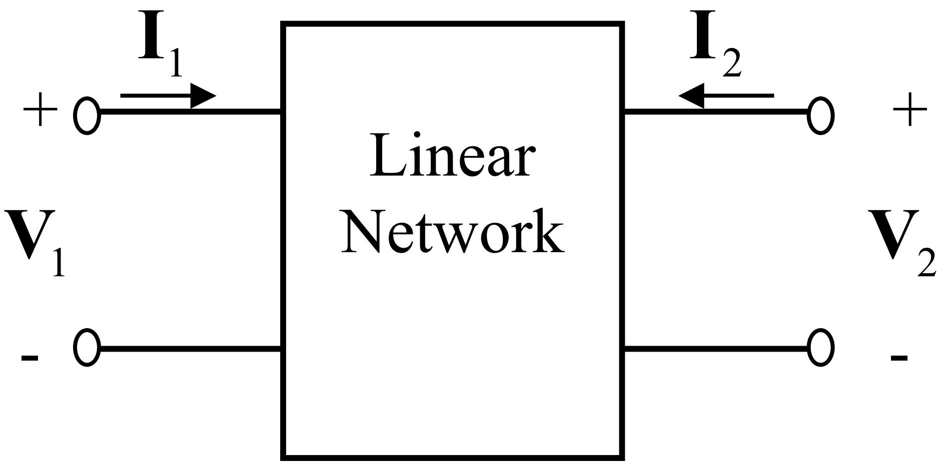
ECSE 210: Circuit Analysis

Lecture #31:

Two-Port Networks

Two-Port Transmission Parameters

→ Commonly referred to as the **ABCD-*parameters***.



→ Note the voltage polarities and current directions.

→ Using the principle of superposition we get:

$$\mathbf{V}_1 = A\mathbf{V}_2 - B\mathbf{I}_2$$

$$\mathbf{I}_1 = C\mathbf{V}_2 - D\mathbf{I}_2$$

ABCD- Parameters

$$\mathbf{V}_1 = A\mathbf{V}_2 - B\mathbf{I}_2$$
$$\mathbf{I}_1 = C\mathbf{V}_2 - D\mathbf{I}_2$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix}$$

- Two equations describe two-port operation.
- *A, B, C and D* are complex constants of proportionality with units of Ohms (Ω), Siemens (S), or unit-less.
- Common in Transmission-line models
- Once *A, B, C and D* are known, the input/output operation of the two-port network is **completely** defined.
- The transmission parameter matrix is:

$$T = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

ABCD - Parameters

$$\mathbf{V}_1 = A\mathbf{V}_2 - B\mathbf{I}_2$$
$$\mathbf{I}_1 = C\mathbf{V}_2 - D\mathbf{I}_2$$

$$\Rightarrow \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix}$$

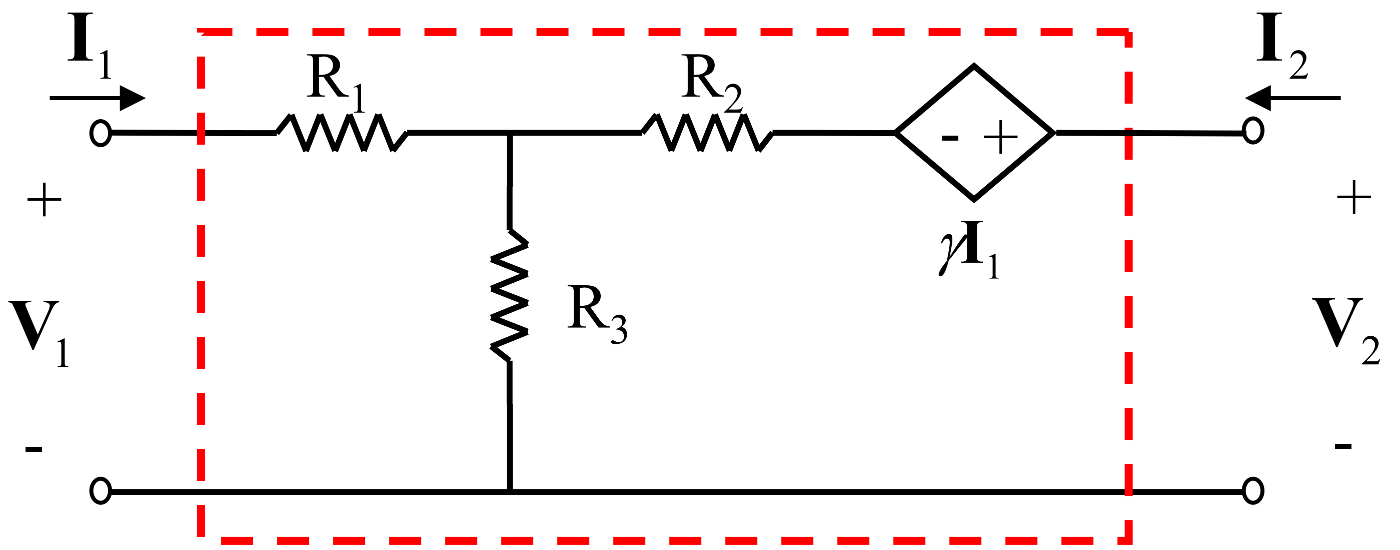
$$\mathbf{V}_2 = 0 \quad \rightarrow \quad \mathbf{V}_1 = -B\mathbf{I}_2 \quad \rightarrow \quad B = -\left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

$$\mathbf{V}_2 = 0 \quad \rightarrow \quad \mathbf{I}_1 = -D\mathbf{I}_2 \quad \rightarrow \quad D = \left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

$$\mathbf{I}_2 = 0 \quad \rightarrow \quad \mathbf{V}_1 = A\mathbf{V}_2 \quad \rightarrow \quad A = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{I}_2 = 0 \quad \rightarrow \quad \mathbf{I}_1 = C\mathbf{V}_2 \quad \rightarrow \quad C = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

ABCD - Parameters



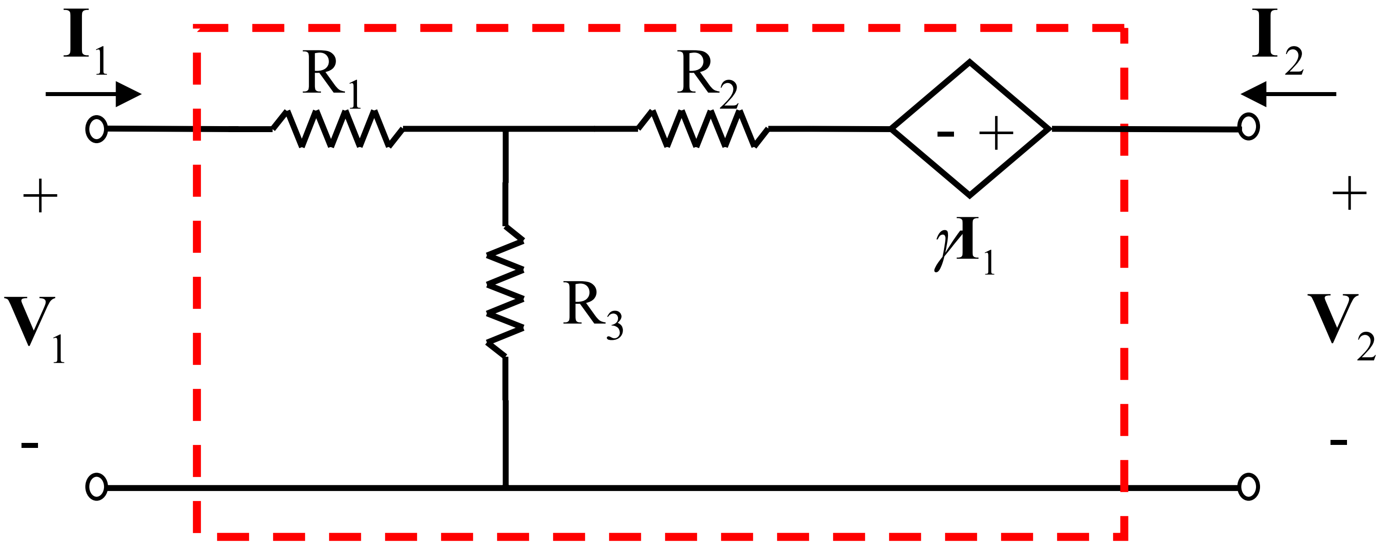
Find the transmission (ABCD) parameters: $\begin{cases} \mathbf{V}_1 = A\mathbf{V}_2 - B\mathbf{I}_2 \\ \mathbf{I}_1 = C\mathbf{V}_2 - D\mathbf{I}_2 \end{cases}$

Open circuit test: $\mathbf{I}_2 = 0$

$$\mathbf{V}_2 = \frac{R_3}{R_1 + R_3}\mathbf{V}_1 + \gamma\mathbf{V}_1 = \frac{R_3}{R_1 + R_3}\mathbf{V}_1 + \gamma\frac{\mathbf{V}_1}{R_1 + R_3}$$

$$\Rightarrow \quad A = \left(\frac{R_3}{R_1 + R_3} + \frac{\gamma}{R_1 + R_3}\right)^{-1} = \frac{R_1 + R_3}{R_3 + \gamma}$$

ABCD - Parameters



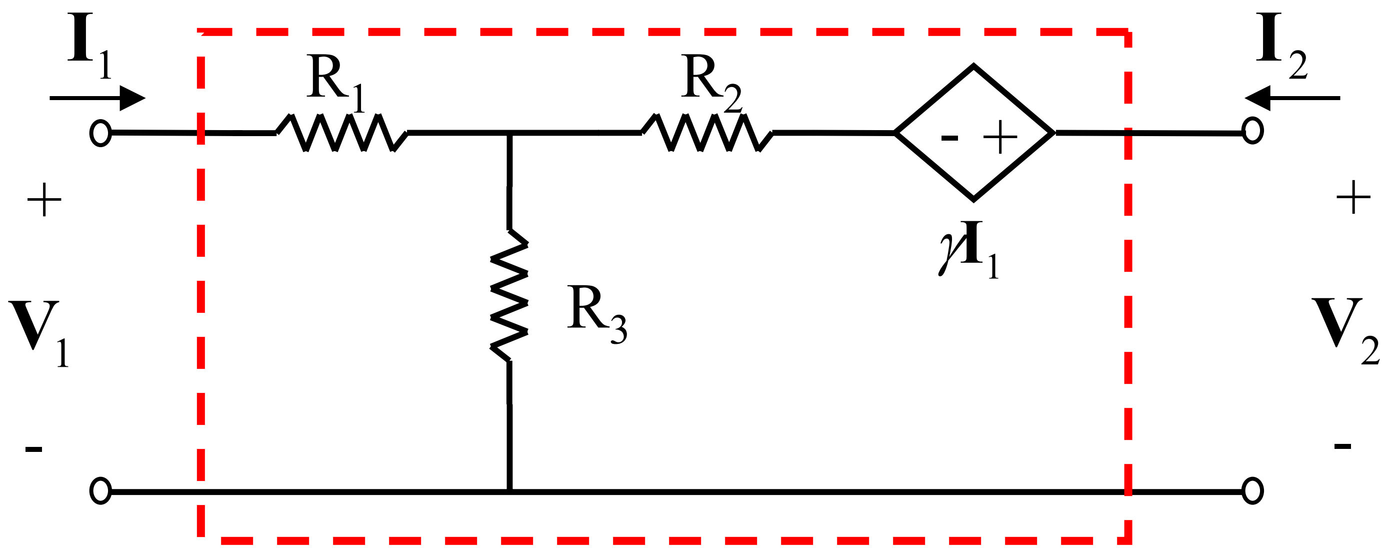
Find the transmission (ABCD) parameters: $\begin{cases} \mathbf{V}_1 = A\mathbf{V}_2 - B\mathbf{I}_2 \\ \mathbf{I}_1 = C\mathbf{V}_2 - D\mathbf{I}_2 \end{cases}$

Open circuit test: $\mathbf{I}_2 = 0$

$$\mathbf{V}_1 = \frac{\mathbf{V}_1}{R_1 + R_3} = \frac{1}{R_1 + R_3} \frac{R_1 + R_3}{R_3 + \gamma} \mathbf{V}_2$$

$$\Rightarrow C = \frac{1}{R_3 + \gamma}$$

ABCD - Parameters



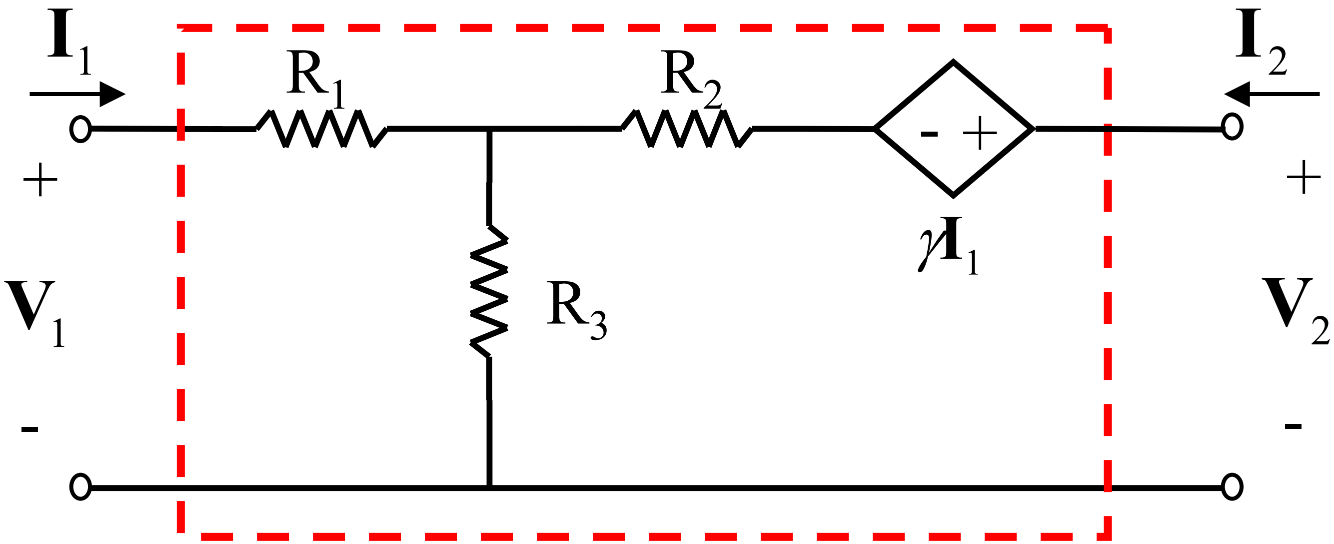
Find the transmission (ABCD) parameters:
$$\begin{cases} \mathbf{V}_1 = A\mathbf{V}_2 - B\mathbf{I}_2 \\ \mathbf{I}_1 = C\mathbf{V}_2 - D\mathbf{I}_2 \end{cases}$$

Short circuit test: $\mathbf{V}_2 = 0$

$$R_3(\mathbf{I}_1 + \mathbf{I}_2) + R_2\mathbf{I}_2 + \gamma\mathbf{I}_1 = 0$$

$$(R_3 + \gamma)\mathbf{I}_1 + (R_3 + R_2)\mathbf{I}_2 = 0 \quad \Rightarrow \quad D = \frac{R_3 + R_2}{R_3 + \gamma}$$

ABCD - Parameters



Find the transmission (ABCD) parameters: $\begin{cases} \mathbf{V}_1 = A\mathbf{V}_2 - B\mathbf{I}_2 \\ \mathbf{I}_1 = C\mathbf{V}_2 - D\mathbf{I}_2 \end{cases}$

Short circuit test: $\mathbf{V}_2 = 0$

$$(R_3 + \gamma)\mathbf{I}_1 + (R_3 + R_2)\mathbf{I}_2 = 0$$

$$\mathbf{V}_1 = \mathbf{I}_1(R_1 + R_3) + \mathbf{I}_2 R_3$$

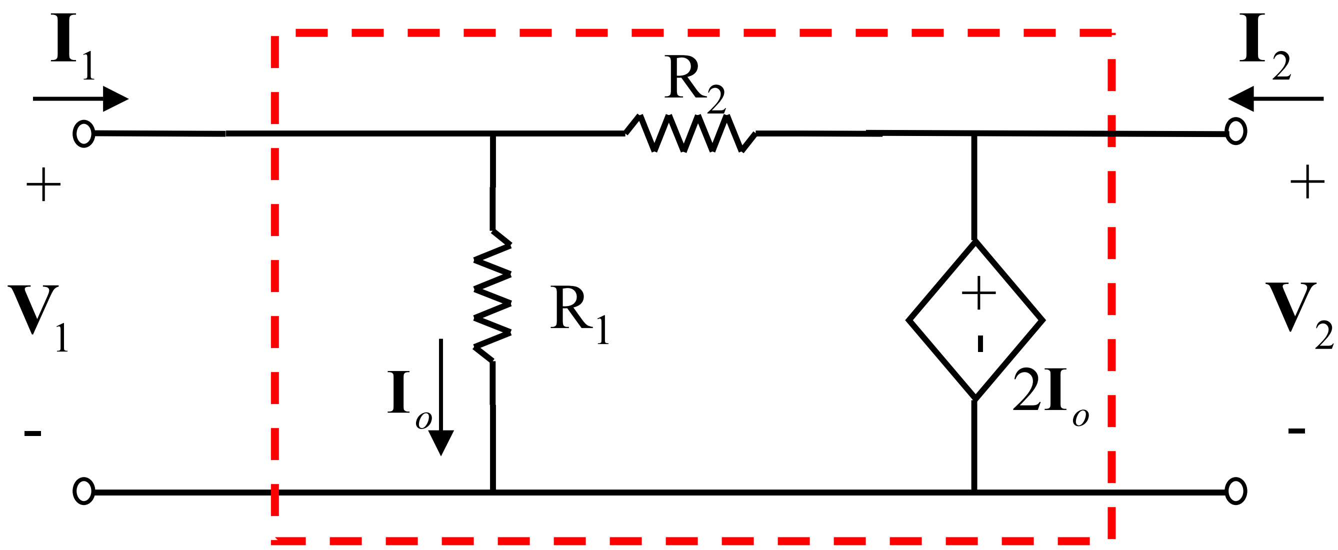
$$\mathbf{V}_1 = -\mathbf{I}_2 \frac{(R_1 + R_3)(R_2 + R_3)}{R_3 + \gamma} + \mathbf{I}_2 R_3$$

$$\Rightarrow \quad B = \frac{(R_1 + R_3)(R_2 + R_3)}{R_3 + \gamma} - R_3$$

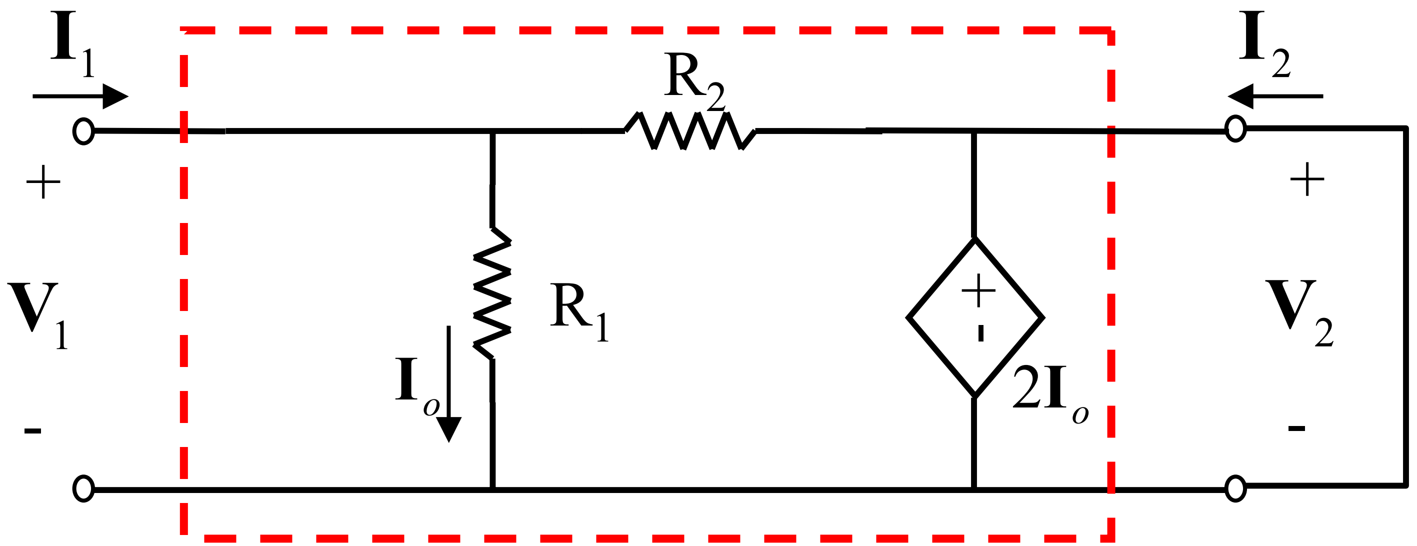
Two-Port Parameters

Note: It is possible that some or all of the two-port parameters for a given network ***may not exist***!

For example, consider the Y-parameters of the following networks:



Two-Port Parameters



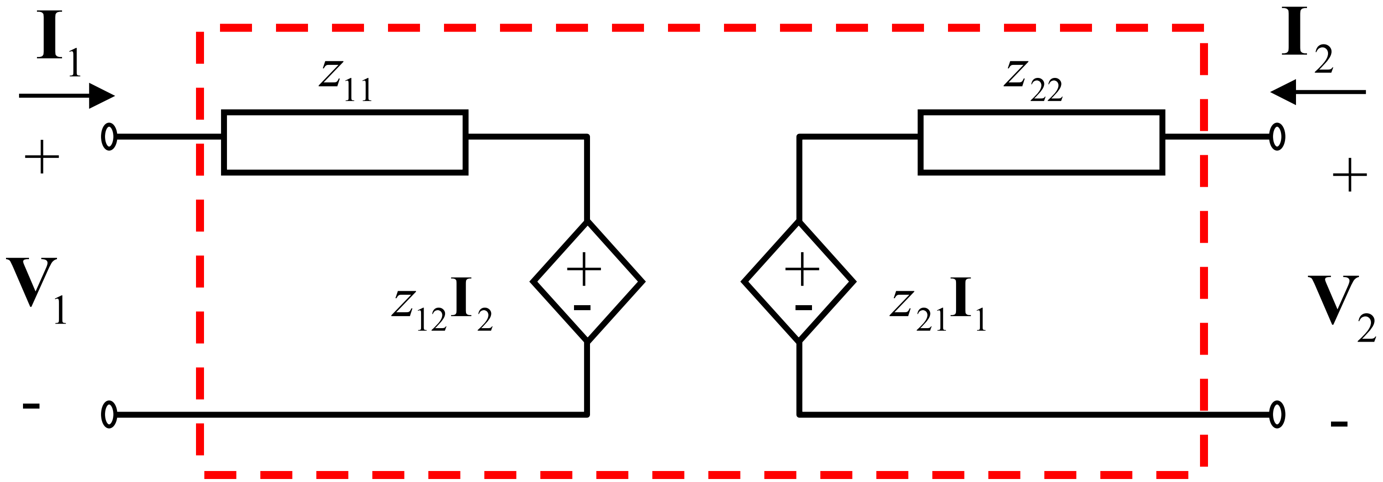
Short circuit test: $V_2 = 0$

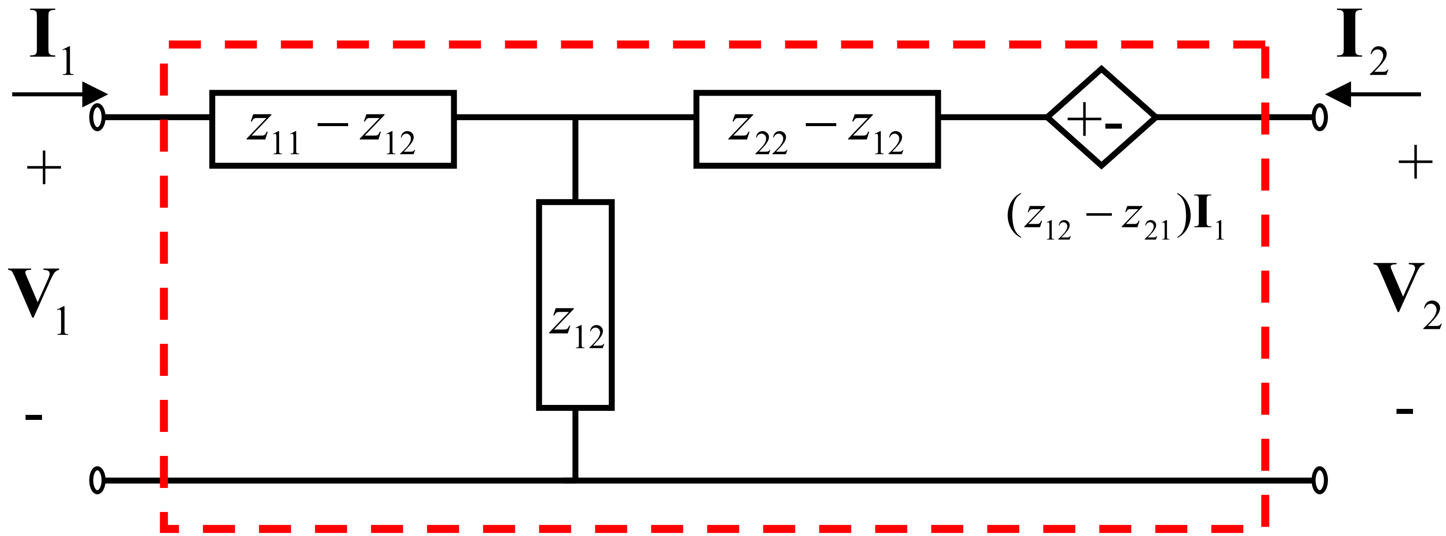
$$I_1 = y_{11}V_1$$

$V_2 = 2I_o = 0$ ⇨ $I_o = 0$!!?

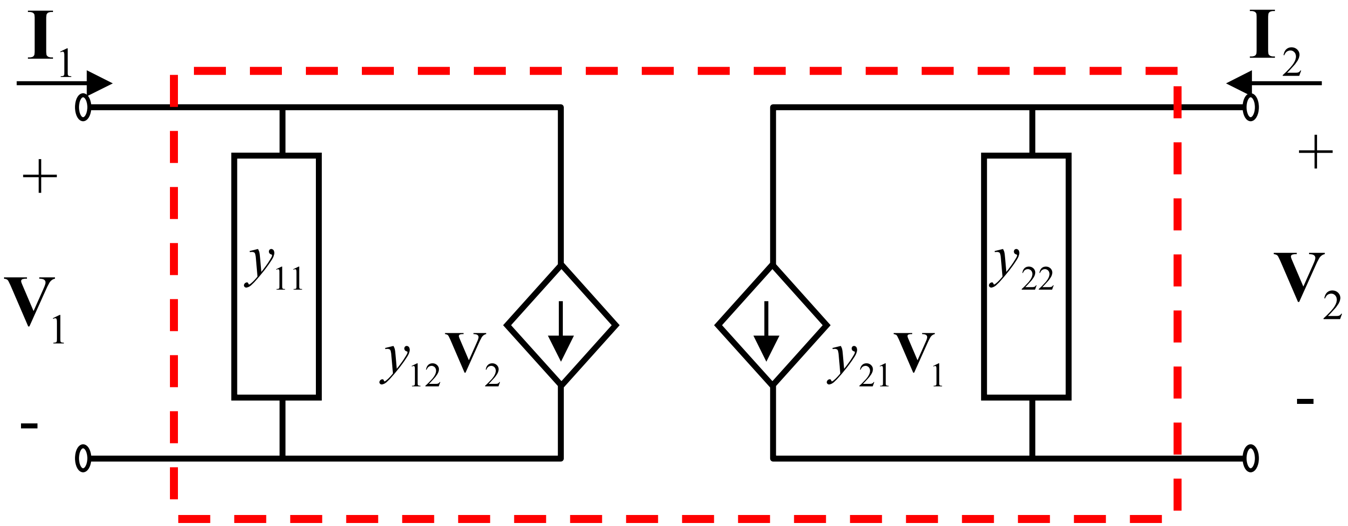
⇨ $V_1 = 0$!!?

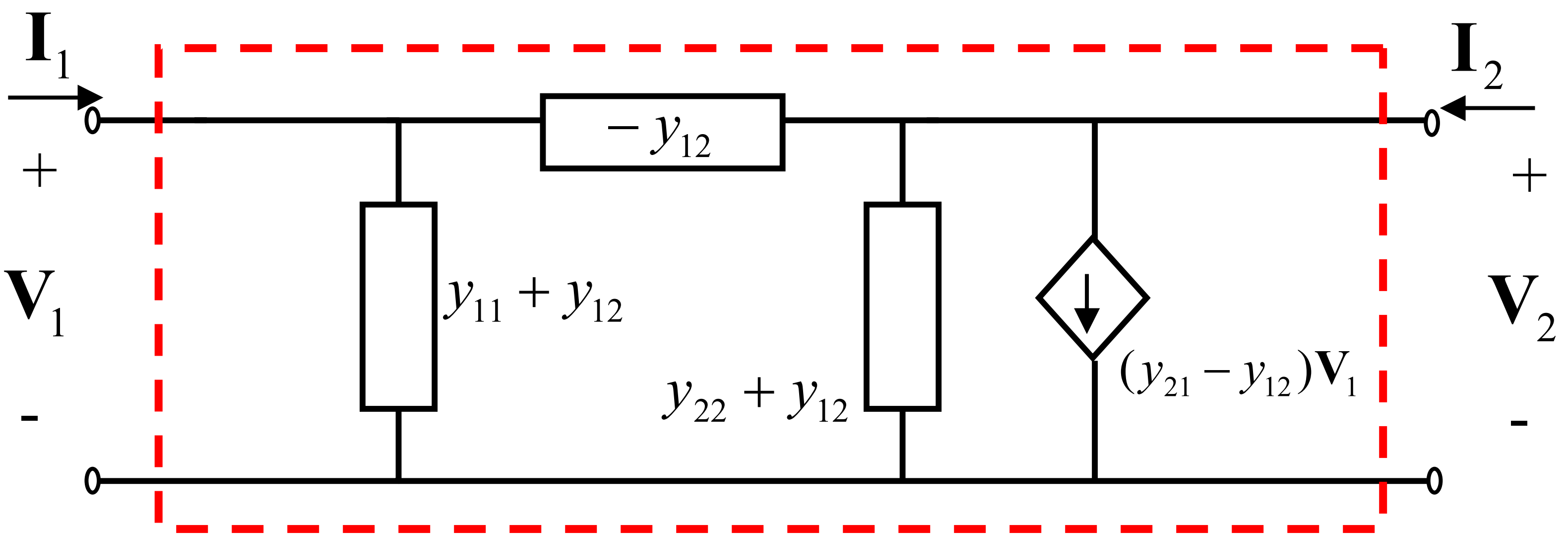
Equivalent Circuits



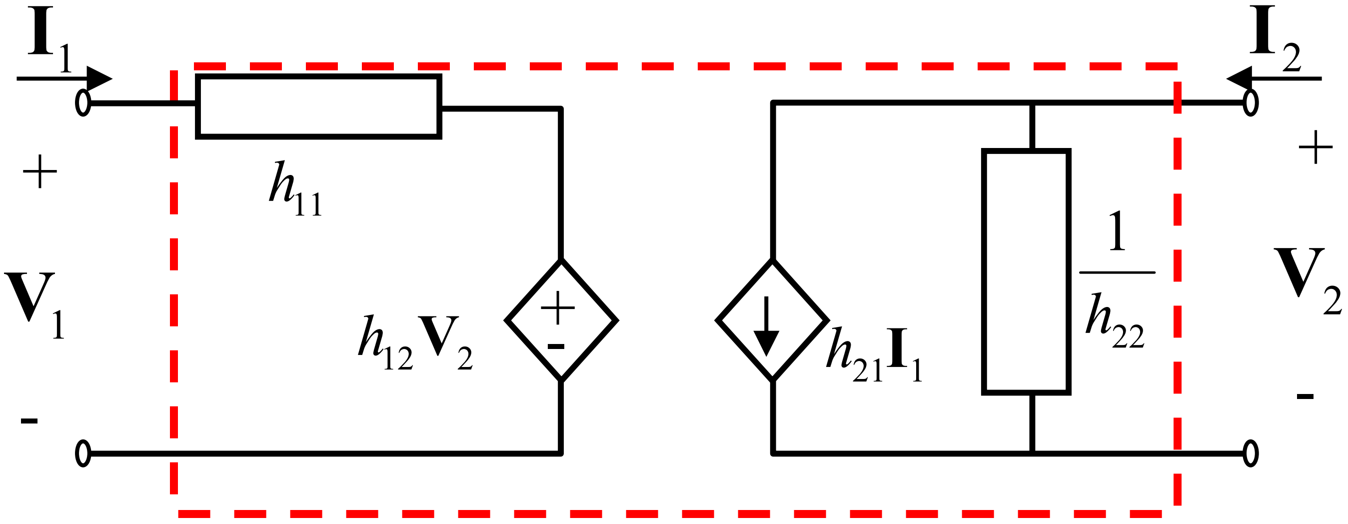


Equivalent Circuits



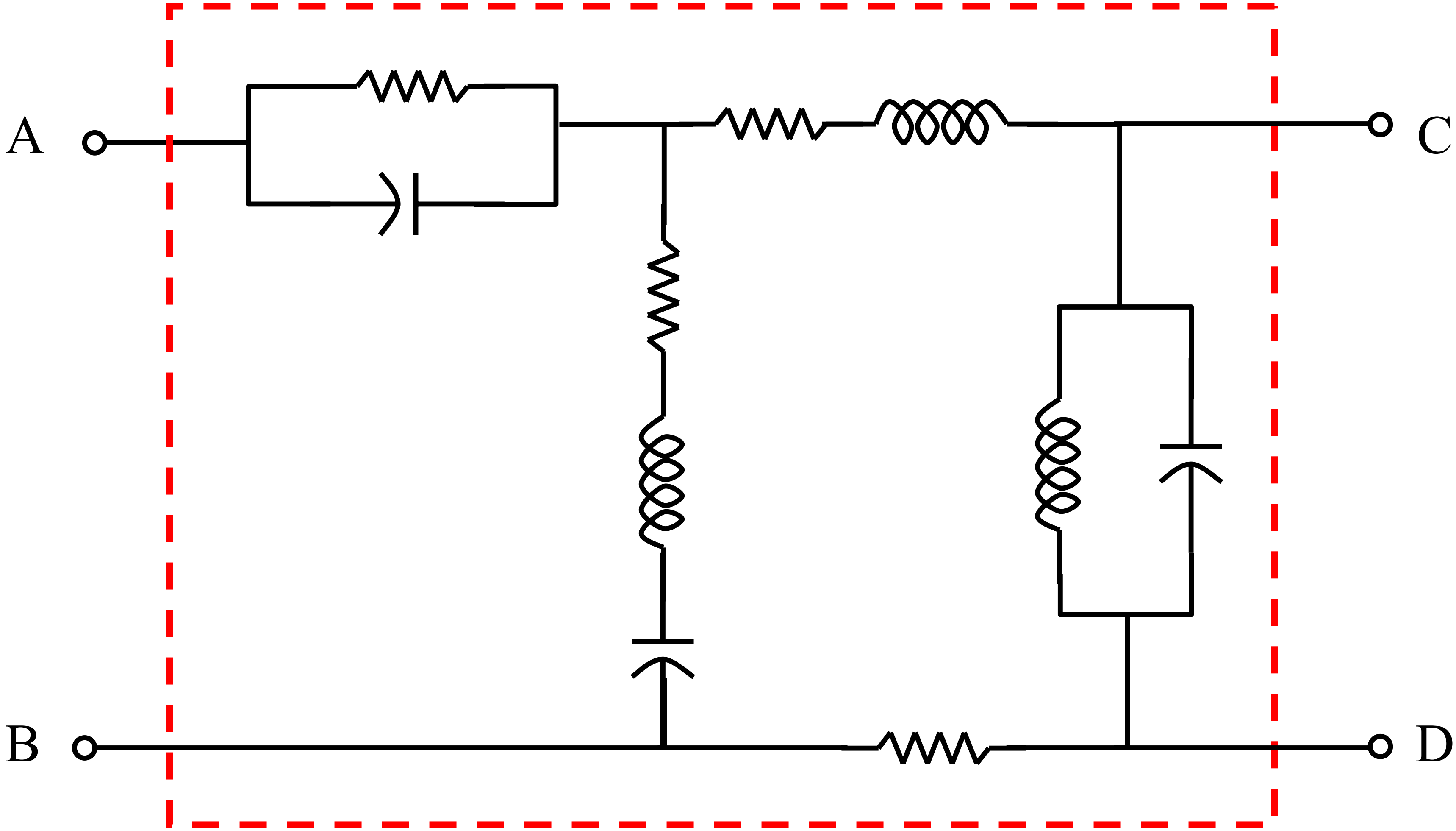


Equivalent Circuits



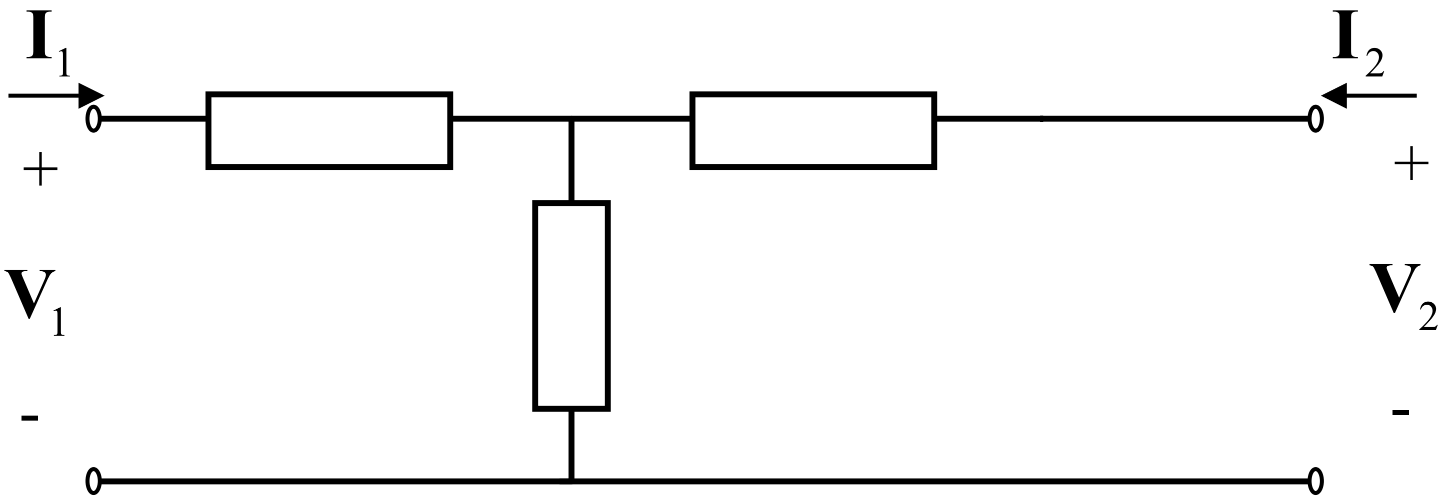
Example: Two-port network

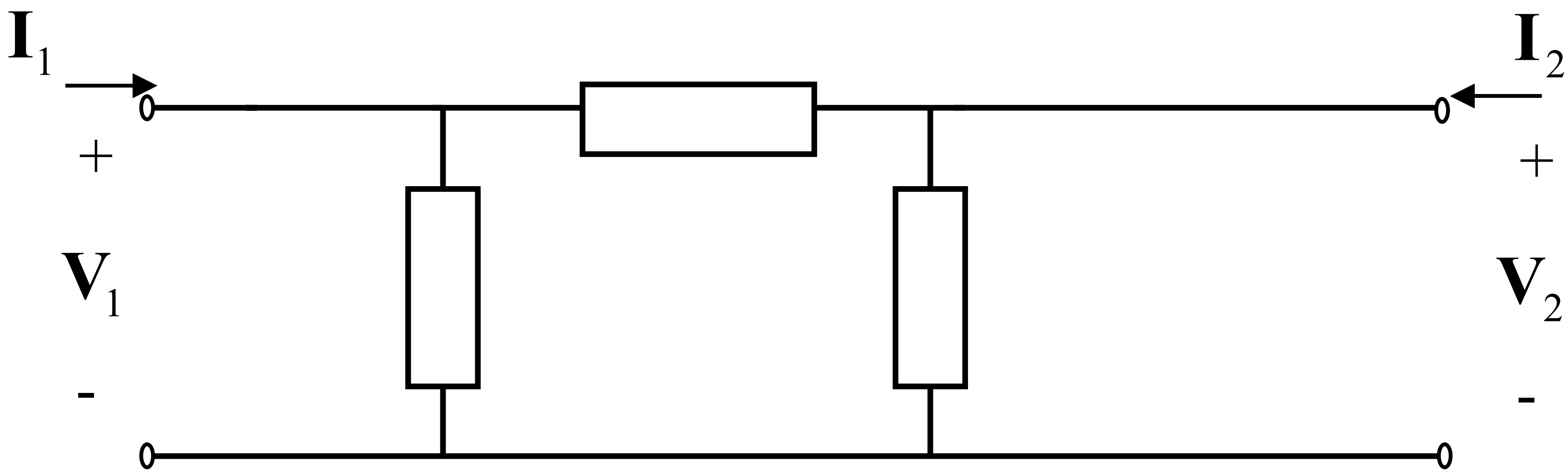
Consider the following two port network:



Example: Two-Port Network

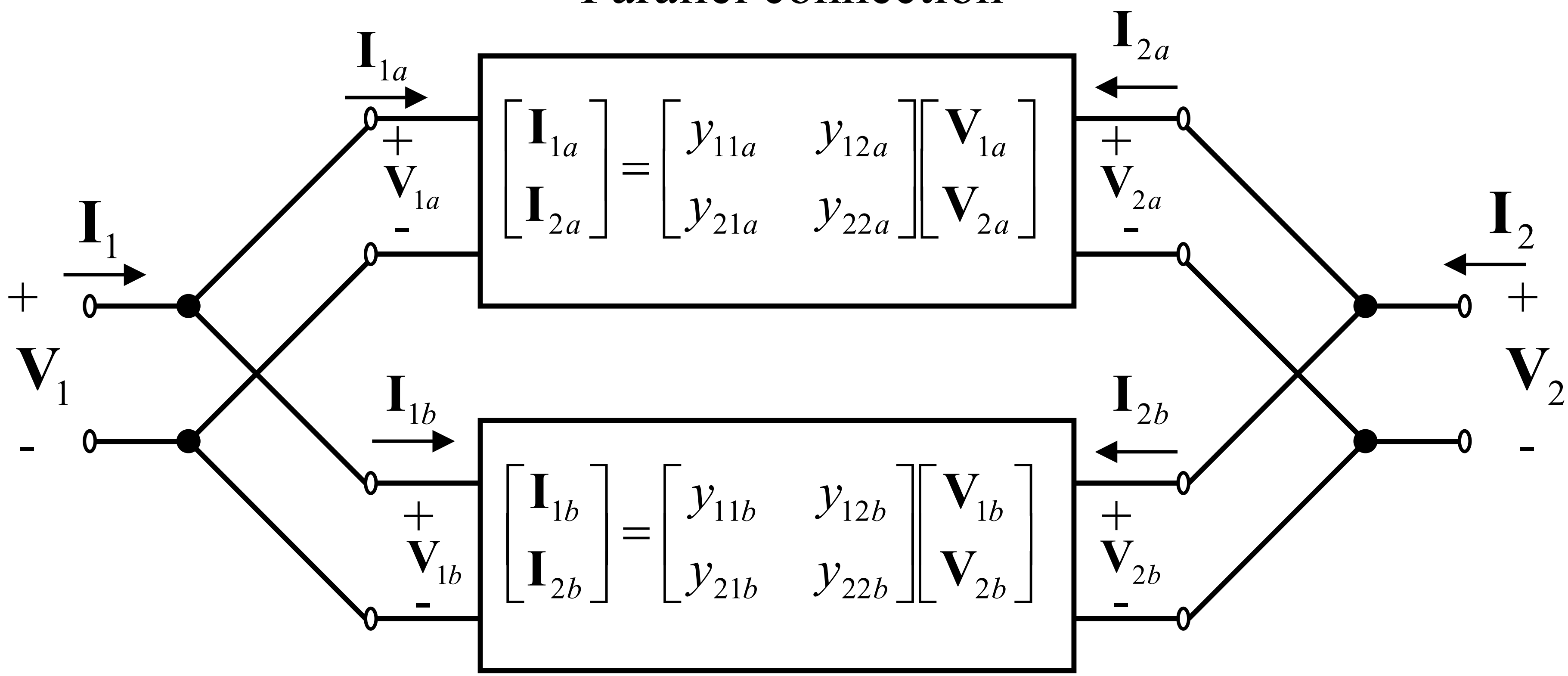
Find an equivalent circuit of the previous network in the form:





Interconnection of Two-Port Networks

Parallel connection



$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b} \qquad \mathbf{V}_1 = \mathbf{V}_{1a} = \mathbf{V}_{1b}$$

$$\mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b} \qquad \mathbf{V}_2 = \mathbf{V}_{2a} = \mathbf{V}_{2b}$$

Interconnection of Two-Port Networks

Parallel Connection

$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b} \qquad \mathbf{V}_1 = \mathbf{V}_{1a} = \mathbf{V}_{1b}$$

$$\mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b} \qquad \mathbf{V}_2 = \mathbf{V}_{2a} = \mathbf{V}_{2b}$$

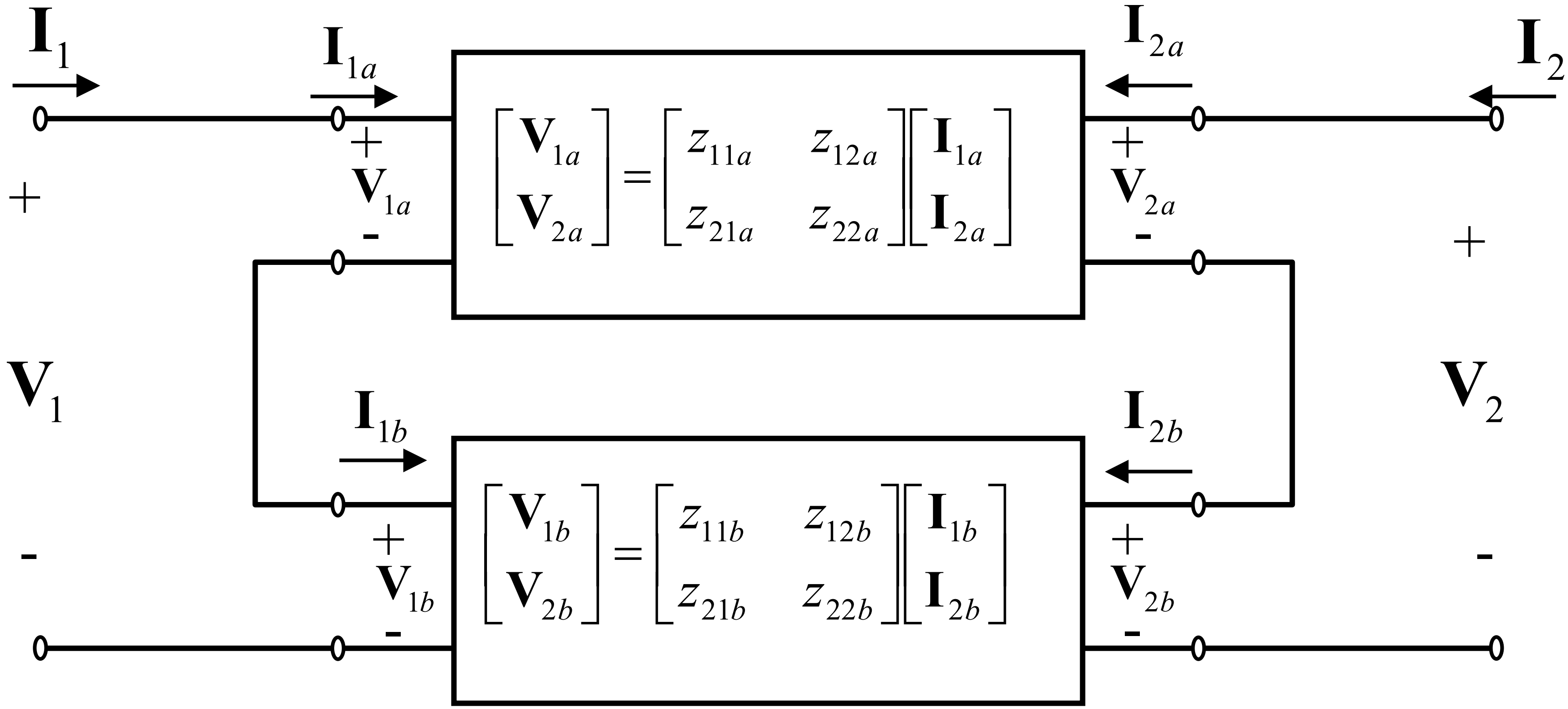
$$\begin{bmatrix} \mathbf{I}_{1a} \\ \mathbf{I}_{2a} \end{bmatrix} = \begin{bmatrix} y_{11a} & y_{12a} \\ y_{21a} & y_{22a} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{1a} \\ \mathbf{V}_{2a} \end{bmatrix} \qquad \begin{bmatrix} \mathbf{I}_{1b} \\ \mathbf{I}_{2b} \end{bmatrix} = \begin{bmatrix} y_{11b} & y_{12b} \\ y_{21b} & y_{22b} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{1b} \\ \mathbf{V}_{2b} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{1a} \\ \mathbf{I}_{2a} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_{1b} \\ \mathbf{I}_{2b} \end{bmatrix} = \left(\begin{bmatrix} y_{11a} & y_{12a} \\ y_{21a} & y_{22a} \end{bmatrix} + \begin{bmatrix} y_{11b} & y_{12b} \\ y_{21b} & y_{22b} \end{bmatrix} \right) \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{Y}_a + \mathbf{Y}_b = \begin{bmatrix} y_{11a} + y_{11b} & y_{12a} + y_{12b} \\ y_{21a} + y_{21b} & y_{22a} + y_{22b} \end{bmatrix}$$

Interconnection of Two-Port Networks

Series connection



$$\begin{bmatrix} \mathbf{V}_{1a} \\ \mathbf{V}_{2a} \end{bmatrix} = \begin{bmatrix} z_{11a} & z_{12a} \\ z_{21a} & z_{22a} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{1a} \\ \mathbf{I}_{2a} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{V}_{1b} \\ \mathbf{V}_{2b} \end{bmatrix} = \begin{bmatrix} z_{11b} & z_{12b} \\ z_{21b} & z_{22b} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{1b} \\ \mathbf{I}_{2b} \end{bmatrix}$$

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} \qquad \mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b}$$

$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} \qquad \mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b}$$

Interconnection of Two-Port Networks

Series connection

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} \qquad \mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b}$$

$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} \qquad \mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b}$$

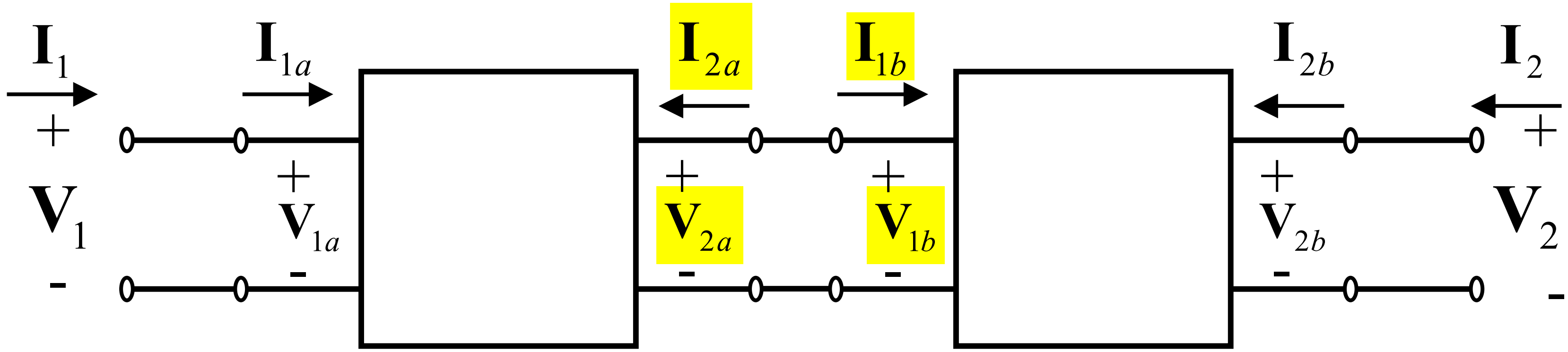
$$\begin{bmatrix} \mathbf{V}_{1a} \\ \mathbf{V}_{2a} \end{bmatrix} = \begin{bmatrix} z_{11a} & z_{12a} \\ z_{21a} & z_{22a} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{1a} \\ \mathbf{I}_{2a} \end{bmatrix} \qquad \begin{bmatrix} \mathbf{V}_{1b} \\ \mathbf{V}_{2b} \end{bmatrix} = \begin{bmatrix} z_{11b} & z_{12b} \\ z_{21b} & z_{22b} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{1b} \\ \mathbf{I}_{2b} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{1a} \\ \mathbf{V}_{2a} \end{bmatrix} + \begin{bmatrix} \mathbf{V}_{1b} \\ \mathbf{V}_{2b} \end{bmatrix} = \left(\begin{bmatrix} z_{11a} & z_{12a} \\ z_{21a} & z_{22a} \end{bmatrix} + \begin{bmatrix} z_{11b} & z_{12b} \\ z_{21b} & z_{22b} \end{bmatrix} \right) \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\mathbf{Z} = \mathbf{Z}_a + \mathbf{Z}_b = \begin{bmatrix} z_{11a} + z_{11b} & z_{12a} + z_{12b} \\ z_{21a} + z_{21b} & z_{22a} + z_{22b} \end{bmatrix}$$

Interconnection of Two-Port Networks

Cascade Connections



$$\begin{bmatrix} \mathbf{V}_{1a} \\ \mathbf{I}_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2a} \\ -\mathbf{I}_{2a} \end{bmatrix} \qquad \begin{bmatrix} \mathbf{V}_{1b} \\ \mathbf{I}_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2b} \\ -\mathbf{I}_{2b} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{V}_{2a} \\ -\mathbf{I}_{2a} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{1b} \\ \mathbf{I}_{1b} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{V}_{1} \\ \mathbf{I}_{1} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{1a} \\ \mathbf{I}_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2b} \\ -\mathbf{I}_{2a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2} \\ -\mathbf{I}_{2} \end{bmatrix}$$