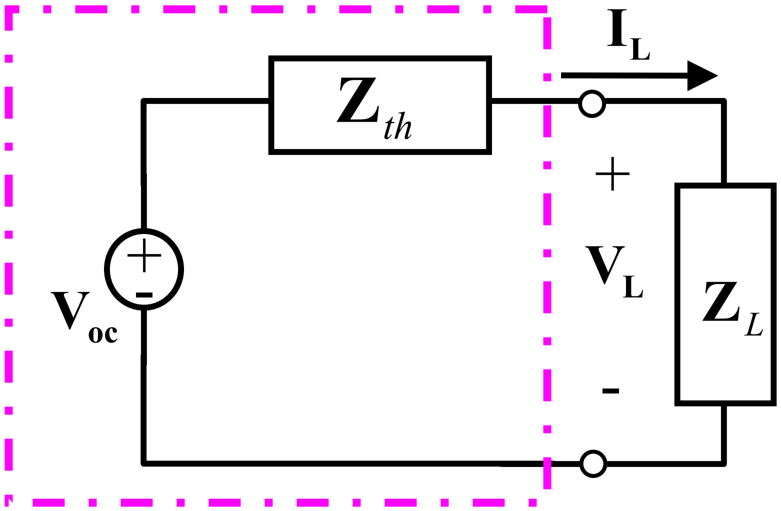
ECSE 210: Circuit Analysis

Lecture#16:

Steady State Power Analysis

Maximum Average Power Transfer

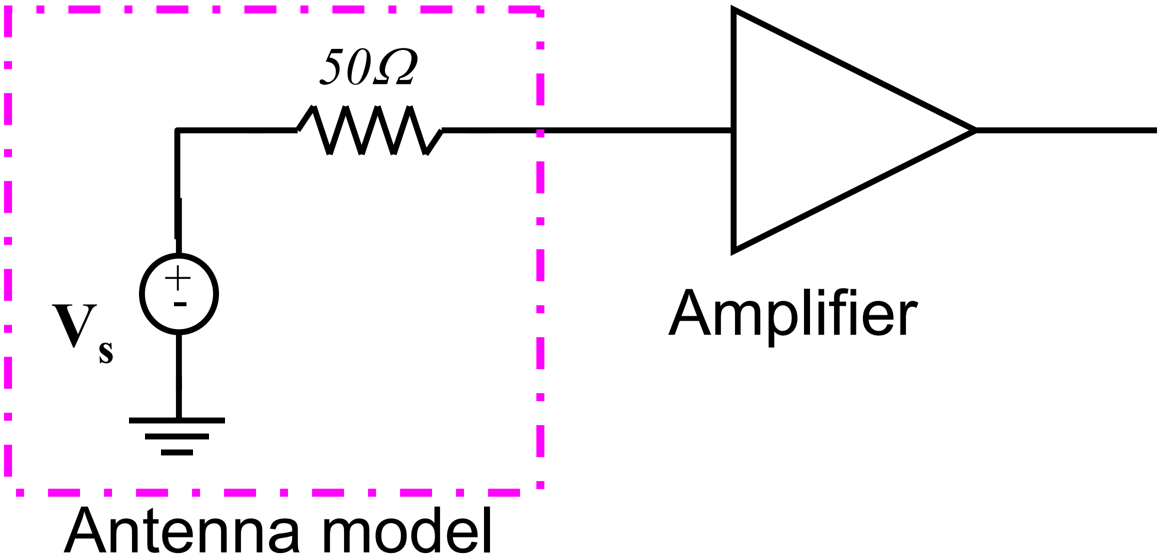


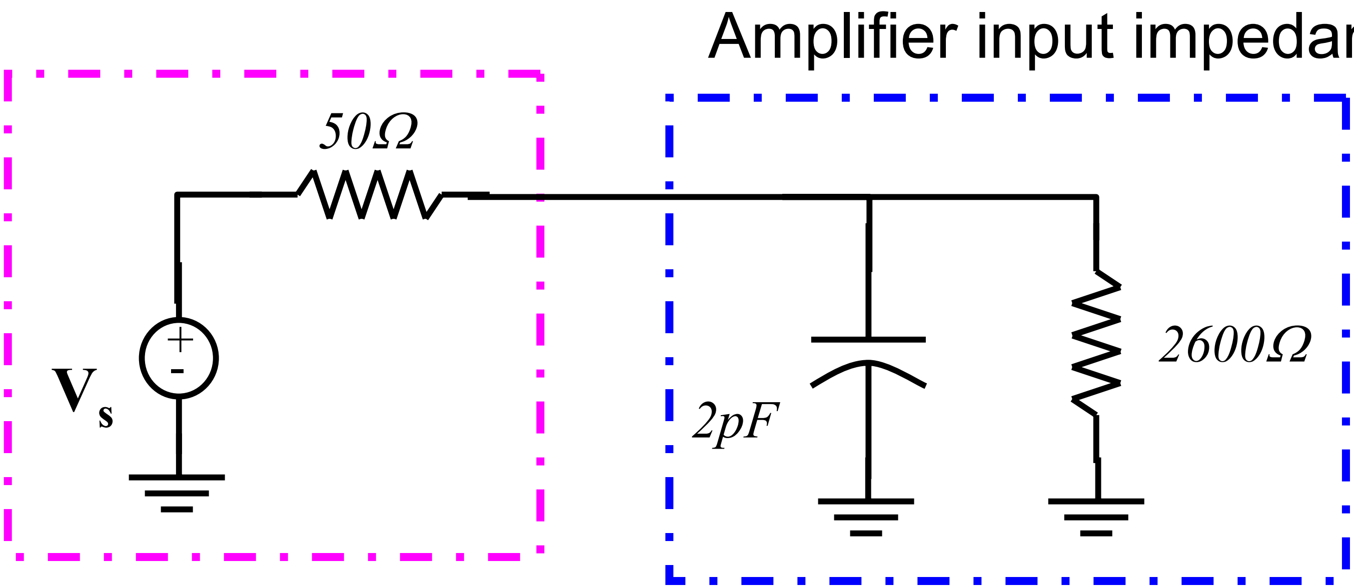
→ The maximum power transfer from a source with a Thevenin impedance $\mathbf{Z}_{th}$ to a load $\mathbf{Z}_L$ occurs when $\mathbf{Z}_L$ is the **complex conjugate** of $\mathbf{Z}_{th}$.

→ In the above case, the source and load impedances are said to be matched.

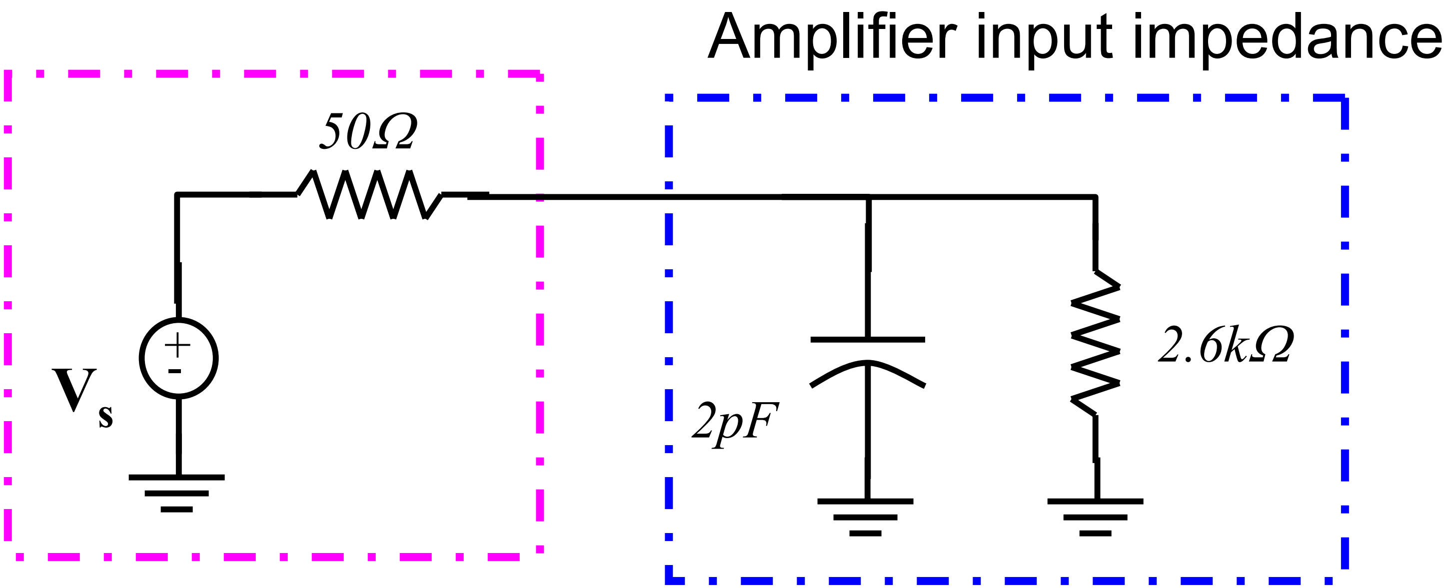
→ Is it possible to “correct” an impedance mismatch?

Example


50Ω
V_s
Amplifier
Antenna model


Amplifier input impedance
50Ω
V_s
2pF
2600Ω

Example



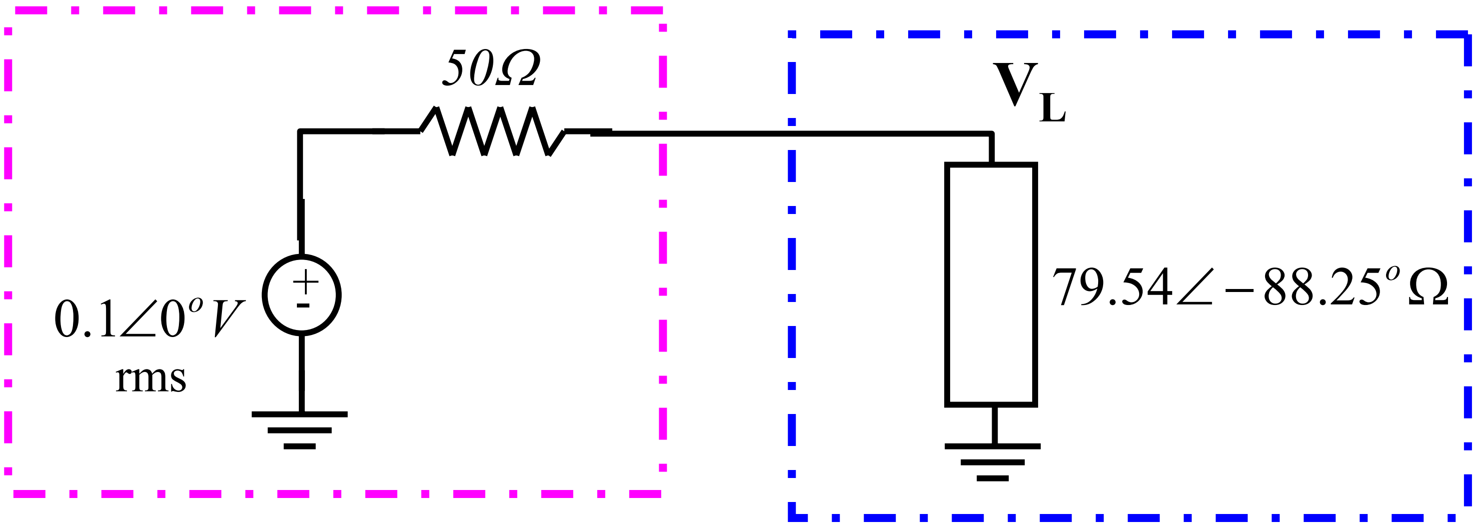
- → If the circuit is operating at **f =1 GHz**, what is the average power supplied to the amplifier?
- → At 1 GHz, the input impedance of the amplifier is:

$$\mathbf{Z}_L = \left(\frac{1}{2600} + j(2\pi \times 1 \times 10^9 \times 2 \times 10^{-12}) \right)^{-1} = 2.43 - j79.5\Omega$$

capacitive load

$$\mathbf{Z}_L = 79.54\angle -88.25^o\,\Omega$$

Example



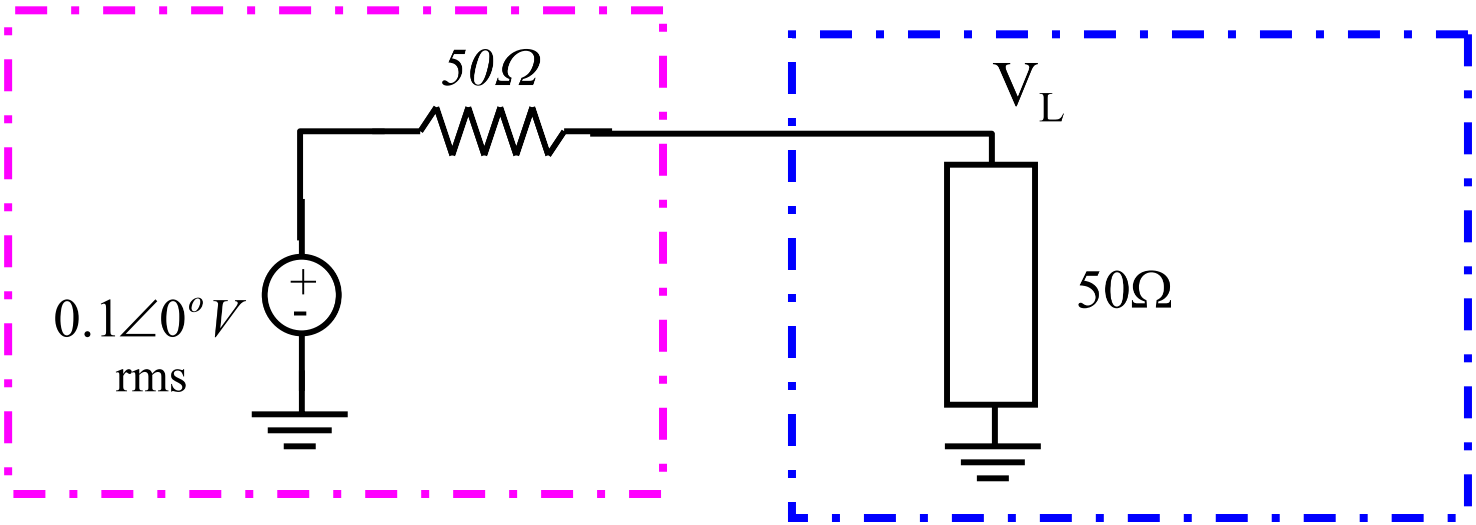
$$\mathbf{V_L} = \frac{79.54\angle -88.25^o\Omega}{79.54\angle -88.25^o\Omega + 50\Omega} 0.1\angle 0^o = 83.52\angle -31.65^o\, mV$$

$$\mathbf{I_L} = \frac{83.52\angle -31.65^o\, mV}{79.54\angle -88.25^o\Omega} = 1.05\angle 56.59^o\, mA$$

$$\mathbf{S} = \mathbf{V_L I_L^*} = \left(83.52\angle -31.65^o\, mV\right)\left(1.05\angle -56.59^o\, mA\right) = 87.7\angle -88.25^o\, \mu VA$$

$$P_{avg} = \mathrm{Re}[\mathbf{S}] = 2.68\mu W$$

Example



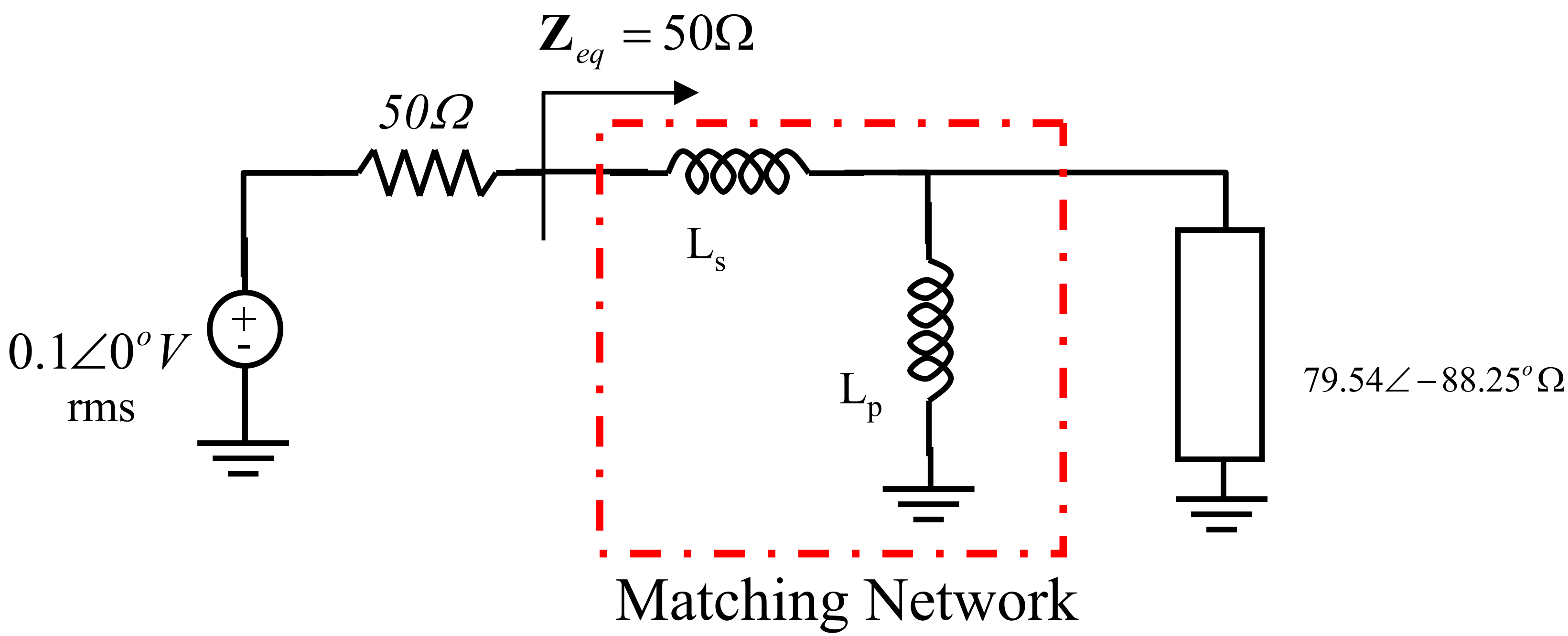
→ What is the maximum power that the source can supply to the load?

→ Maximum power is delivered when the impedances are matched (complex conjugates).

$$\mathbf{V_L} = \frac{50}{50+50} 0.1\angle 0^o V = 0.05\angle 0V \qquad \mathbf{I_L} = \frac{0.05\angle 0V}{50} = 1\angle 0^o mA$$

$$\mathbf{S} = (1\angle 0^o mA)(0.05\angle 0^o V) = 50\times 10^{-6}\angle 0^o VA \qquad P_{avg} = 50\mu W$$

Impedance Matching

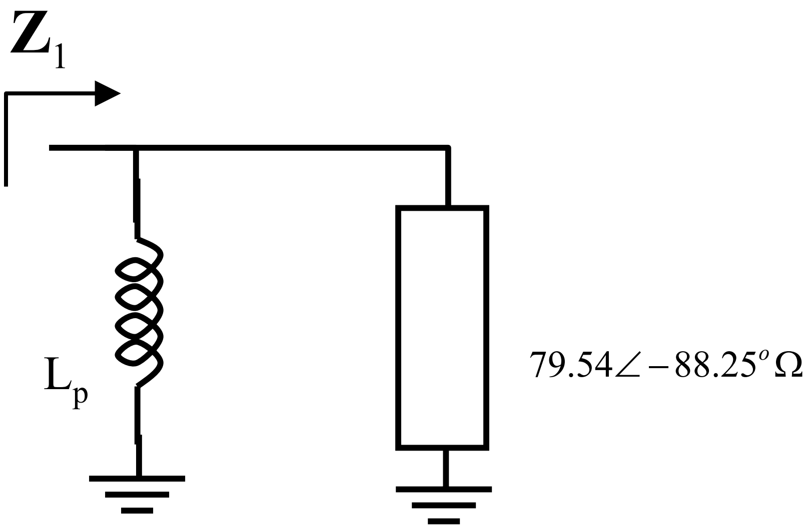


Add a matching network to transform the load impedance, such that the impedance $\mathbf{Z}_{eq}$ "seen" by the source is 50Ω.

Impedance Matching: Strategy

→ Choose the value of the first inductor such that the parallel combination of the inductor and load results in an impedance of the form:

$$\mathbf{Z}_1 = 50 + jX_1\Omega$$



→ The above impedance should have a real part of 50Ω and a **negative reactive part** (capacitive).

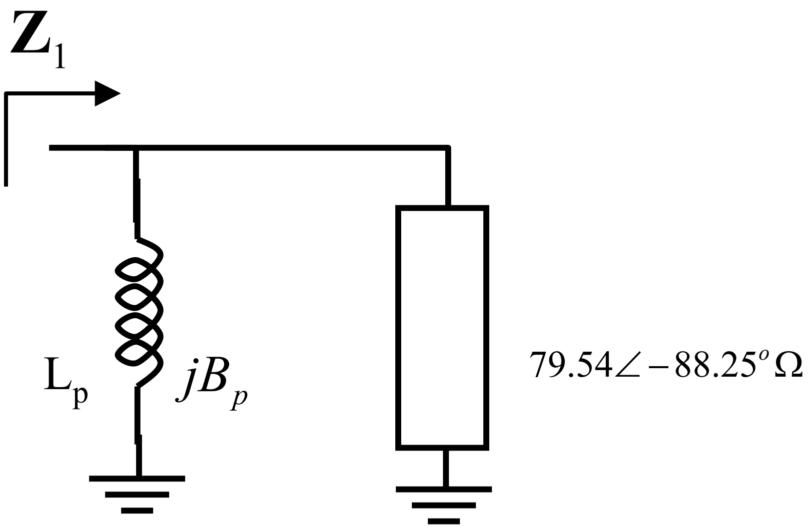
→ Now choose the value of the series inductor to **cancel** the reactive component of the above impedance.

Impedance Matching: Strategy

$$\mathbf{Z}_L = 79.54\angle -88.25^o\,\Omega$$

$$\mathbf{Y}_L = \frac{1}{79.54\angle -88.25^o\,\Omega} = 0.384 + j12.57mS$$

$$\mathbf{Y}_1 = 0.384\times 10^{-3} + j(12.57\times 10^{-3} + B_p)$$



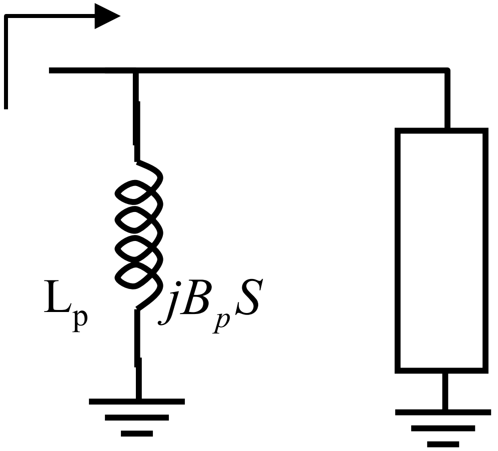
$$\mathbf{Z}_1 = \frac{1}{0.384\times 10^{-3} + j(12.57\times 10^{-3} + B_p)} = \frac{0.384\times 10^{-3} - j(12.57\times 10^{-3} + B_p)}{(0.384\times 10^{-3})^2 + (12.57\times 10^{-3} + B_p)^2}$$

$$\mathrm{Re}[\mathbf{Z}_1] = \frac{0.384\times 10^{-3}}{(0.384\times 10^{-3})^2 + (12.57\times 10^{-3} + B_p)^2} = 50$$

$$7.68\times 10^{-6} = (0.384\times 10^{-3})^2 + (12.57\times 10^{-3} + B_p)^2$$

quadratic

Impedance Matching: Strategy



$$7.68\times10^{-6} = (0.384\times10^{-3})^2 + (12.57\times10^{-3} + B_p)^2$$

$$7.68\times10^{-6} = 147.5\times10^{-9} + 158\times10^{-6} + 25.14\times10^{-3}B_p + B_p^{\;2}$$

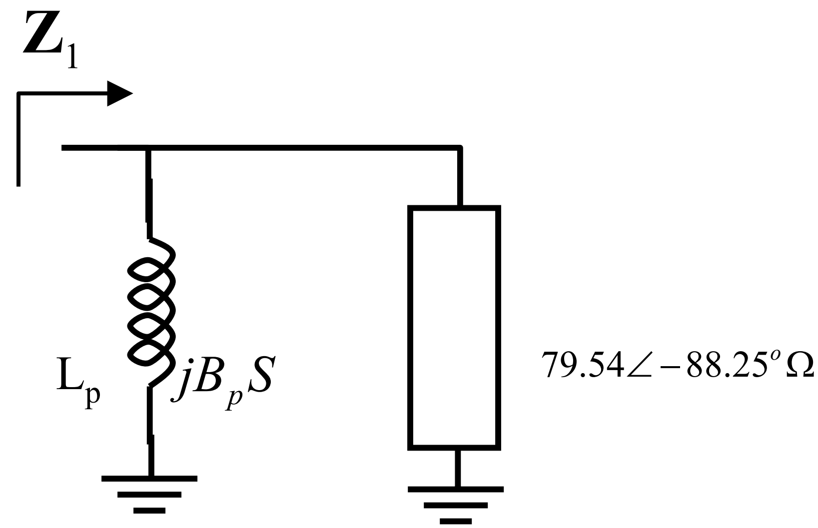
$$B_p^{\;2} + 25.14\times10^{-3}B_p + 150.5\times10^{-6} = 0$$

$$B_p = \frac{-25.14\times10^{-3} \pm \sqrt{632\times10^{-6} - 4\times150.5\times10^{-6}}}{2}$$

$$\begin{cases} B_p = -15.31\times10^{-3}\,S \\ B_p = -9.83\times10^{-3}\,S \end{cases}$$

Impedance Matching: Strategy

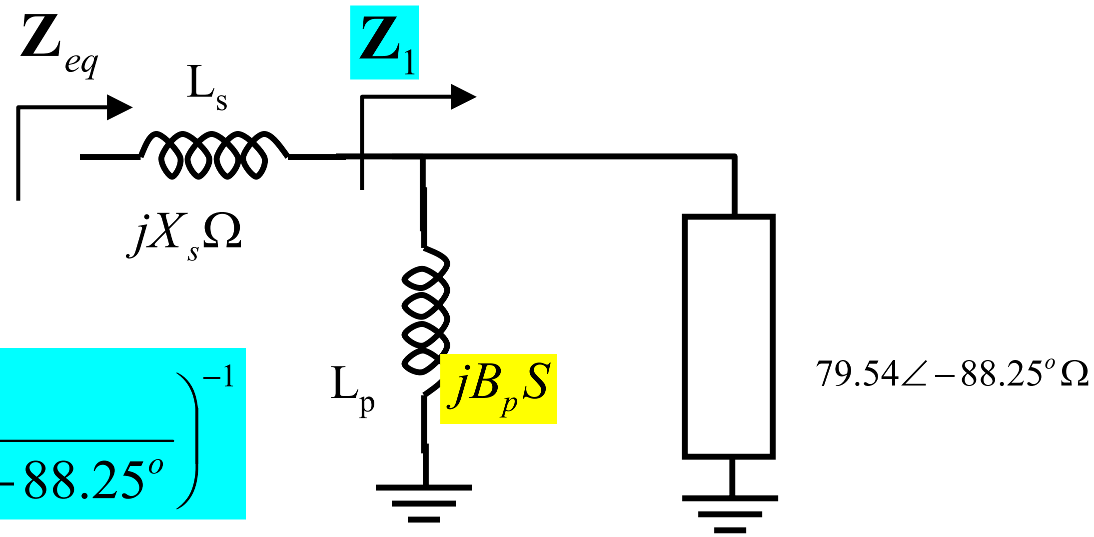
$$\begin{cases} B_p = -15.31\times10^{-3}\,S \\ \mathbf{Z}_1 = \left(-j15.31\times10^{-3} + \dfrac{1}{79.54\angle-88.25^o}\right)^{-1} \\ \quad = 49.86 + j356.9\Omega \end{cases}$$



$$\begin{cases} B_p = -9.83\times10^{-3}\,S \\ \mathbf{Z}_1 = \left(-j9.83\times10^{-3} + \dfrac{1}{79.54\angle-88.25^o}\right)^{-1} \\ \quad = 50.28 - j358.4\Omega \end{cases}$$

→ **Choose second case (capacitive, negative reactance)**.

Impedance Matching: Strategy



$$B_p = -9.83\times10^{-3}\,S$$

$$\mathbf{Z}_1 = \left(-j9.83\times10^{-3} + \frac{1}{79.54\angle -88.25^o}\right)^{-1} = 50.28 - j358.4\Omega$$

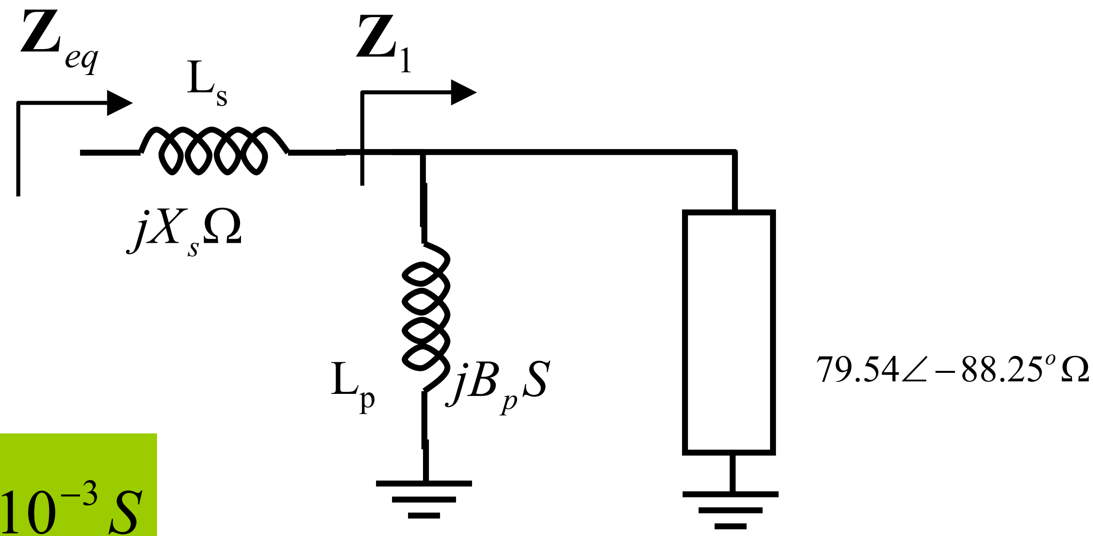
→ Use series inductor to cancel reactive part:

$$\mathbf{Z}_{eq} = 50.28 - j358 + jX_s \quad \rightarrow \quad X_s = 358\Omega$$

→ $\mathbf{Z}_{eq} = 50.28 \cong 50\Omega$

Impedance Matching: Strategy

$$\begin{cases} B_p = -9.83\times10^{-3}\,S \\ X_s = 358\Omega \end{cases}$$



→ **Find inductances:**

$$jB_p = \frac{1}{j\omega L_p} = -j9.83\times10^{-3}\,S$$

$$L_p = \frac{1}{\left(2\pi\times10^9\right)\left(9.83\times10^{-3}\right)} = 16.2nH$$

$$jX_s = j\omega L_s = j358\Omega$$

$$L_s = \frac{358}{2\pi\times10^9} = 57nH$$

Impedance Matching: Summary

1. Any load can be matched to a 50Ω source.
2. This matching is only valid at one frequency.
3. The topology we have used (a parallel inductor and a series inductor) does not work for all cases. We will not worry about the choice of topologies.
4. The matching network is purely reactive and does not consume any average power.