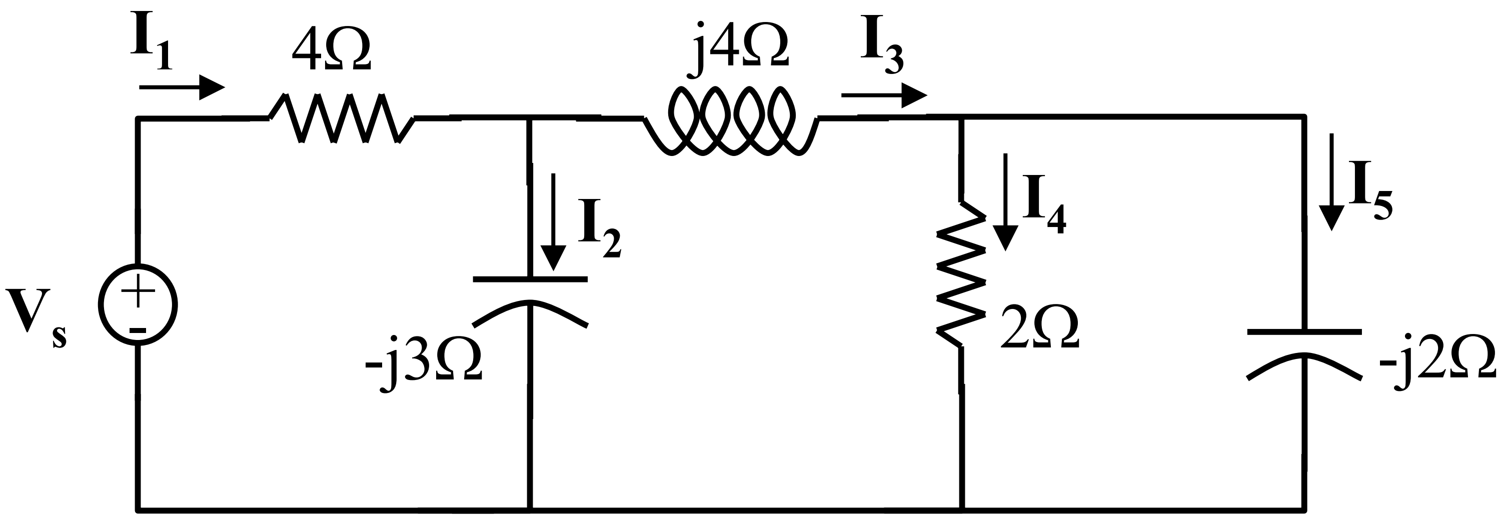
ECSE 210: Circuit Analysis

Lecture #13:

AC Steady State Analysis

Example 1: Linearity



Use linearity to determine the phasor $\mathbf{I_4}$ and the time domain current $i_4(t)$ given that:

$$v_s(t) = 12\cos(377t + 30)V$$

→ The phasor for $v_s(t)$ is given by:

$$\mathbf{V_s} = 12\angle 30^o V$$

Example 1: Linearity

Solve using linearity:

Assume: $\mathbf{I_4} = 1\angle 0$ → Solve for $\mathbf{V_s}$

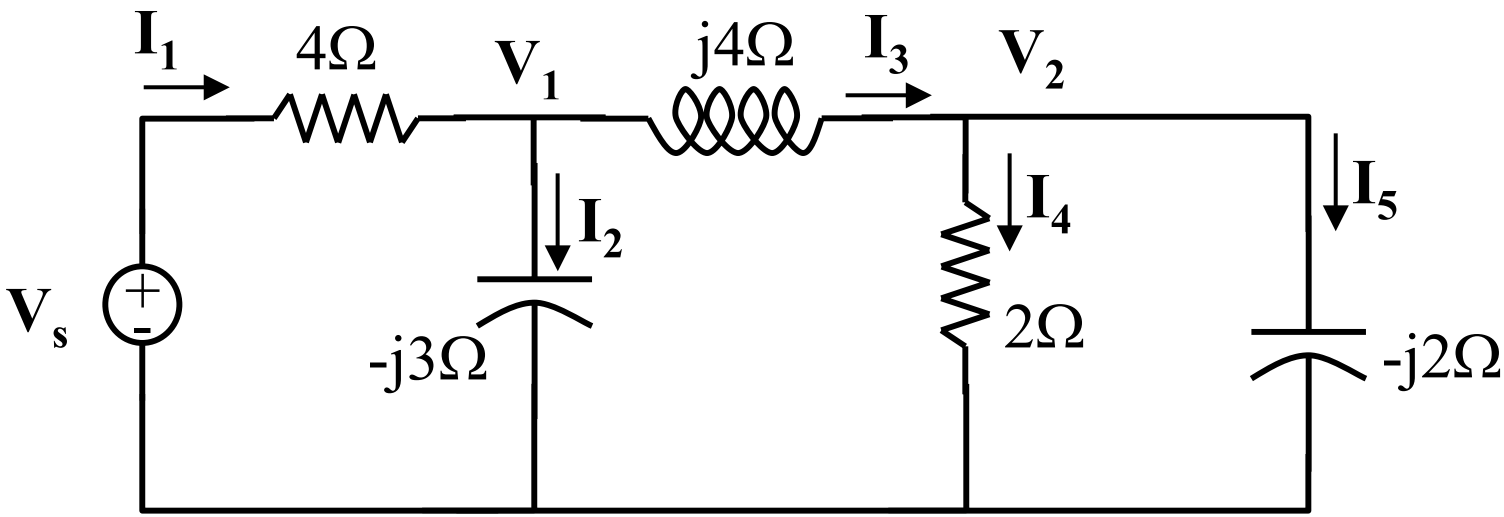
→ Using linearity calculate $\mathbf{I_4}$ for the given value of $\mathbf{V_s}$

For example:

- Assume that for $\mathbf{I_4}$=1A we can solve for $\mathbf{V_s}$=3V.
- Then we could immediately state (using linearity) that if
  $\mathbf{V_s}$=1.5 then I_4= (1.5/3)1 = 0.5A

→ Apply above procedure to the circuit on previous slide.

Example 1: Linearity



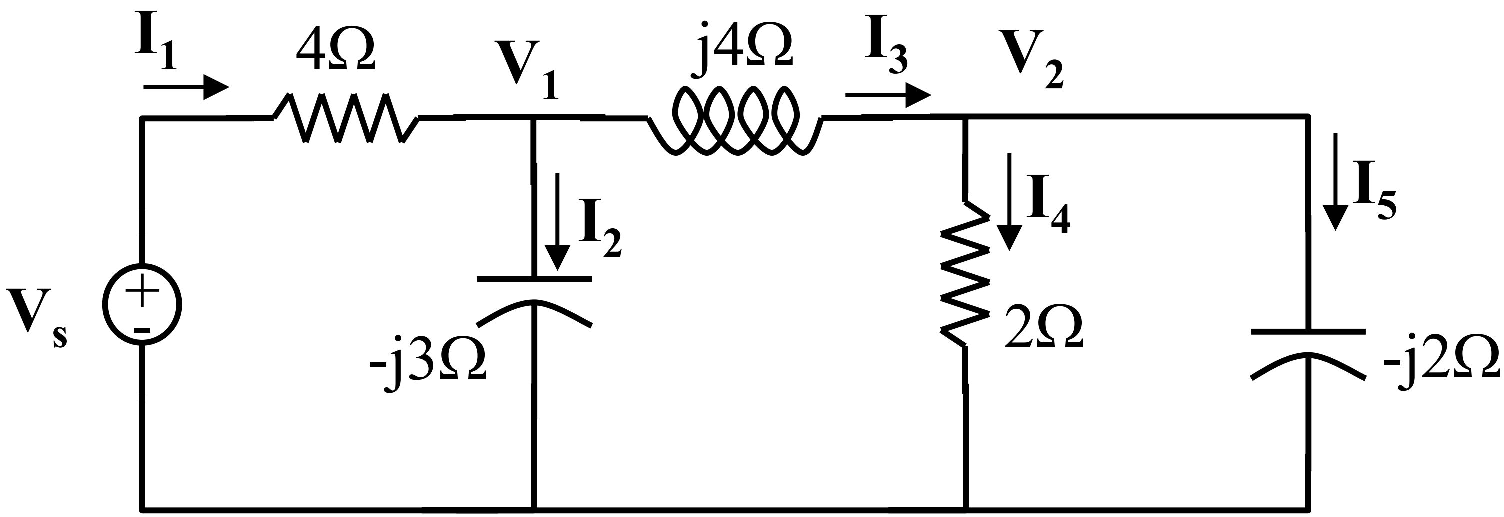
$\mathbf{I_4} = 1\angle 0^o\, A$ → Calculate $\mathbf{V_s}$

$\mathbf{I_4} = 1\angle 0^o\, A$ → $\mathbf{V_2} = (1\angle 0^o\, A)(2\Omega) = 2V$

$$\rightarrow \quad \mathbf{I_5} = \frac{2V}{-j2\Omega} = j$$

$\rightarrow \mathbf{I_3} = \mathbf{I_4} + \mathbf{I_5} = 1 + j \quad \text{A}$

Example 1: Linearity

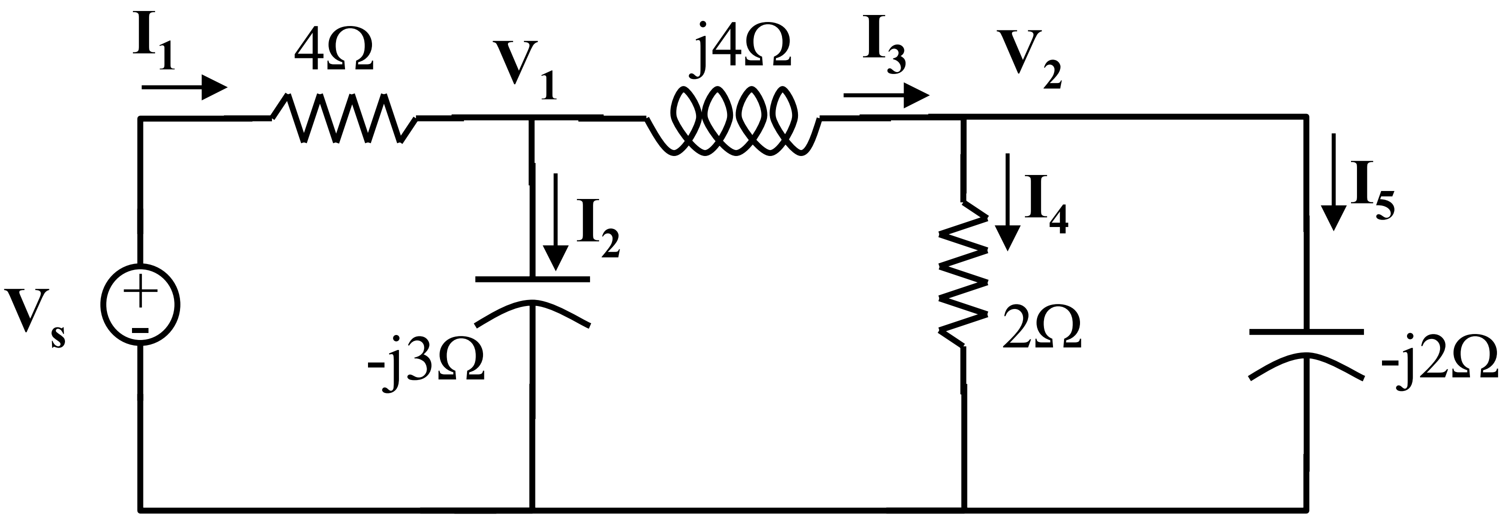


$$\mathbf{V}_1 = \mathbf{I}_3\left(j4\Omega\right) + \mathbf{V}_2 = (1+j)(j4) + 2 = -2 + 4j \quad \text{V}$$

$$\rightarrow \quad \mathbf{I}_2 = \frac{-2+4j}{-j3} = \frac{-4}{3} - j\frac{2}{3}$$

$$\rightarrow \quad \mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_3 = -\frac{4}{3} - j\frac{2}{3} + 1 + j = -\frac{1}{3} + j\frac{1}{3}$$

Example 1: Linearity



→ $$\mathbf{V_s} = \mathbf{I_1}(4\Omega) + \mathbf{V_1} = \left(-\frac{1}{3} + j\frac{1}{3}\right)(4\Omega) + (-2 + 4j)$$

→ $$\mathbf{V_s} = -\frac{10}{3} + j\frac{16}{3} = 6.29\angle 122^o \quad \text{V}$$

Example 1: Linearity

$$\mathbf{I_4} = 1\angle 0^o \quad \rightarrow \quad \mathbf{V_s} = -\frac{10}{3} + j\frac{16}{3} = 6.29\angle 122^o$$

Using linearity:

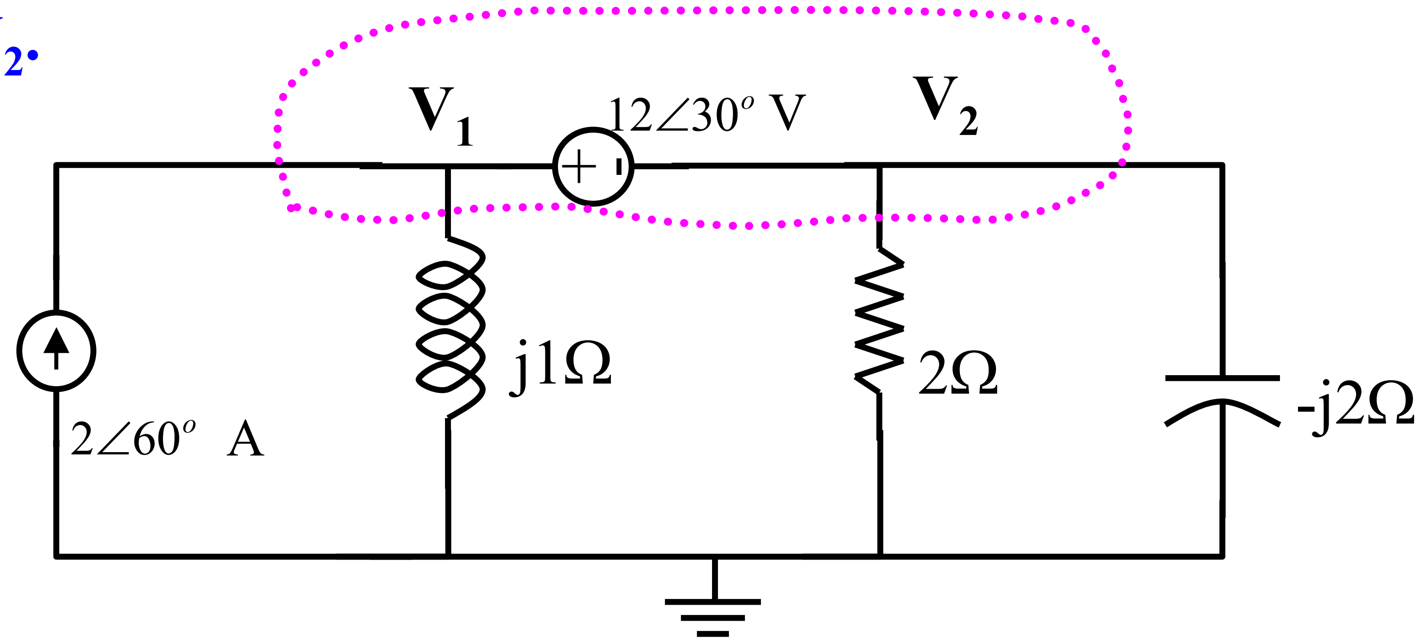
$$\mathbf{V_s} = 12\angle 30^o \quad \rightarrow \quad \mathbf{I_4} = \frac{(12\angle 30^o)(1\angle 0^o)}{6.29\angle 122^o} = 1.91\angle -92^o$$

In the time domain:

$$i_4(t) = 1.91\cos(377t - 92^o) \quad \text{A}$$

Example 2: Nodal Analysis

Find $\mathbf{V}_1$ and $\mathbf{V}_2$.



KCL at supernode:

$$\frac{\mathbf{V}_1}{j1}+\frac{\mathbf{V}_2}{2}+\frac{\mathbf{V}_2}{-j2}=2\angle 60^o$$

For the supernode:

$$\mathbf{V}_2=\mathbf{V}_1-12\angle 30^o$$

$$\frac{\mathbf{V}_1}{j1}+\frac{\mathbf{V}_1-12\angle 30}{2}+\frac{\mathbf{V}_1-12\angle 30}{-j2}=2\angle 60^o$$

Example 2: Nodal Analysis

$$\frac{\mathbf{V_1}}{j1} + \frac{\mathbf{V_1} - 12\angle 30}{2} + \frac{\mathbf{V_1} - 12\angle 30}{-j2} = 2\angle 60^o$$

$$\mathbf{V_1}\left(\frac{1}{j} + \frac{1}{2} + \frac{1}{-2j}\right) = 2\angle 60^o + \frac{12\angle 30^o}{2} - \frac{12\angle 30^o}{j2}$$

$$\mathbf{V_1} = 14.69\angle 117.14^o \quad \text{V}$$

<u>Also</u>

$$\mathbf{V_2} = \mathbf{V_1} - 12\angle 30^o = 14.69\angle 117.14^o - 12\angle 30^o$$

$$= -6.70 + j13.07 - (10.4 + j6)$$

$$= -17.1 + j7.07 = 18.5\angle 157.54^o \quad \text{V}$$

Example 3: Nodal Analysis 2

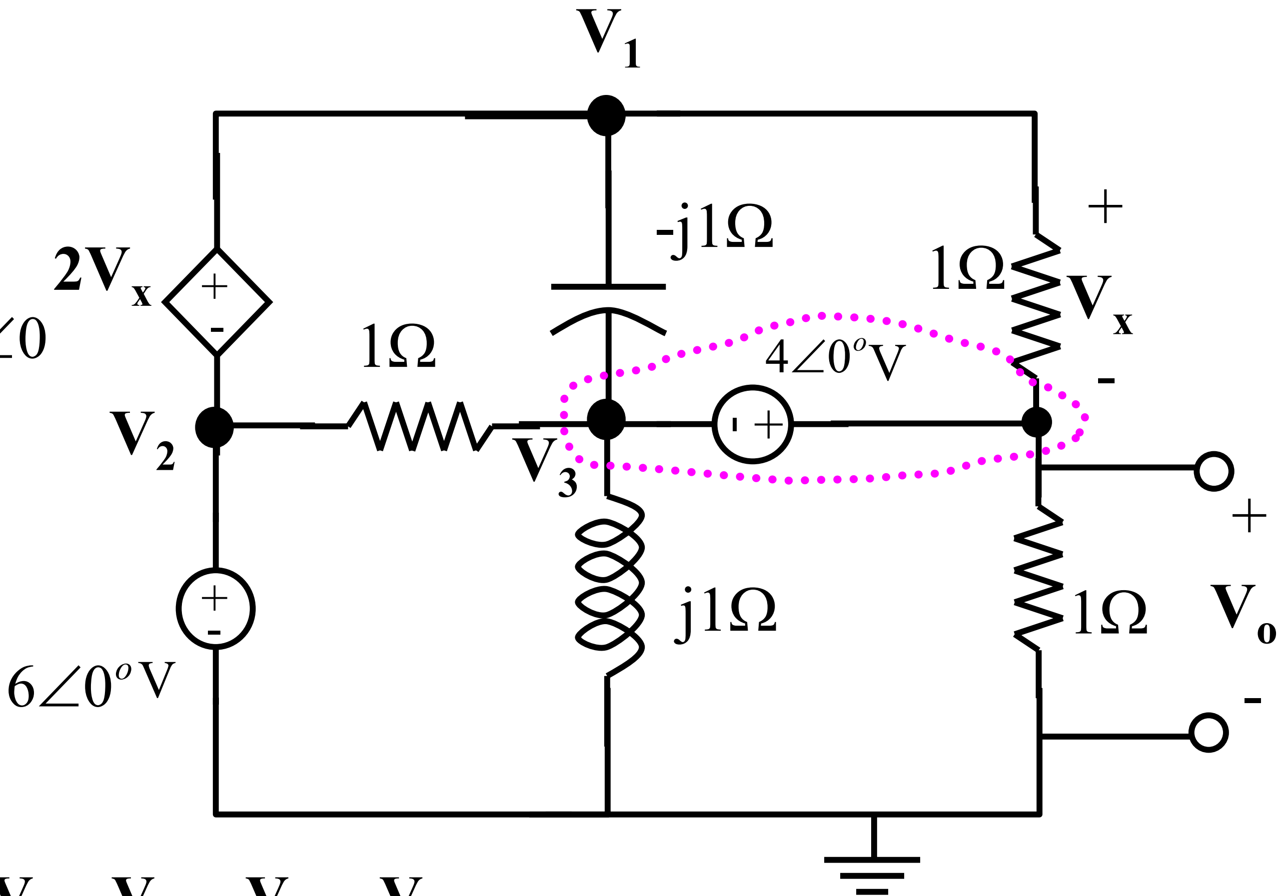
Find V_o.

$$V_2 = 6\angle 0$$

$$V_x = V_o - 6\angle 0$$

$$V_1 = V_o + V_x = 2V_o - 6\angle 0$$

$$V_3 = V_o - 4\angle 0$$



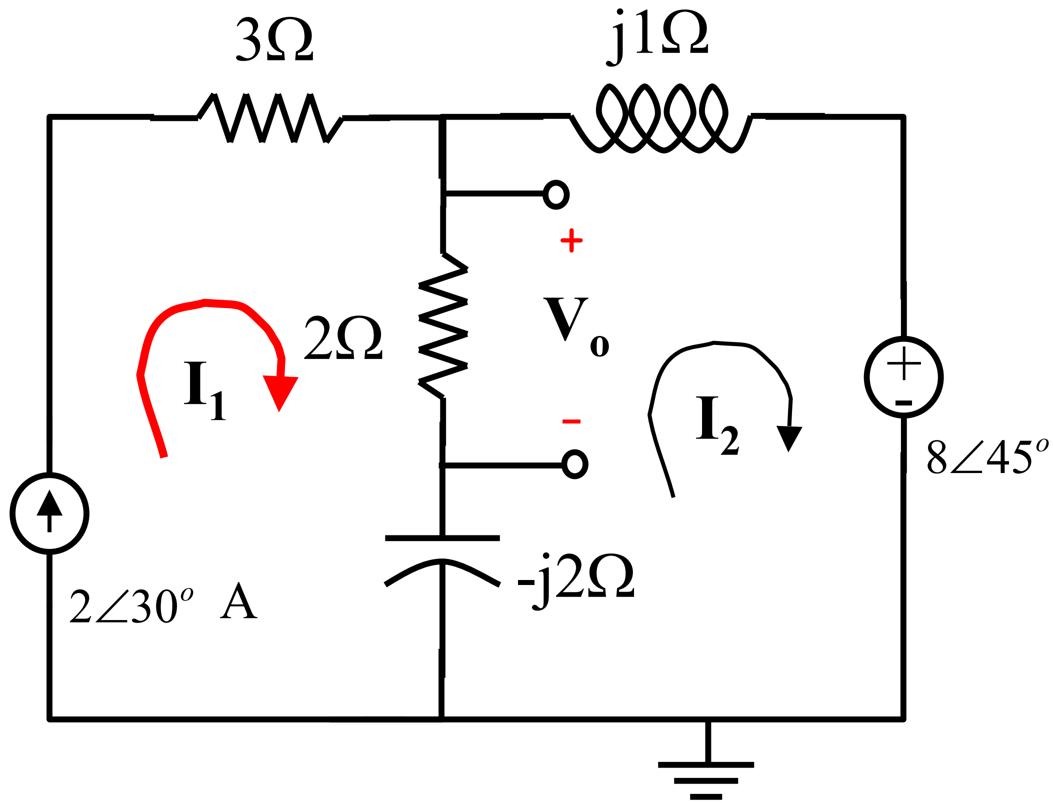
KCL at supernode:

$$\frac{V_o - V_1}{1} + \frac{V_3 - V_1}{-j} + \frac{V_3 - V_2}{1} + \frac{V_3}{j} + \frac{V_o}{1} = 0$$

→ Substitute for V_1, V_2, V_3: $\quad V_o = 3.225\angle 7.125^o$ V

Example 4: Mesh Analysis

Find $\mathbf{V_o}$.



$$\mathbf{I_1} = 2\angle 30^o$$

KVL

$$(\mathbf{I_2} - \mathbf{I_1})(2-2j) + \mathbf{I_2}(j1) + 8\angle 45^o = 0$$

$$\left(-2\angle 30^o\right)(2-2j) + I_2(2-1j) + 8\angle 45^o = 0$$

$$\mathbf{I_2} = \frac{\left(2\angle 30^o\right)(2-2j) - 8\angle 45^o}{2-1j} = 3.18\angle -64.96^o$$

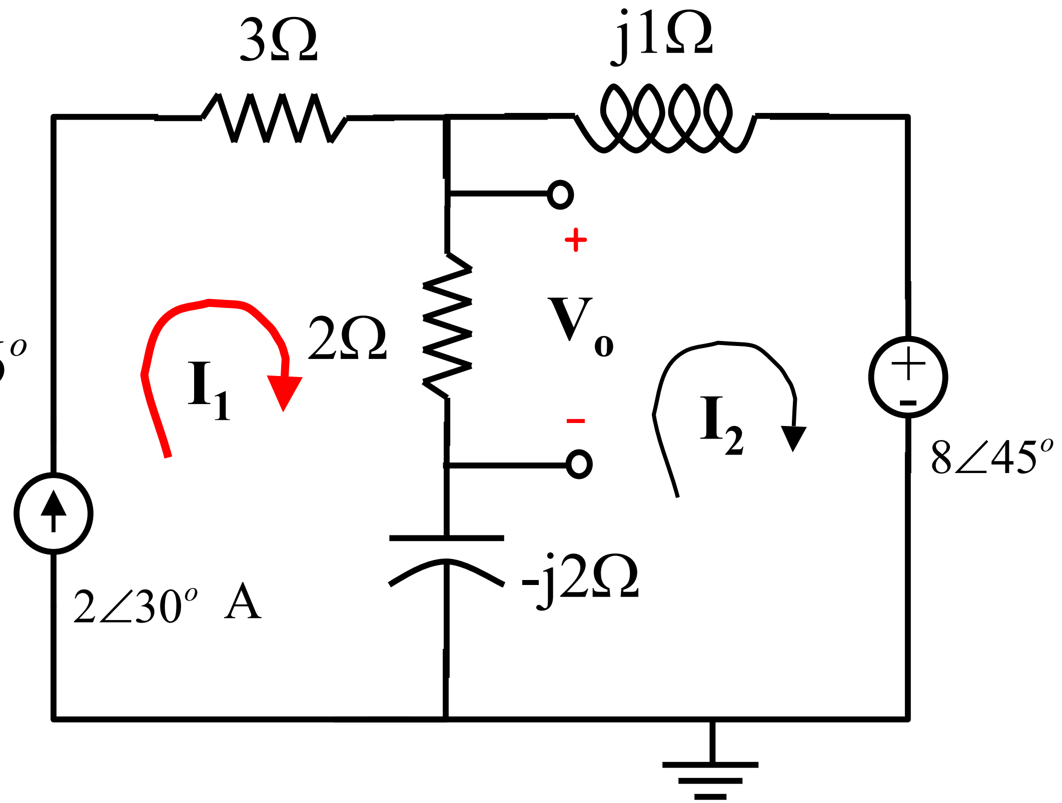
Example 4: Mesh Analysis

$$\mathbf{I_1} = 2\angle 30^o$$

$$\mathbf{I_2} = 3.18\angle -64.96^o$$

$$\mathbf{I_1} - \mathbf{I_2} = 2\angle 30^o - 3.18\angle -64.96^o$$

$$= 0.38 + j3.88 \quad \text{A}$$

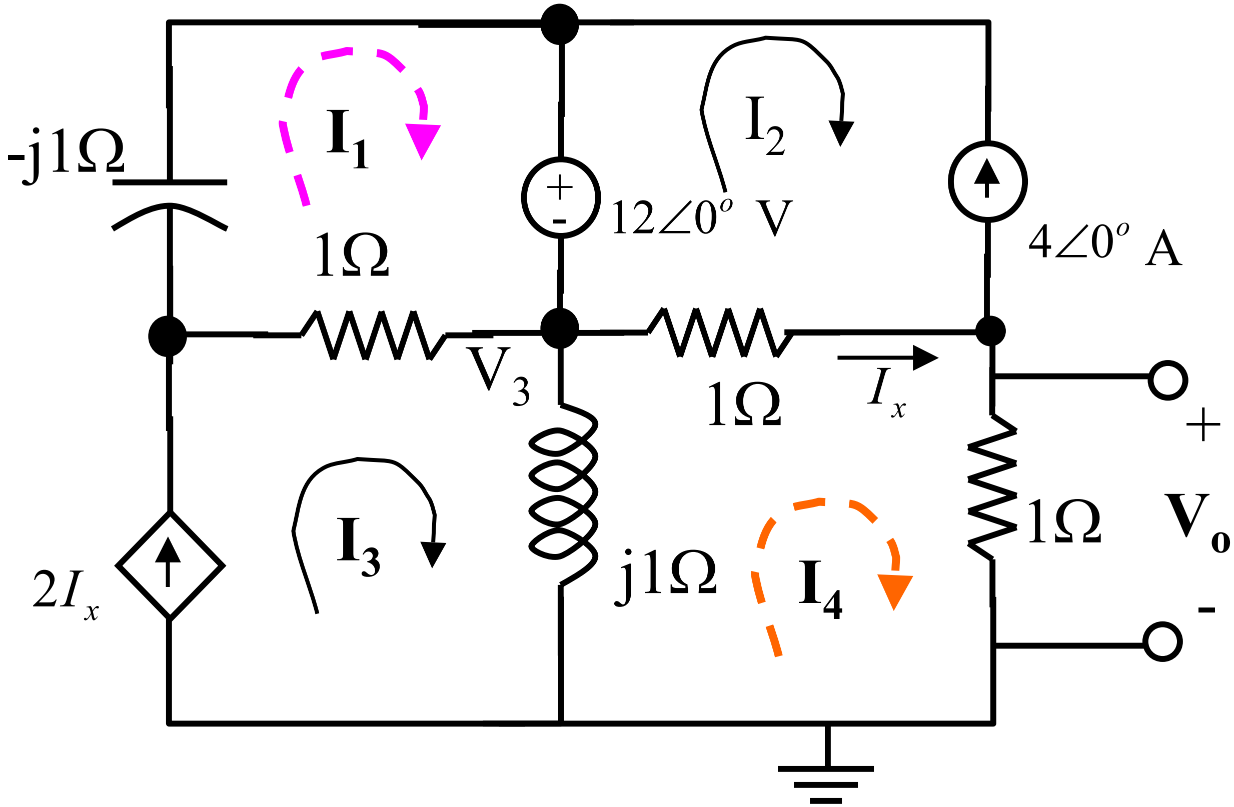


$$\mathbf{V_o} = 2(\mathbf{I_1} - \mathbf{I_2}) = 0.76 + j7.76 \quad \text{V}$$

Example 5: Mesh Analysis 2

Objective:

Find $\mathbf{I}_4$ then $\mathbf{V}_o$.



Constraint Equations

$$\mathbf{I}_2 = -4\angle 0$$

$$\mathbf{I}_x = \mathbf{I}_4 - \mathbf{I}_2 = \mathbf{I}_4 + 4$$

$$\mathbf{I}_3 = 2\mathbf{I}_x = 2(\mathbf{I}_4 + 4)$$

Mesh Equations

$$-j\mathbf{I}_1 + 1(\mathbf{I}_1 - \mathbf{I}_3) = -12\angle 0$$

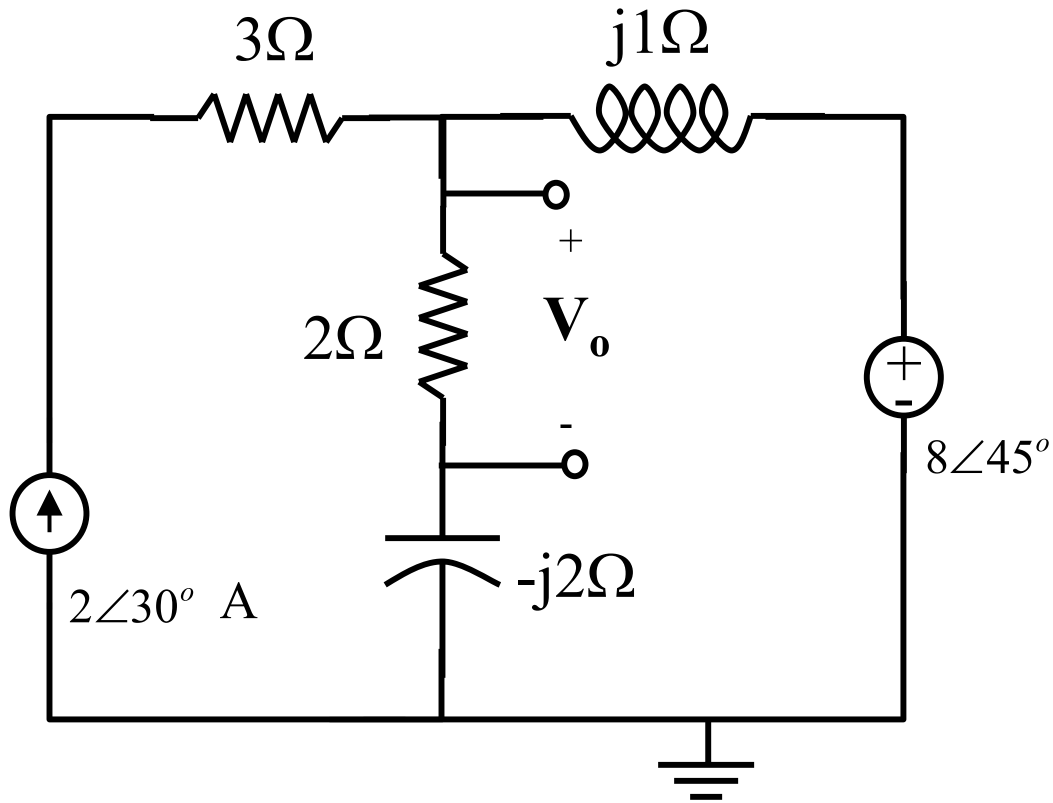
$$j(\mathbf{I}_4 - \mathbf{I}_3) + 1(\mathbf{I}_4 - \mathbf{I}_2) + 1\mathbf{I}_4 = 0$$

$$\mathbf{I}_4 = -4\angle -36.87^o \quad \text{A}$$

$$\mathbf{V}_o = -4\angle -36.87^o \quad \text{V}$$

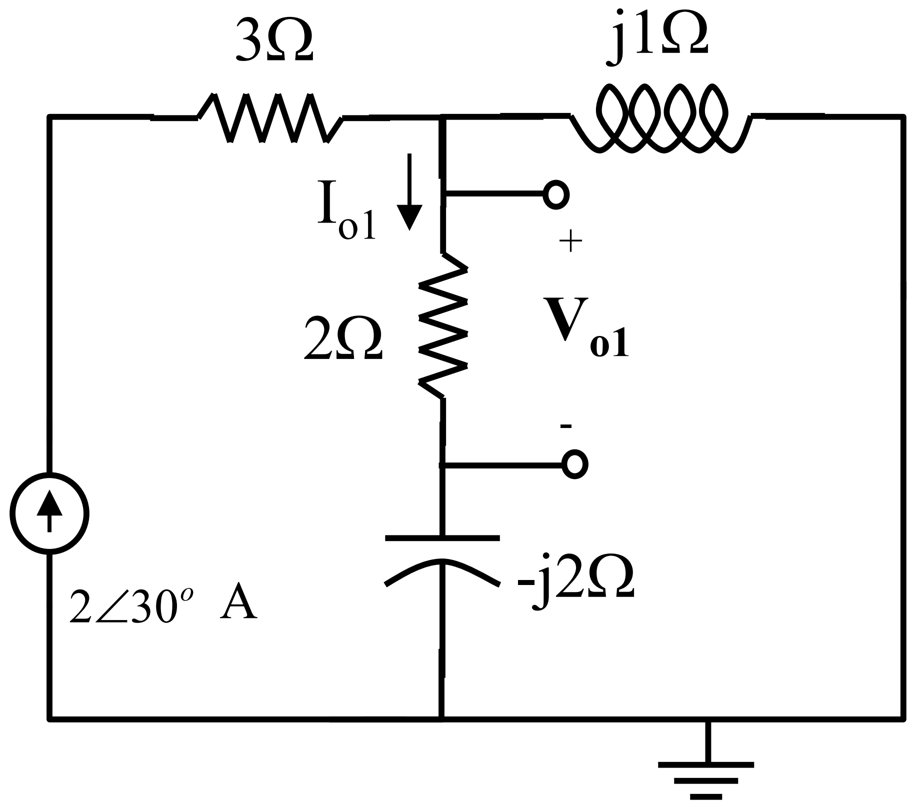
Example 6: Linear Superpostion

Calculate V_o using superposition:



Example 6: Linear Superpostion

Short circuit voltage source and calculate V_{o1}.



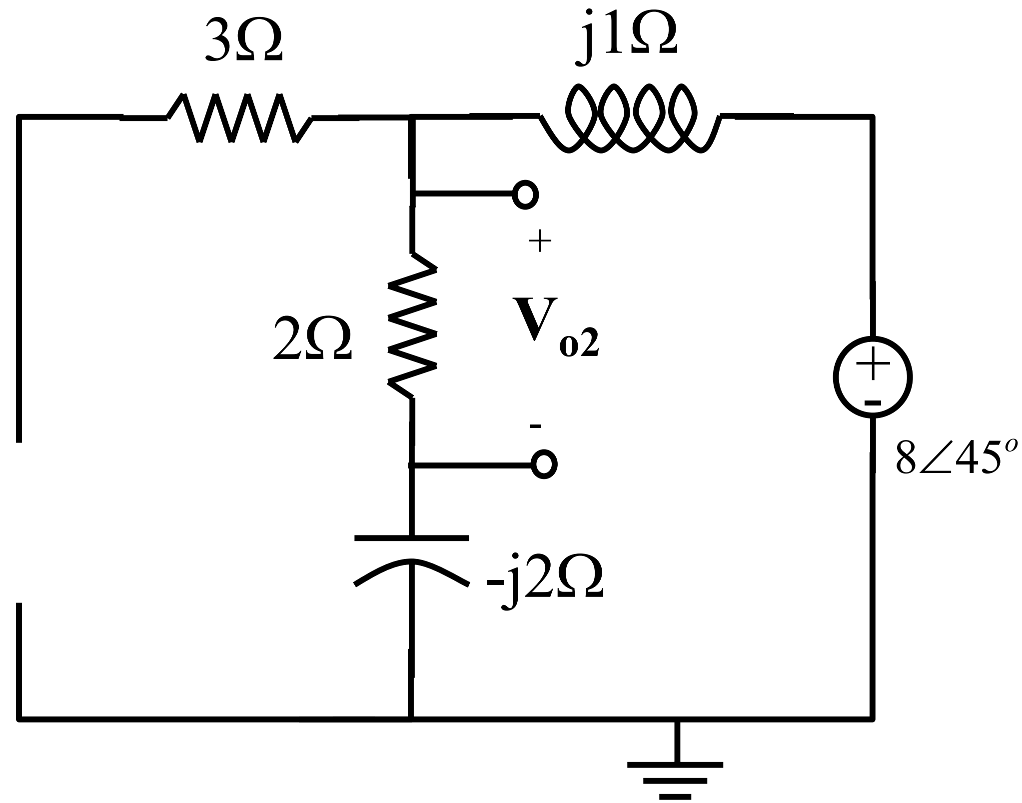
Current divider:

$$\mathbf{I_{o1}} = \frac{j1\Omega}{2 - j2 + j1}\left(2\angle 30^o\right)$$

$$\mathbf{V_{o1}} = \frac{j1}{2 - j2 + j1}\left(2\angle 30^o\right)(2) = 1.79\angle 146.57^o$$

Example 6: Linear Superpostion

Open circuit current source and calculate $\mathbf{V}_{o2}$.



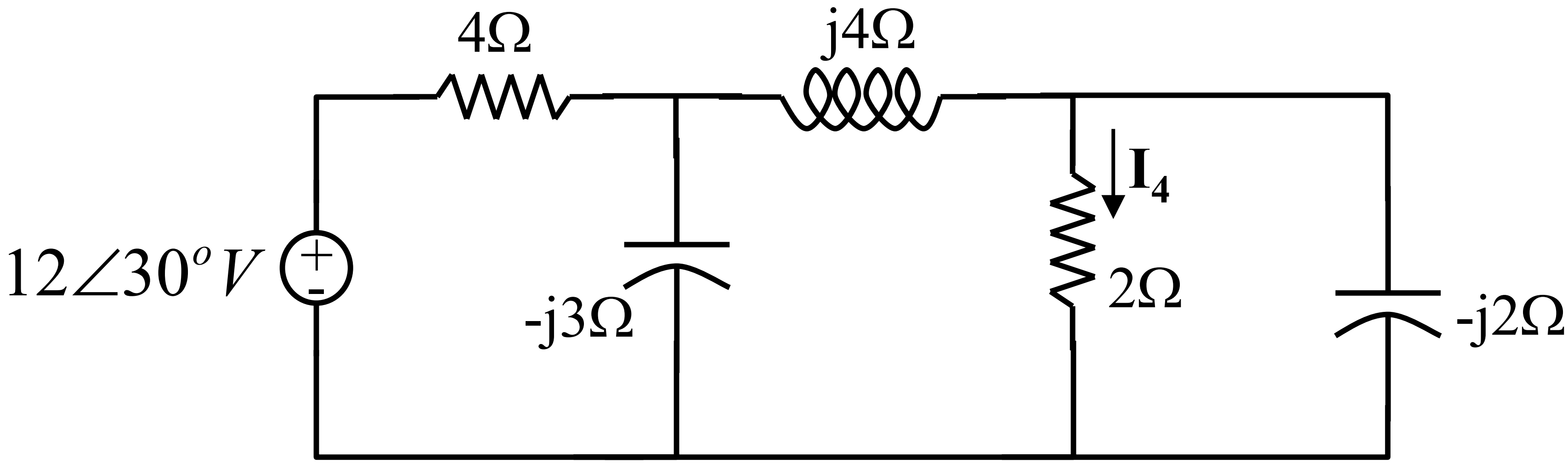
Voltage divider:

$$\mathbf{V}_{o2} = \frac{2}{-j2+2+j1} 8\angle 45^o = 7.14\angle 71.57^o$$

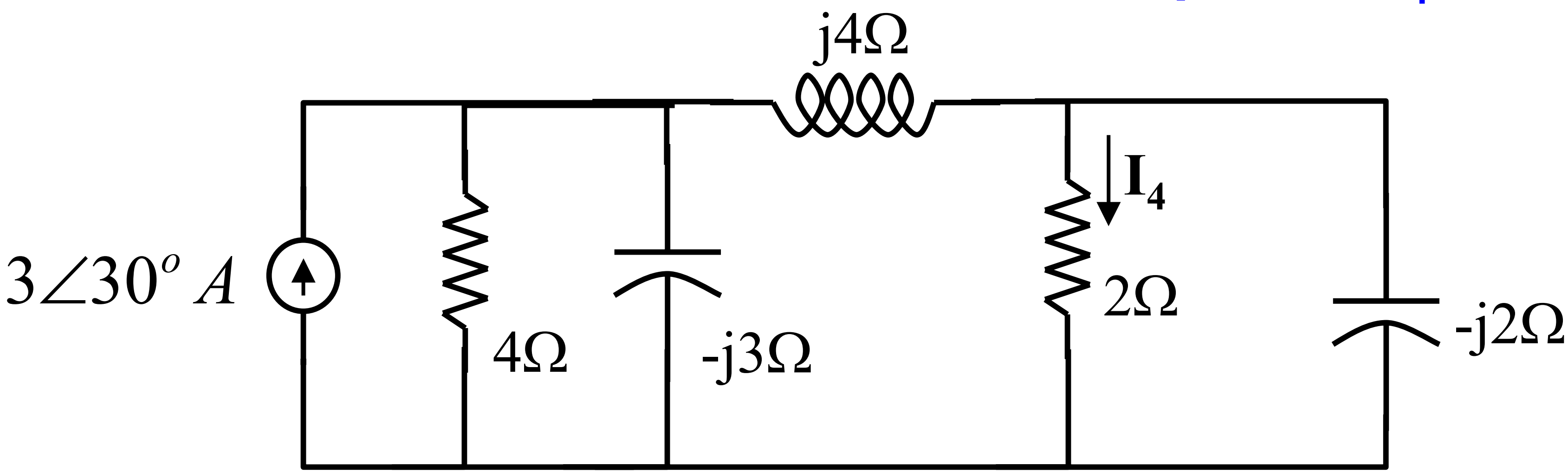
By superposition: $\mathbf{V}_o = \mathbf{V}_{o1} + \mathbf{V}_{o2} = 1.79\angle 146.57^o + 7.14\angle 71.57^o$

$$= 0.76 + j7.76 \quad \text{V}$$

Example 7: Source Transformation



Use source transformation to determine the phasor I_4.



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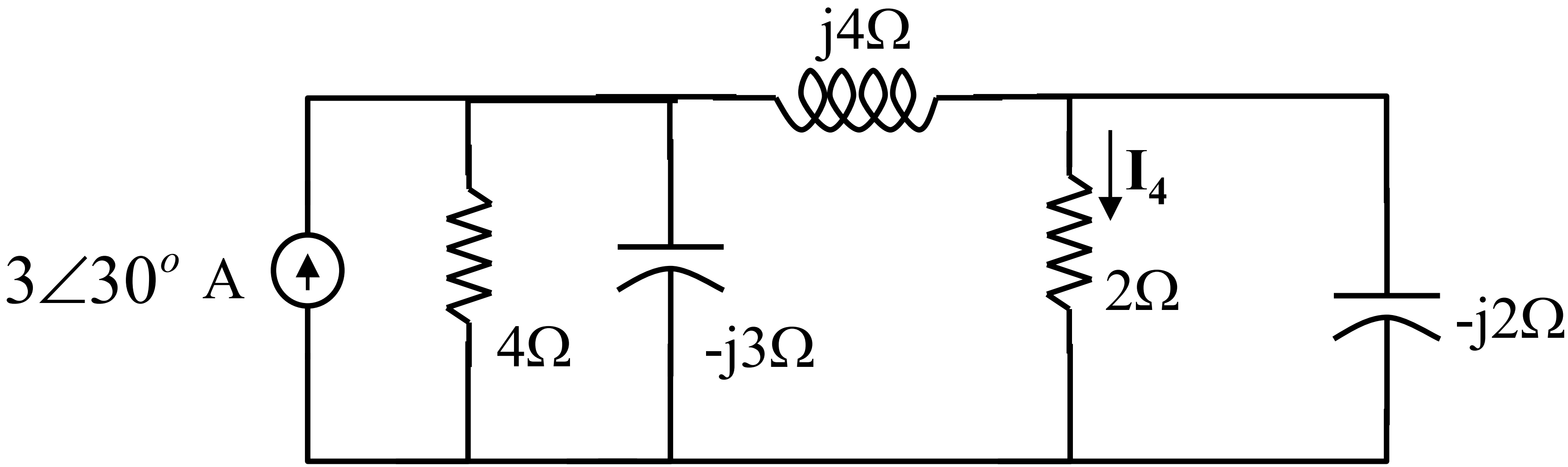
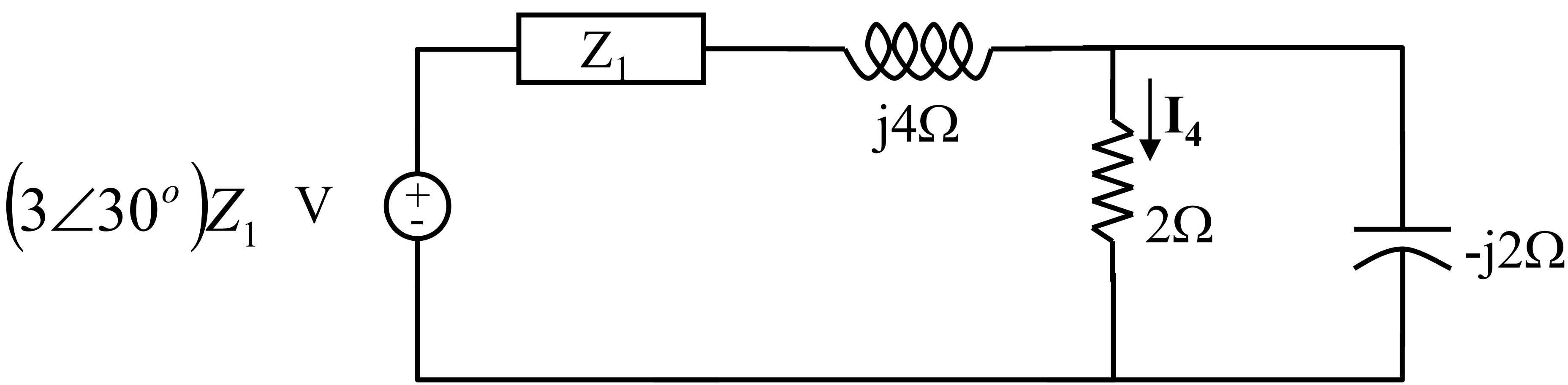
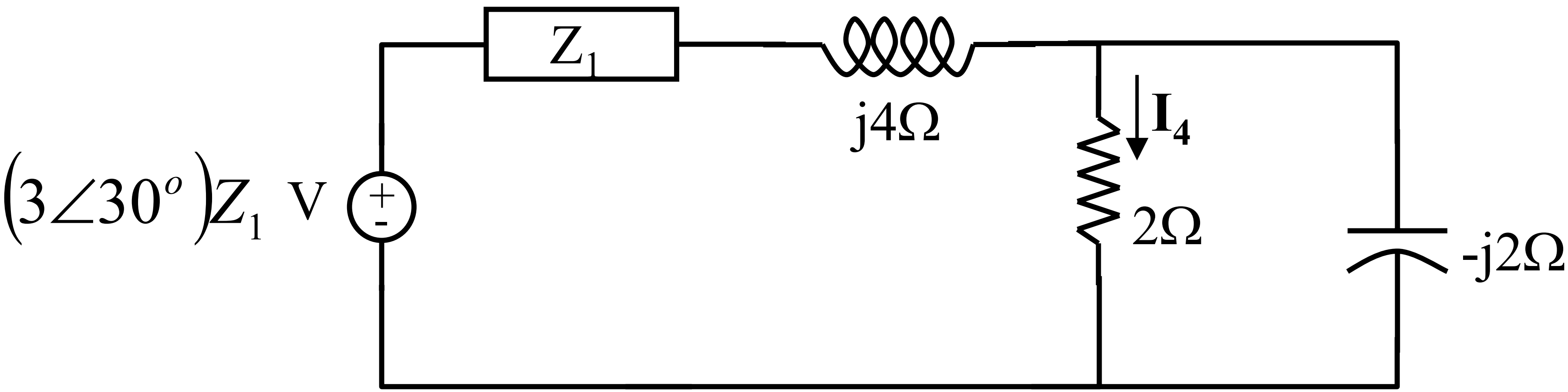
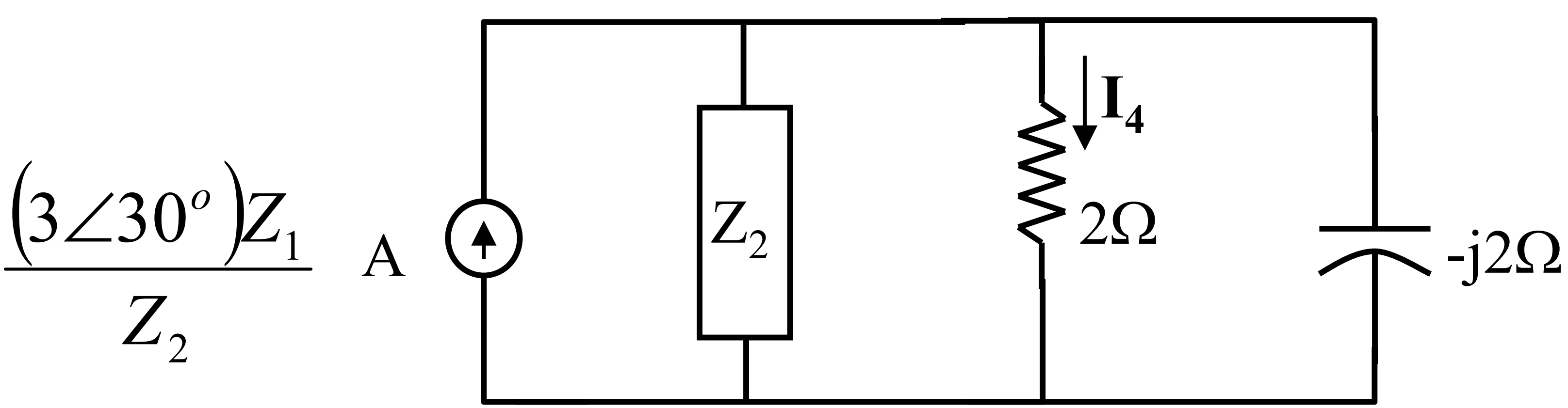




$$Z_1 = \frac{(4)(-j3)}{4-j3} = \frac{-j12}{4-j3}$$

Example 7: Source Transformation

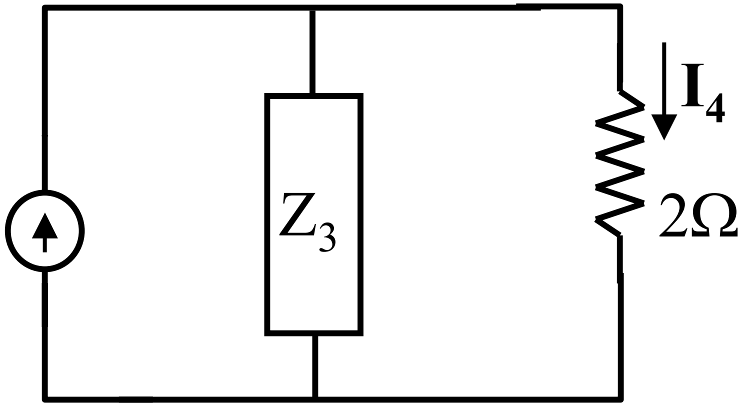




$$Z_2 = Z_1 + j4 = \frac{12 + j4}{4 - j3} \quad \Omega$$

Example 7: Source Transformation

$$\frac{(3\angle 30^o)Z_1}{Z_2}$$



$$Z_3 = \frac{(-j2)(Z_2)}{-j2+Z_2}$$

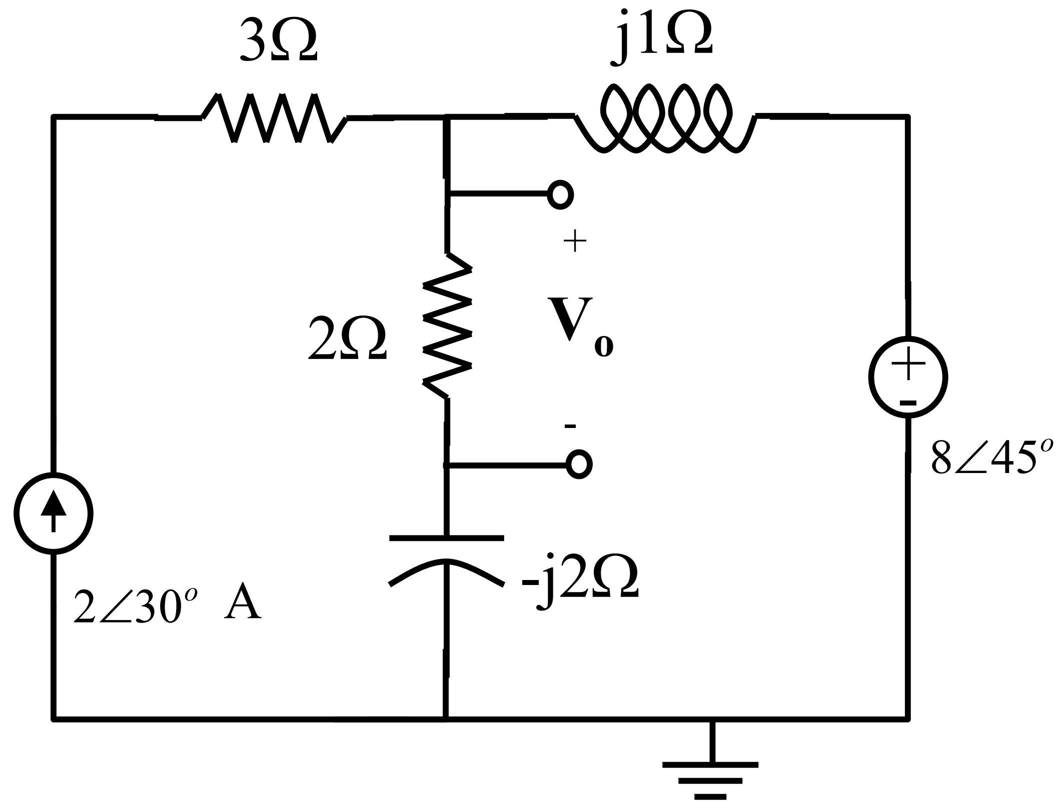
$$= \frac{4-j12}{3-j2} \quad \Omega$$

$$\frac{(3\angle 30^o)Z_1}{Z_2} = 2.85\angle -78.43^o \quad \text{A}$$

$$\mathbf{I_4} = \frac{\left(\frac{4-j12}{3-j2}\right)}{2+\left(\frac{4-j12}{3-j2}\right)} 2.85\angle -78.43^o = 1.91\angle -78.43^o \quad \text{A}$$

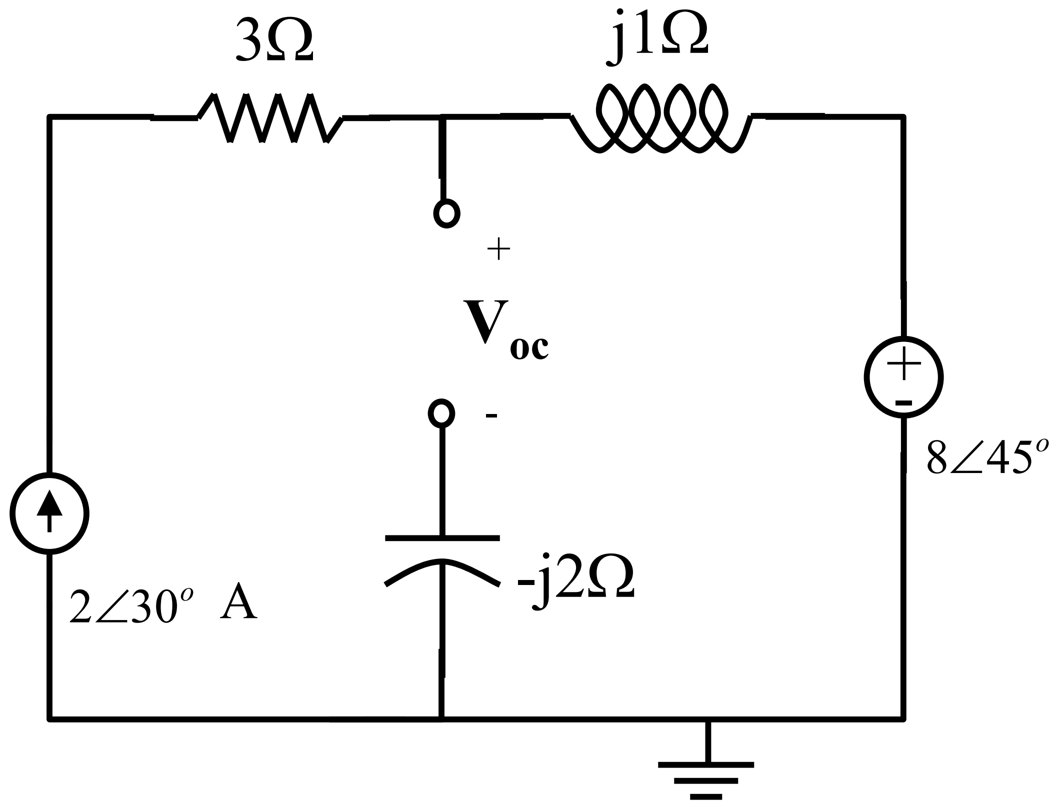
Example 8: Thevenin's Theorem

Calculate V_o using Thevenin's Theorem.



Example 8: Thevenin's Theorem

Calculate V_o using Thevenin's Theorem.

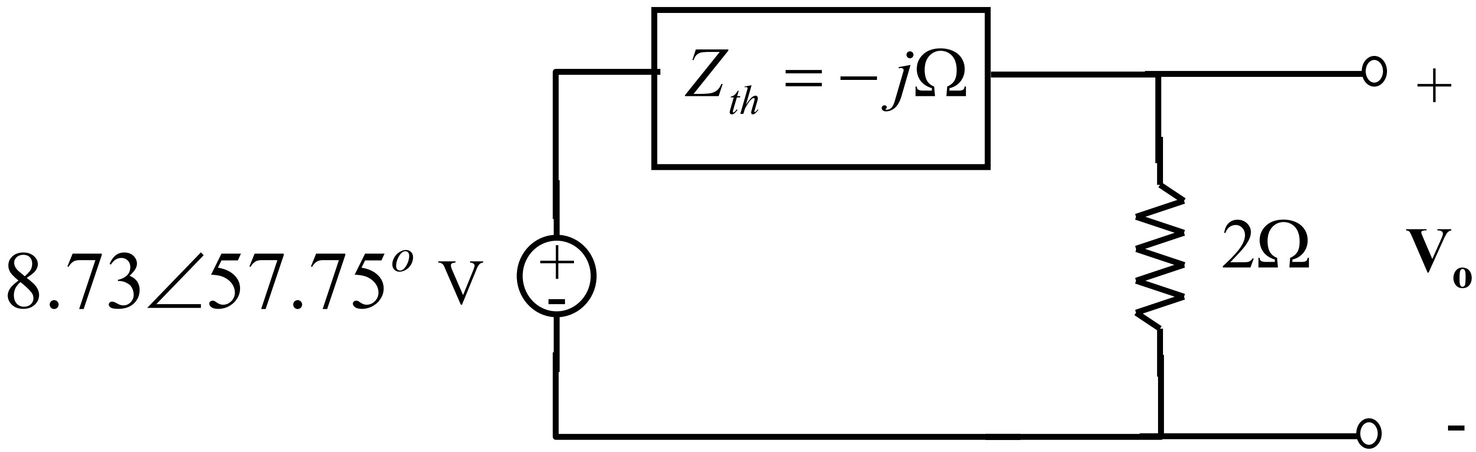


$$\mathbf{V_{oc}} = (2\angle 30^o)(j1) + 8\angle 45^o = 8.73\angle 57.75^o \text{ V}$$

$$Z_{th} = -j2 + j1 = -j\Omega$$

$$\mathbf{V_{th}} = \mathbf{V_{oc}} = 8.73\angle 57.75^o \text{ V}$$

Example 8: Thevenin's Theorem



$$\mathbf{V_0} = \frac{2}{2-j} 8.73\angle 57.75^o = 0.76 + j7.76 \quad \text{V}$$

Summary

1. All the methods derived for solving ***dc steady-state*** circuits are also valid for ***ac steady-state*** circuit analysis in the frequency domain. These methods include:

   → Nodal analysis.
   → Mesh and loop analysis.
   → Superposition.
   → Source transformation.
   → Thevenin and Norton's theorems

2. The principle of linearity is valid for the analysis of ***ac steady-state*** circuit analysis in the frequency domain.