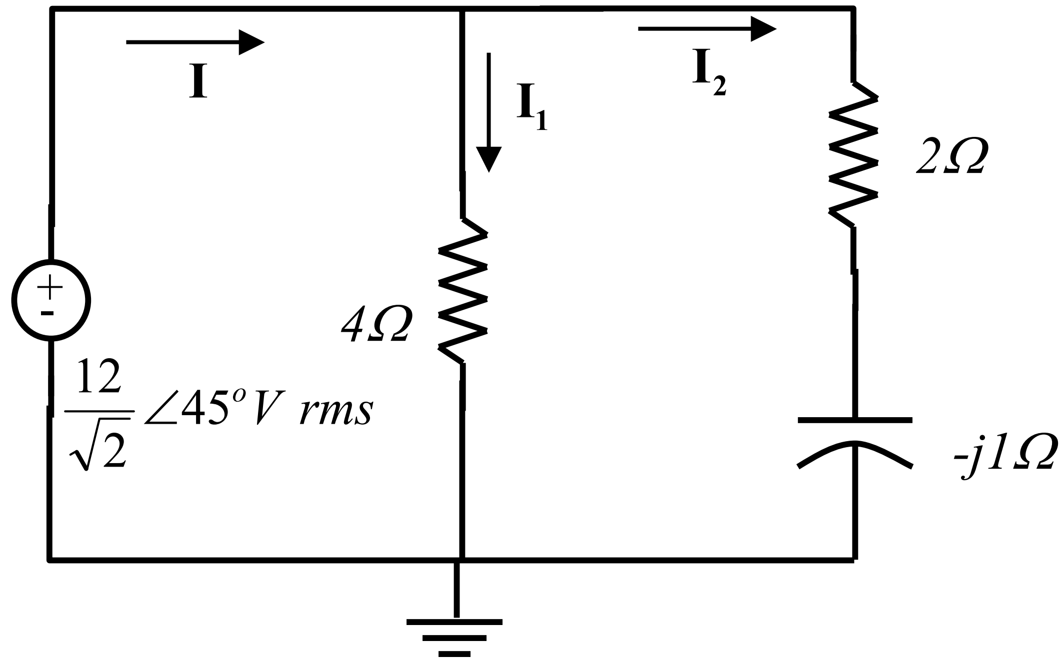
$$P_2 = \frac{1}{2}(5.37)^2 = 28.8W$$

Conservation of Power

$$\mathbf{I}_1 = \frac{\frac{12}{\sqrt{2}}\angle 45^o}{4} = \frac{3}{\sqrt{2}}\angle 45^o\,A\,rms$$

$$\mathbf{I}_2 = \frac{\frac{12}{\sqrt{2}}\angle 45^o}{2-j} = \frac{5.37}{\sqrt{2}}\angle 71.57^o\,A\,rms$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \frac{8.16}{\sqrt{2}}\angle 62.08^o\,A\,rms$$



Total power supplied by source:

$$P_s = \left(\frac{12}{\sqrt{2}}\right)\left(\frac{8.16}{\sqrt{2}}\right)\cos(45-62.08) = 46.8W$$

→ Average power is conserved $P_2 + P_4 = 18.8 + 18 = 46.8 = P_s$

→ We can also verify that complex power is conserved.

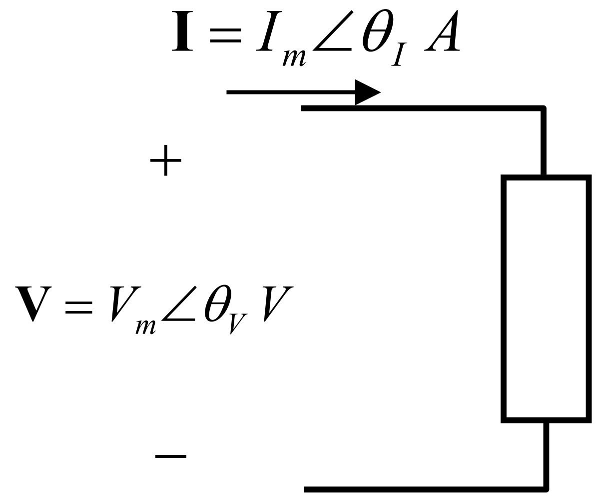
Summary

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_I)$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2} V_m I_m \angle(\theta_V - \theta_I)$$

$$P_{avg} = \text{Re}[\mathbf{S}]$$

$$pf =_m \cos(\theta_v - \theta_I)$$

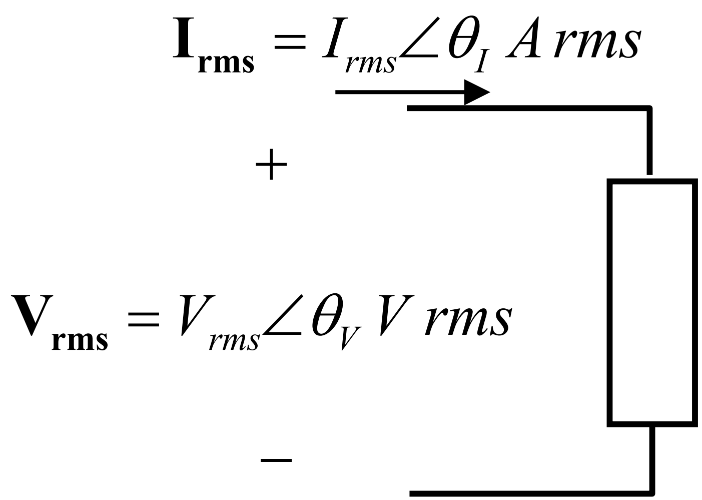


$$P_{ave} = V_{rms} I_{rms} \cos(\theta_v - \theta_I)$$

$$\mathbf{S} = \mathbf{V_{rms}}\mathbf{I^*_{rms}} = V_{rms} I_{rms} \angle(\theta_V - \theta_I)$$

$$P_{avg} = \text{Re}[\mathbf{S}]$$

$$pf =_m \cos(\theta_v - \theta_I)$$



Summary

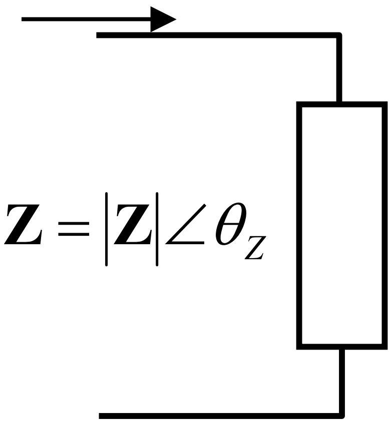
$$\mathbf{V} = \mathbf{I}\mathbf{Z}$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}\mathbf{I}\,\mathbf{I}^*\mathbf{Z} = \frac{1}{2}I_m^2\mathbf{Z}$$

$$P_{avg} = \mathrm{Re}[\mathbf{S}] = \frac{1}{2}I_m^2|\mathbf{Z}|\cos(\theta_Z)$$

$$pf = \cos(\theta_v - \theta_I) = \cos(\theta_Z)$$

$$\mathbf{I} = I_m\angle\theta_I\ A$$



$$\mathbf{Z} = |\mathbf{Z}|\angle\theta_Z$$

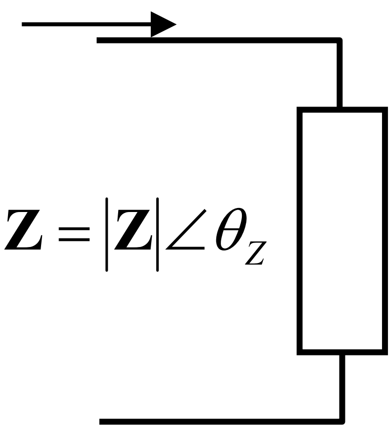
$$\mathbf{V_{rms}} = \mathbf{I_{rms}}\mathbf{Z}$$

$$\mathbf{S} = \mathbf{V_{rms}}\mathbf{I^*_{rms}} = \mathbf{I_{rms}}\,\mathbf{I^*_{rms}}\mathbf{Z} = I_{rms}^2\mathbf{Z}$$

$$P_{avg} = \mathrm{Re}[\mathbf{S}] = I_{rms}^2|\mathbf{Z}|\cos(\theta_Z)$$

$$pf = \cos(\theta_v - \theta_I) = \cos(\theta_Z)$$

$$\mathbf{I_{rms}} = I_{rms}\angle\theta_I\ A\ rms$$



$$\mathbf{Z} = |\mathbf{Z}|\angle\theta_Z$$

Summary

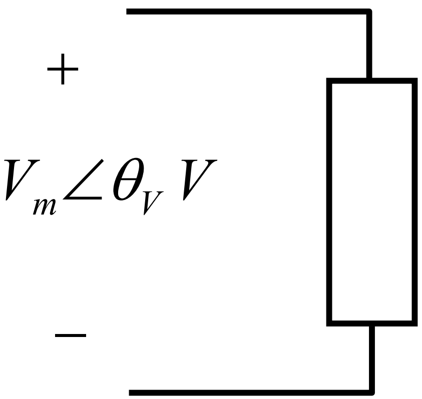
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}}$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}\frac{\mathbf{V}\mathbf{V}^*}{\mathbf{Z}^*} = \frac{1}{2}\frac{V_m^2}{\mathbf{Z}^*} = \frac{1}{2}\frac{V_m^2}{|\mathbf{Z}|}\angle\theta_Z$$

$$P_{ave} = \mathrm{Re}[\mathbf{S}] = \frac{1}{2}\frac{V_m^2}{|\mathbf{Z}|}\cos(\theta_Z)$$

$$pf = \cos(\theta_v - \theta_I) = \cos(\theta_Z)$$

$$\mathbf{V} = V_m\angle\theta_V\ V$$



$$\mathbf{Z} = |\mathbf{Z}|\angle\theta_Z$$

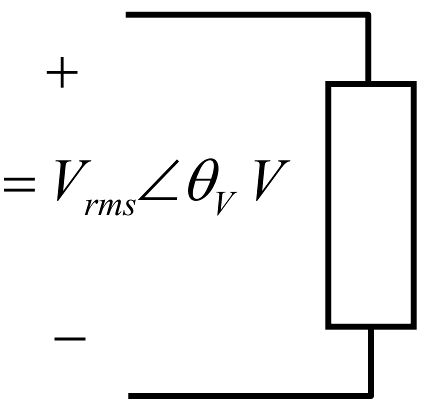
$$\mathbf{I}_{\mathbf{rms}} = \frac{\mathbf{V}_{\mathbf{rms}}}{\mathbf{Z}}$$

$$\mathbf{S} = \mathbf{V}_{\mathbf{rms}}\mathbf{I}^*_{\mathbf{rms}} = \frac{\mathbf{V}_{\mathbf{rms}}\mathbf{V}^*_{\mathbf{rms}}}{\mathbf{Z}^*} = \frac{V_{rms}^2}{\mathbf{Z}^*} = \frac{V_{rms}^2}{|\mathbf{Z}|}\angle\theta_Z$$

$$P_{avg} = \mathrm{Re}[\mathbf{S}] = \frac{V_{rms}^2}{|\mathbf{Z}|}\cos(\theta_Z)$$

$$pf = \cos(\theta_v - \theta_I) = \cos(\theta_Z)$$

$$\mathbf{V}_{\mathbf{rms}} = V_{rms}\angle\theta_V\ V$$



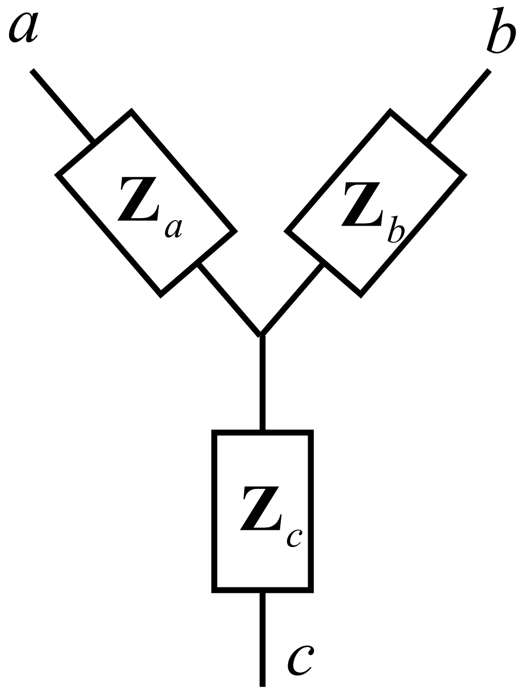
$$\mathbf{Z} = |\mathbf{Z}|\angle\theta_Z$$

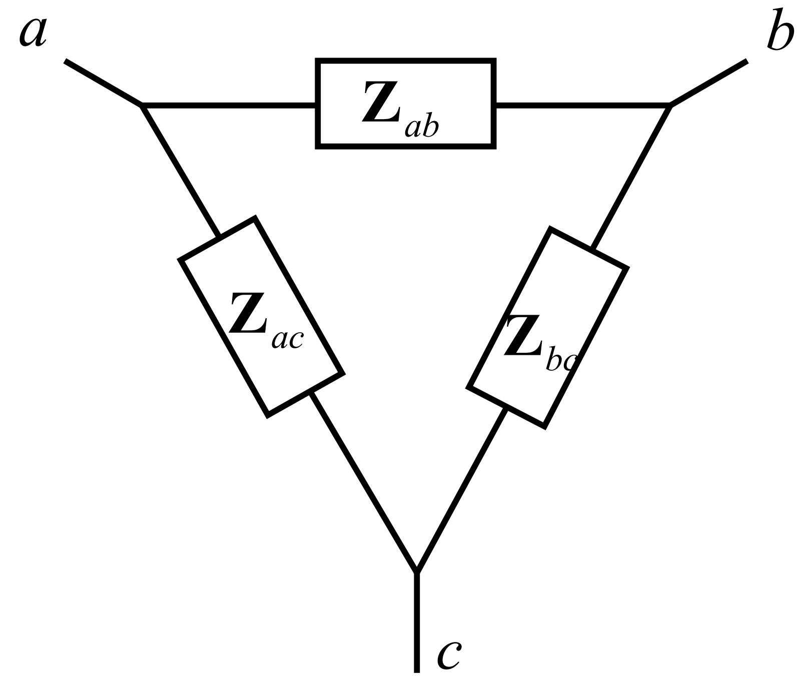
Summary

Other key points:

1. Power factor correction.
2. RMS value of general periodic signals.
3. Conservation of power.
4. Do not apply superposition to power.
5. Maximum average power transfer and impedance matching.
6. Capacitors and inductors are lossless elements (do not consume average power).

Wye-Delta Transformation





$$\mathbf{Z}_a = \frac{\mathbf{Z}_{ab}\mathbf{Z}_{ac}}{\mathbf{Z}_{ab} + \mathbf{Z}_{ac} + \mathbf{Z}_{bc}}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_{ab}\mathbf{Z}_{bc}}{\mathbf{Z}_{ab} + \mathbf{Z}_{ac} + \mathbf{Z}_{bc}}$$

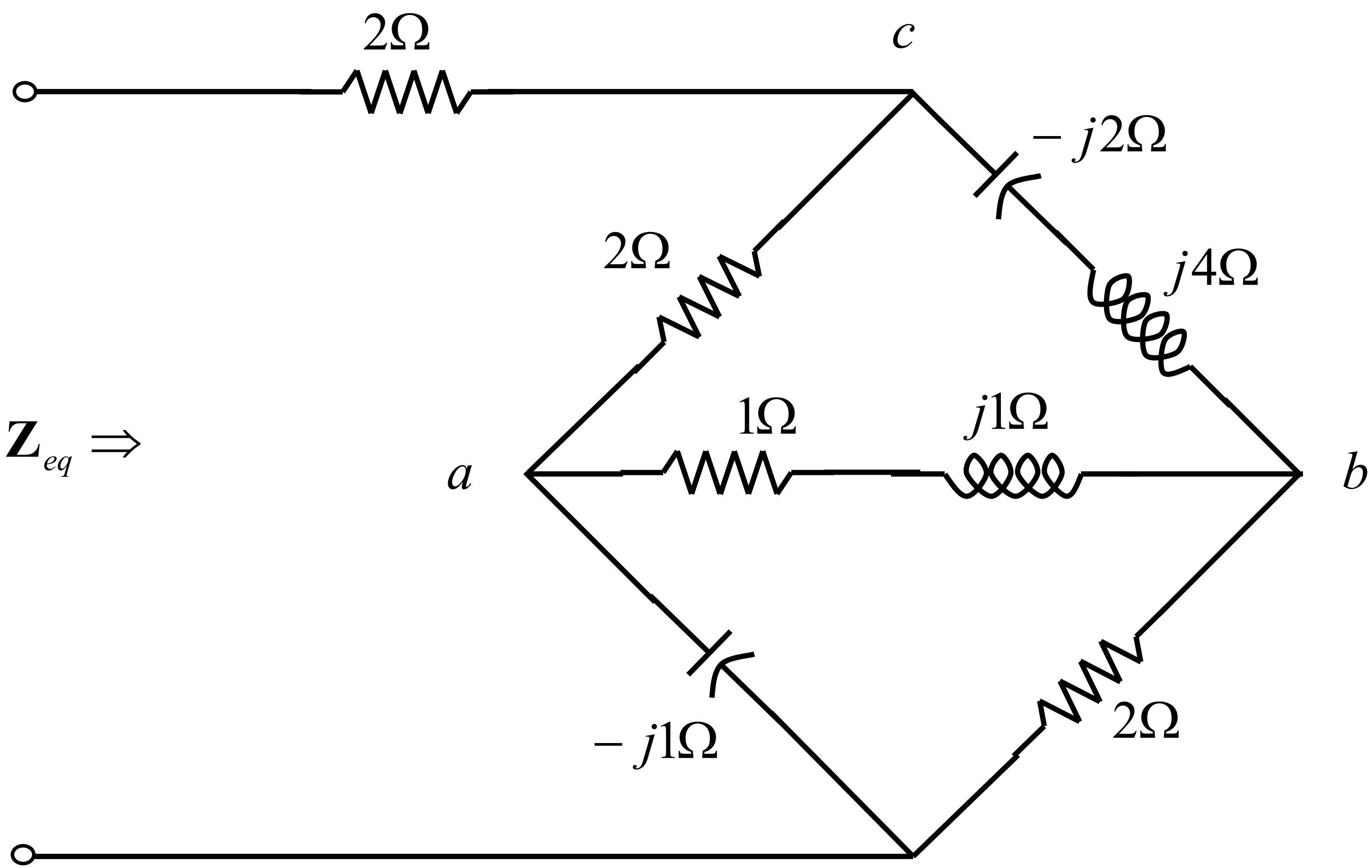
$$\mathbf{Z}_c = \frac{\mathbf{Z}_{ac}\mathbf{Z}_{bc}}{\mathbf{Z}_{ab} + \mathbf{Z}_{ac} + \mathbf{Z}_{bc}}$$

$$\mathbf{Z}_{ab} = \frac{\mathbf{Z}_a\mathbf{Z}_b + \mathbf{Z}_b\mathbf{Z}_c + \mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_c}$$

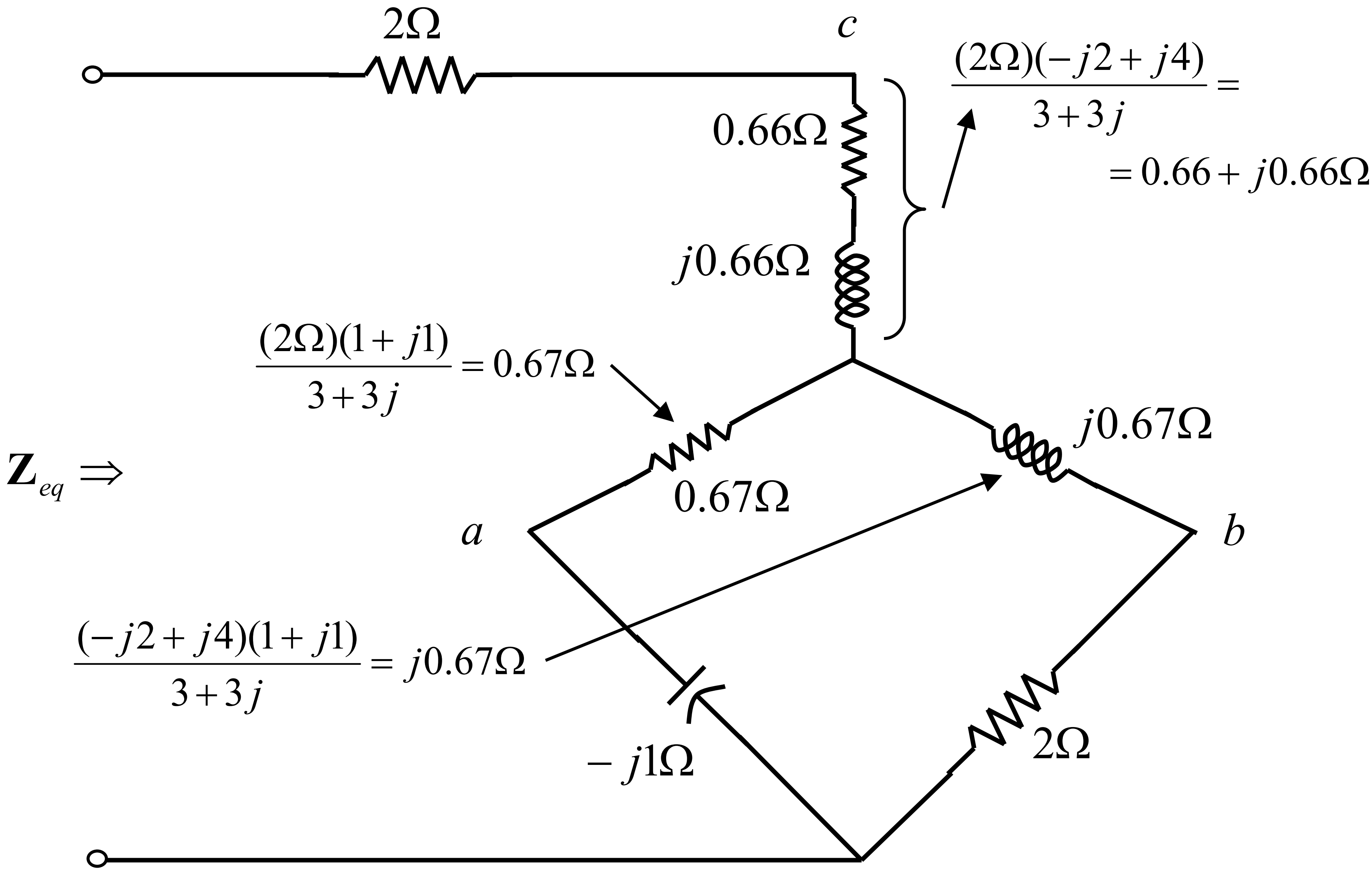
$$\mathbf{Z}_{ac} = \frac{\mathbf{Z}_a\mathbf{Z}_b + \mathbf{Z}_b\mathbf{Z}_c + \mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_b}$$

$$\mathbf{Z}_{bc} = \frac{\mathbf{Z}_a\mathbf{Z}_b + \mathbf{Z}_b\mathbf{Z}_c + \mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a}$$

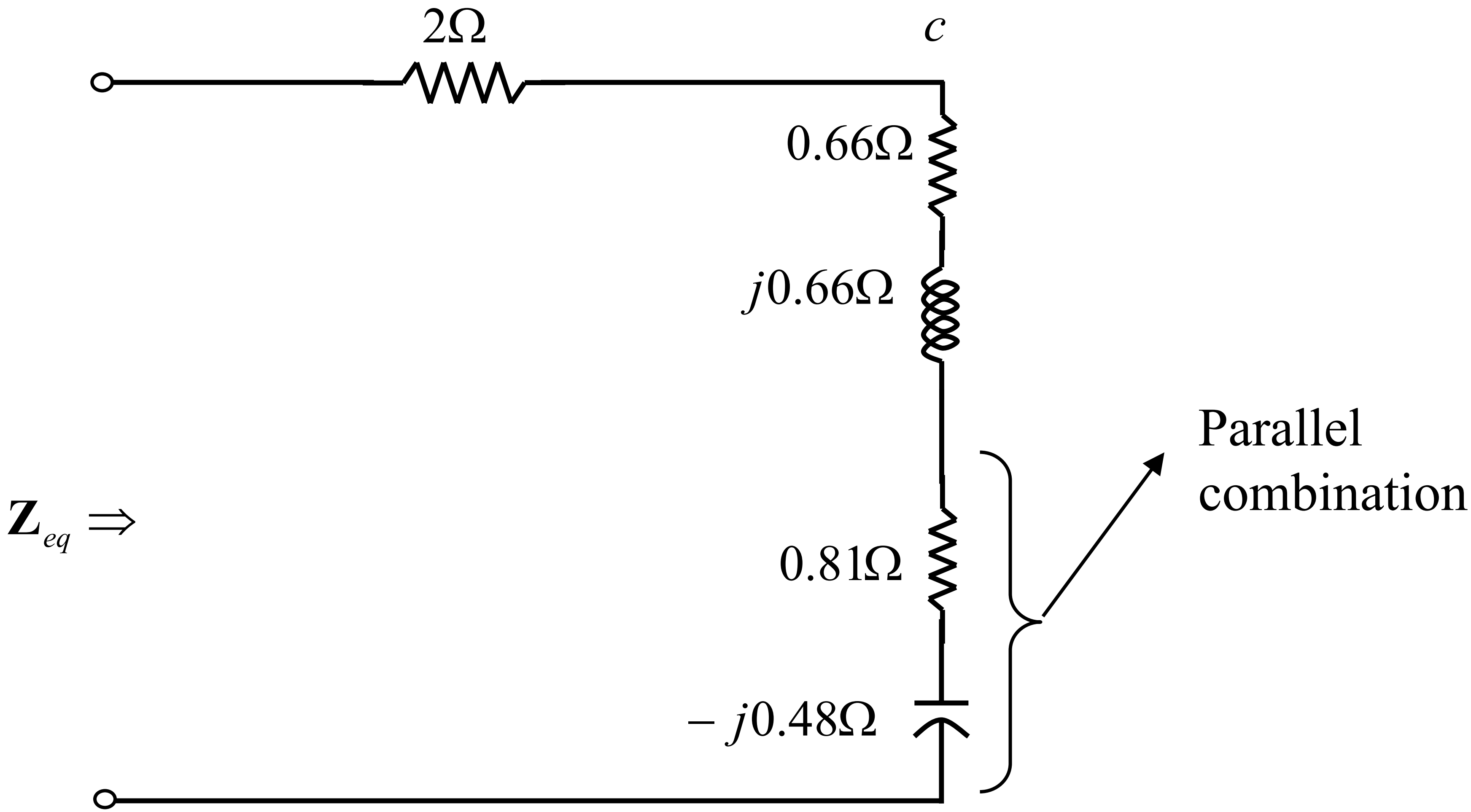
Wye-Delta Example



Wye-Delta Example

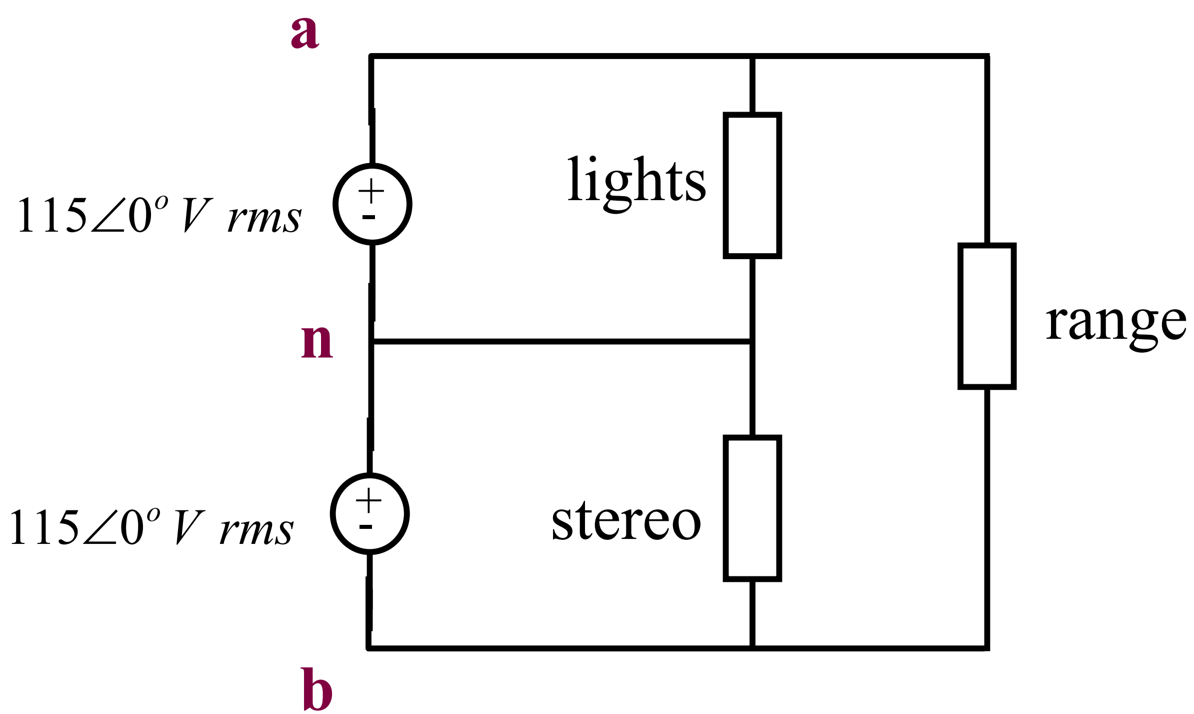


Wye-Delta Example



$$\mathbf{Z}_{eq} = 3.47 + j0.18\Omega$$

Single-Phase Three-Wire Circuits



1. Typical for home power.
2. Two sources ***in phase***.
3. Large voltage is used for high power elements such as water heaters and ranges.

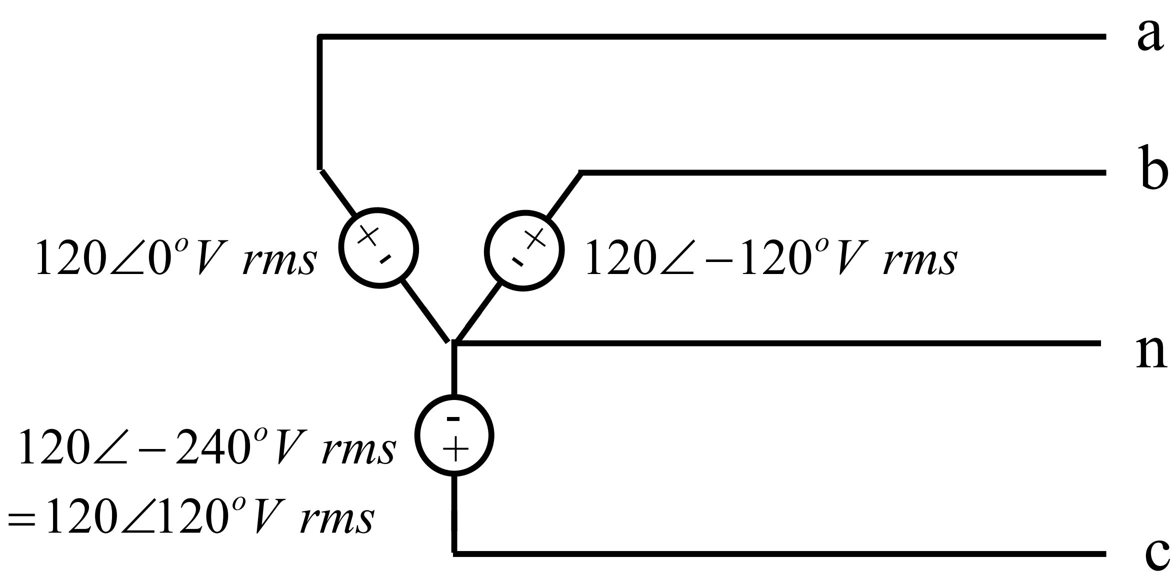
Poly-Phase Circuits

- → Multiple sources with *different phases.*
- → For practical reasons, most electric power is transmitted in poly-phase electric circuits.

Terminology:

- ➔ Balanced Three-Phase voltages:
  - → Each voltage has the *same magnitude and frequency*.
  - → Each voltage is 120^{o} out of phase with the other two.
- ➔ Balanced Three-Phase currents:
  - → Each current has the *same magnitude and frequency*.
  - → Each current is 120^{o} out of phase with the other two.
- ➔ Balanced Three-Phase circuits:
  - → Three-phase voltage system is balanced
  - → Resulting three-phase currents are balanced.

Three-Phase Circuits



→ Standard double subscript notation is used. $\mathbf{V_{ij}}$ is the voltage at point i with respect to the voltage at point j. $\mathbf{V_{ij} = V_i - V_j}$

Instantaneous Voltage of Balanced Circuits

The phasor three phase voltages:

$$\mathbf{V_{an}} = 120\angle 0^o\ V\ rms$$

$$\mathbf{V_{bn}} = 120\angle -120^o\ V\ rms$$

$$\mathbf{V_{cn}} = 120\angle -240^o\ V\ rms$$

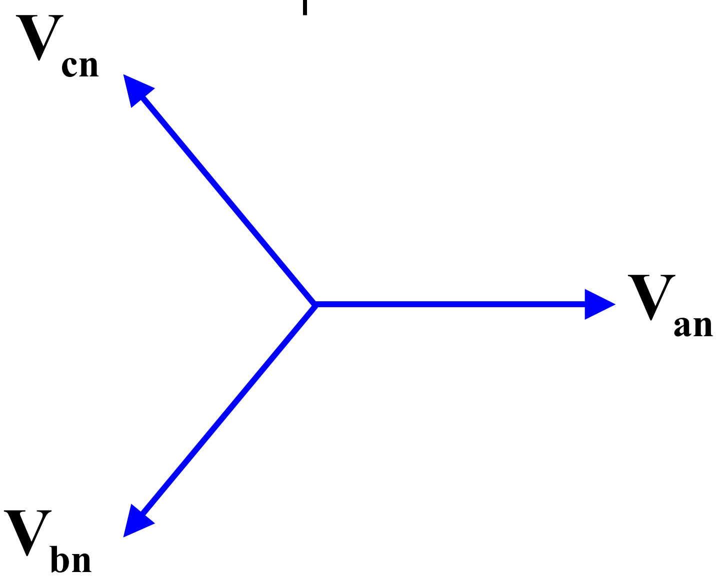
$$= 120\angle 120^o\ V\ rms$$

Three phase time-domain voltages:

$$v_{an}(t) = 120\sqrt{2}\cos\omega t\ V$$

$$v_{bn}(t) = 120\sqrt{2}\cos(\omega t - 120)\ V$$

$$v_{cn}(t) = 120\sqrt{2}\cos(\omega t + 120)\ V$$



Instantaneous Current of Balanced Circuits

Balanced currents:

$$\mathbf{I_a} = I_{rms}\angle -\theta\, A\ rms$$

$$\mathbf{I_b} = I_{rms}\angle(-\theta - 120^o)\, A\ rms$$

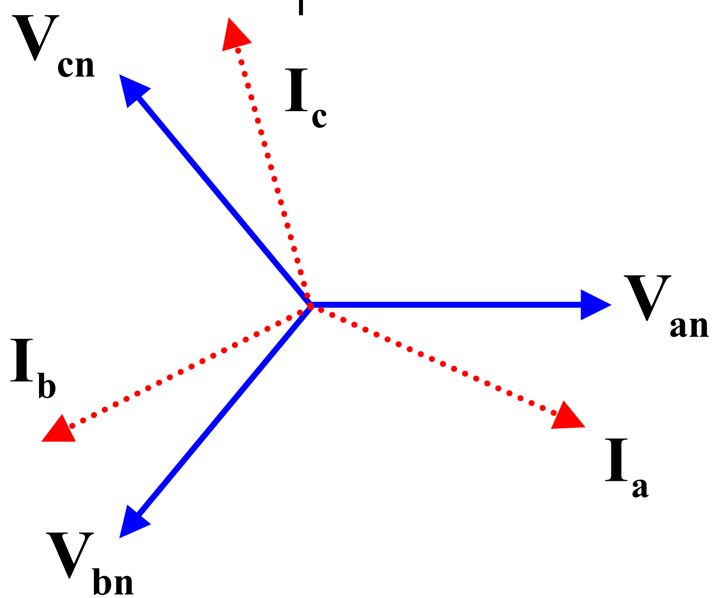
$$\mathbf{I_c} = I_{rms}\angle(-\theta + 120^o)\, A\ rms$$
$$= 120\angle 120^o\ V\ rms$$

Three phase time-domain currents:

$$i_a(t) = I_{rms}\sqrt{2}\cos(\omega t - \theta)\ A$$

$$i_b(t) = I_{rms}\sqrt{2}\cos(\omega t - \theta - 120)\ A$$

$$i_c(t) = I_{rms}\sqrt{2}\cos(\omega t - \theta + 120)\ A$$



Instantaneous Power of Balanced Circuits

$$p(t) = p_a(t) + p_b(t) + p_c(t)$$

$$p_a(t) = (V_{rms}\sqrt{2})(I_{rms}\sqrt{2})\cos(\omega t)\cos(\omega t - \theta)$$

$$= 2V_{rms}I_{rms}\cos(\omega t)\cos(\omega t - \theta)$$

$$p_b(t) = 2V_{rms}I_{rms}\cos(\omega t - 120)\cos(\omega t - \theta - 120)$$

$$p_c(t) = 2V_{rms}I_{rms}\cos(\omega t + 120)\cos(\omega t - \theta + 120)$$

$$p(t) = 2V_{rms}I_{rms}[\cos(\omega t)\cos(\omega t - \theta) + \cos(\omega t - 120)\cos(\omega t - \theta - 120) + \cos(\omega t + 120)\cos(\omega t - \theta + 120)]$$

Instantaneous Power of Balanced Circuits

$$p(t) = 2V_{rms}I_{rms}[\cos(\omega t)\cos(\omega t - \theta)$$
$$+\cos(\omega t - 120)\cos(\omega t - \theta - 120)$$
$$+\cos(\omega t + 120)\cos(\omega t - \theta + 120)]$$

Recall:

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$p(t) = V_{rms}I_{rms}[3\cos(\theta) + \cos(2\omega t - \theta)$$
$$+\cos(2\omega t - \theta - 120) + \cos(2\omega t - \theta + 120)]$$

$$p(t) = 3V_{rms}I_{rms}\cos(\theta)$$ ⇒ **Constant !**