ECSE 210: Circuit Analysis

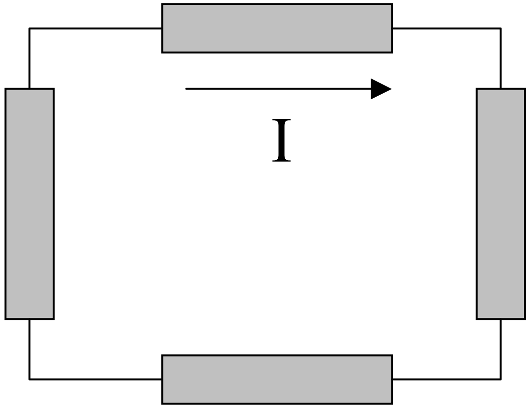
Lecture #1: Introduction

Circuit Analysis

- **Design** → **Creation of a Circuit**
- **Analysis** → **Evaluation of a Circuit**

Electric Circuit:

An interconnection of electrical elements linked together in a closed path so that electric current may flow.



Analogy: water flow

Linear Electric Circuits

- **A linear circuit satisfies the properties of superposition and homogeneity.**

<u>Example:</u>

Circuit with input i and output v

$$i \longrightarrow v$$

- Superposition:

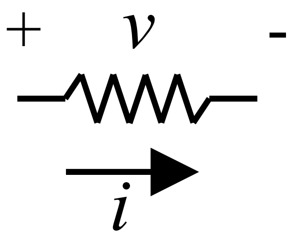
$$\left.\begin{array}{l} i_1 \longrightarrow v_1 \\ i_2 \longrightarrow v_2 \end{array}\right\} \longrightarrow \quad i_1 + i_2 \longrightarrow v_1 + v_2$$

- Homogeneity:

$$i \longrightarrow v \qquad \longrightarrow \qquad ki \longrightarrow kv$$

Linear Electric Circuits

- **Example of a linear element: The resistor**



$$v = iR$$

- Superposition:

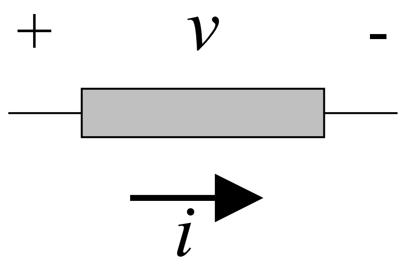
$$i_1 \longrightarrow v_1 = i_1 R$$
$$i_2 \longrightarrow v_2 = i_2 R$$
$$\Rightarrow \quad i_1 + i_2 \longrightarrow v = (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$$

- Homogeneity:

$$i \longrightarrow v = iR \quad \Rightarrow \quad ki \longrightarrow kiR = kv$$

Linear Electric Circuits

- **Example of a nonlinear element:**



$$v = e^{i}$$

- Superposition:

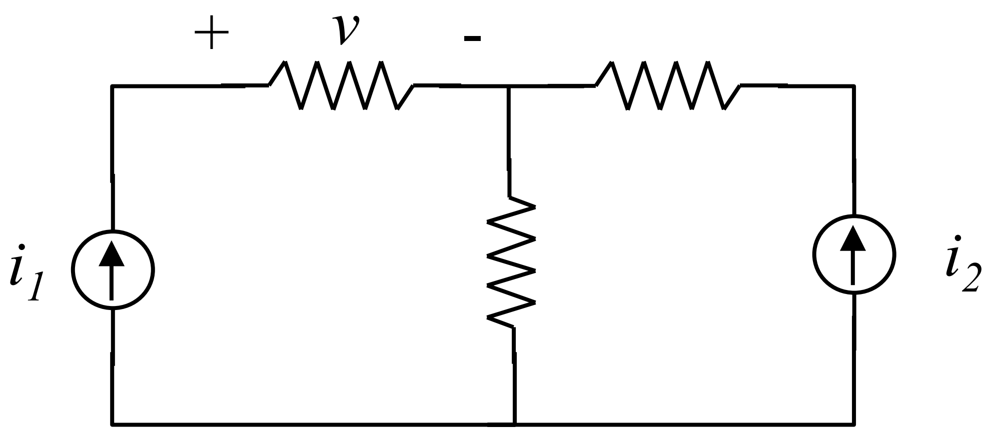
$$i_1 \rightarrow v_1 = e^{i_1}$$
$$i_2 \rightarrow v_2 = e^{i_2}$$
$$\rightarrow \quad i_1 + i_2 \rightarrow v = e^{i_1 + i_2} \neq e^{i_1} + e^{i_2}$$

- Homogeneity:

$$i \rightarrow v = e^{i} \quad \rightarrow \quad ki \rightarrow e^{ki} \neq k e^{i}$$

Linear Electric Circuits

- **Example of superposition**



- Remove i_2 and calculate v
- Remove i_1 and calculate v
- Add the two solutions

Circuit Analysis – Basic steps

Given Circuit (Design)

Analysis

1. Identify element models (e.g., V=IR for resistor).
2. Define analysis variables (e.g., currents, voltages).
3. Assemble circuit equations (e.g., KCL, KVL).
4. Solve circuit equations (e.g., matrix methods, computer…).
5. Evaluate circuit performance (e.g., power delivery, dynamic response, frequency response)

Interpretation of results

Circuit Analysis

Note: Circuit analysis yields the "physical" performance of *idealized* electric circuits, without concern for the actual systems represented by the circuits.

- Circuit models are used to *approximate* real systems;
- Circuit analysis determines the performance of circuit *models*.
- Electrical engineers use circuits to *estimate* the performance of real systems and devices.

Question: Do circuit models provide the exact performance of real electric circuits?

Some SI Units

Quantity	Unit	Symbol
length	Meter	m
time	second	s
charge	Coulomb	C
current	Ampere	A
voltage	Volt	V
resistance	Ohm	Ω
capacitance	Farad	F
inductance	Henry	H
energy	Joule	J
power	Watt	W

Standard SI prefixes

Prefix	Symbol	Multiplier
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
kilo	k	10^{+3}
mega	M	10^{+6}
giga	G	10^{+9}
tera	T	10^{+12}

Standard SI prefixes

Examples:

0.0015 Amperes should be written 1.5mA
3,500,000 Watts should be written 3.5MW
0.0012 mA should be written as 1.2μA

→ Concept of significant figures vs. decimal places

Standard SI prefixes

Examples:

0.0015 Amperes should be written 1.5mA
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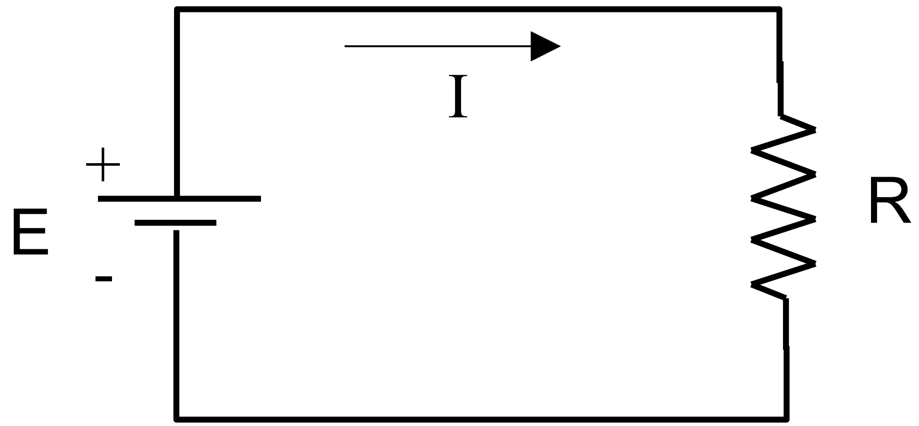
➡ Concept of significant figures vs. decimal places

More Basics - Current

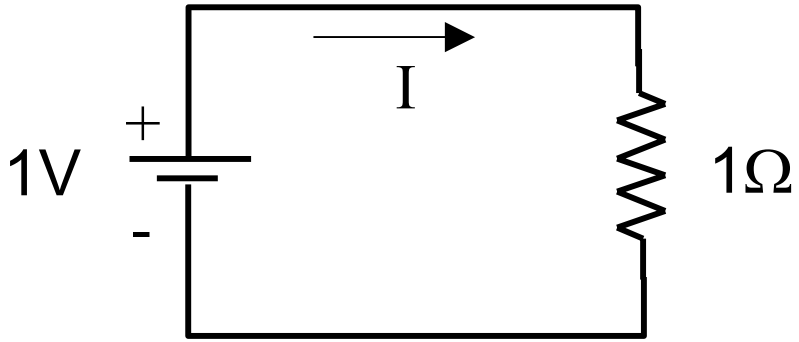
Circuit analysis *current variables* must be defined with prescribed *directions*.



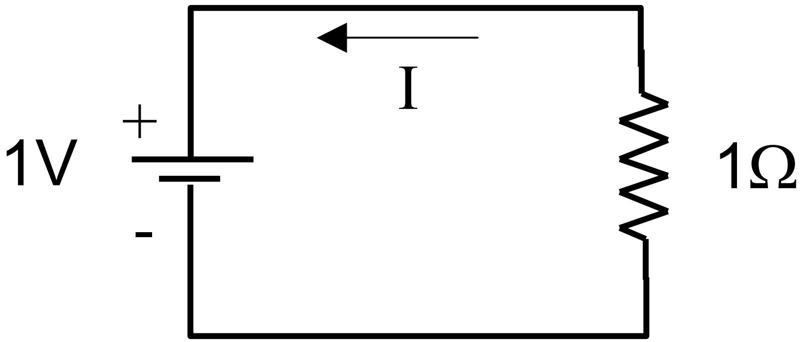
Does it matter which direction we assign to the current?

More Basics - Current

Current variable need not correspond to *physical* current flow.



I=1A

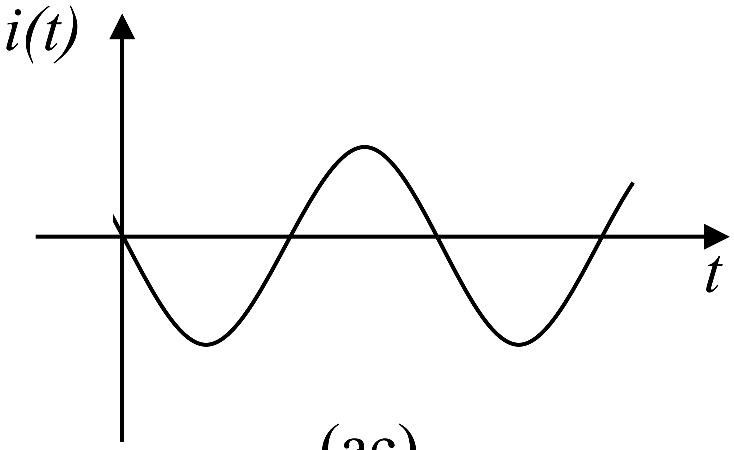


I=–1A

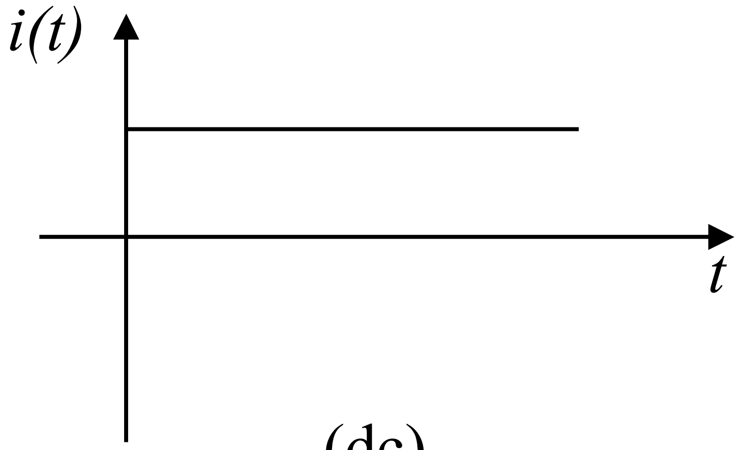
More Basics - Current

There are two types of current:

- **Time-invariant (stationary), commonly known as *direct current* (dc).**
- **Time-varying, e.g, the sinusoidal *alternating current* (ac)**



(ac)



(dc)

Voltage or Potential

Charge in motion → Energy Transfer

- **The *voltage between two points* in a circuit is defined as the difference in energy level of a positive unit charge located at each of the two points.**
- **In other words, the voltage between two points is the energy required to move a positive unit charge between the two points.**
- **Circuit analysis voltage variables must be defined with prescribed orientation – to identify the point at higher potential.**
- **The "+" and "-" signs are used: "+" marks the point of higher potential and "-" marks the point of lower potential.**

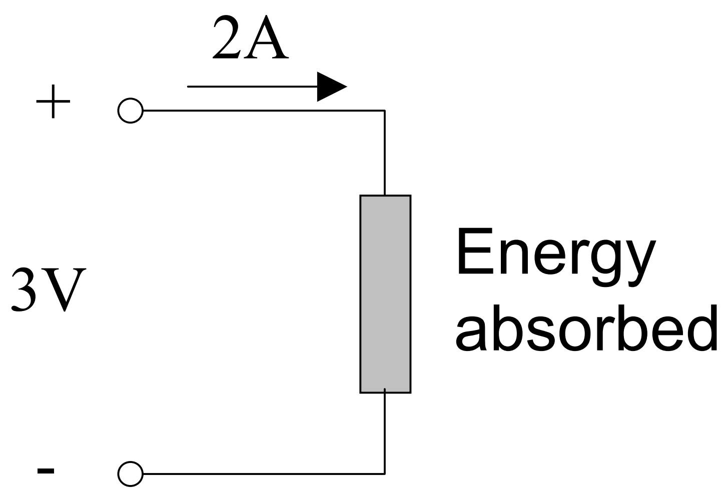
Voltage or Potential

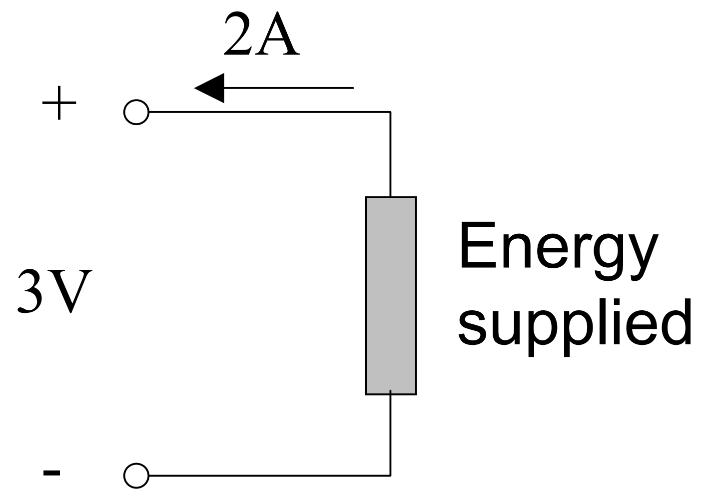
Does it make a difference which orientation we assign to a voltage variable?

Resist mixing variables and values.

Be consistent.

Energy Transfer





By definition (passive sign convention)

- Positive current into positive terminal → energy absorbed.
- Positive current into negative terminal → energy supplied.

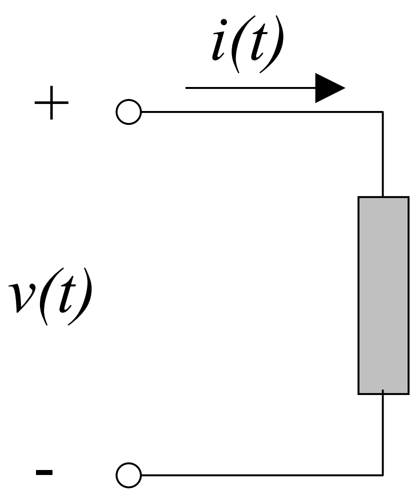
Question: Where does the absorbed (supplied) energy come from (go to)?

Power

Power flow in an electric circuit is defined by the "rate of movement of energy" in the circuit.

$$P = VI \qquad \text{or} \qquad p(t) = v(t)i(t)$$

with the passive sign convention (psc) assumed.



Under these conditions (psc), if *p(t)* is ***positive*** then power is ***absorbed*** by the element; if *p(t)* is ***negative***, then power is ***supplied*** by the element.

Circuit Elements

- In circuit analysis, physical circuit elements are represented by abstract mathematical models which describe their behavior.

- When we refer to a circuit element we actually mean the mathematical model that describes its behavior.

- In this course, all the circuit elements are terminal devices, completely characterized by the current through the element and/or the voltage across it.

Passive vs. Active Elements

Two types of elements: ***active*** and ***passive***:

- Active elements can *generate* energy; e.g., batteries and generators are active.

- Passive elements *can't* generate energy (but some can *store* energy); resistors, capacitors and inductors are passive.

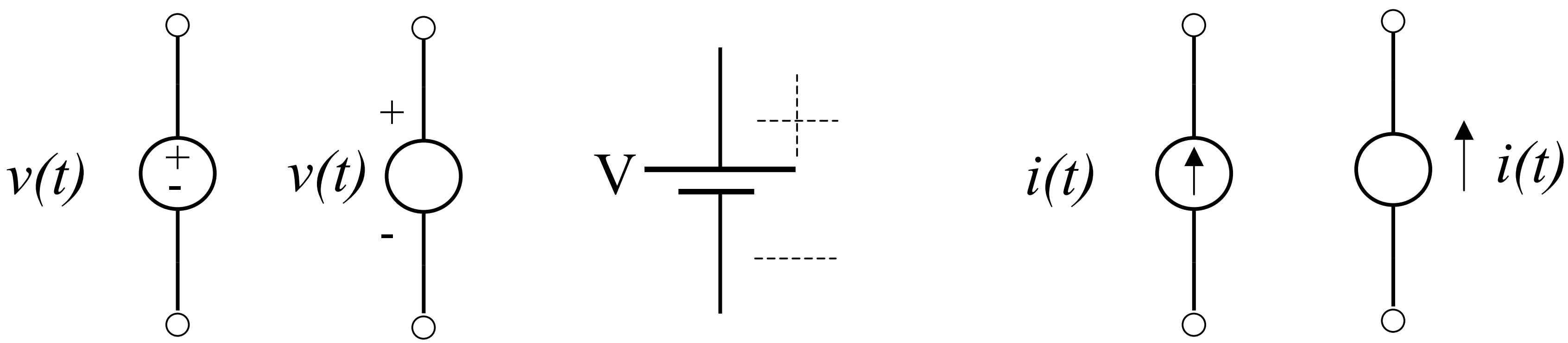
Four Basic Active Elements

Independent Voltage Source:

Two terminal element that maintains a specified voltage between its terminals *regardless* of the current through it.

Independent Current Source:

Two terminal element that maintains a specified current flow *regardless* of the voltage across its terminals.



Four Basic Active Elements

Dependent (controlled) Voltage Source:

Two terminal element that maintains a specified voltage between its terminals that is determined by another voltage or current elsewhere in the circuit.

Dependent (controlled) Current Source:

Two terminal element that maintains a specified current flow that is determined by another current or voltage elsewhere in the circuit.

Four Basic Active Elements