ECSE 210: Circuit Analysis

Lecture #28:

Passive Filter Networks

Passive Filter Networks

Filters are designed to *pass* signals in a given frequency range and *reject* or *attenuate* signals outside that range.

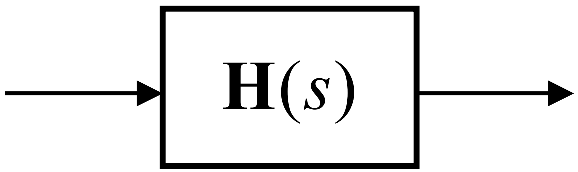
→ **"Pass"** frequency range is called "*passband*".
→ **"Reject"** frequency range is called the *"rejection band"*

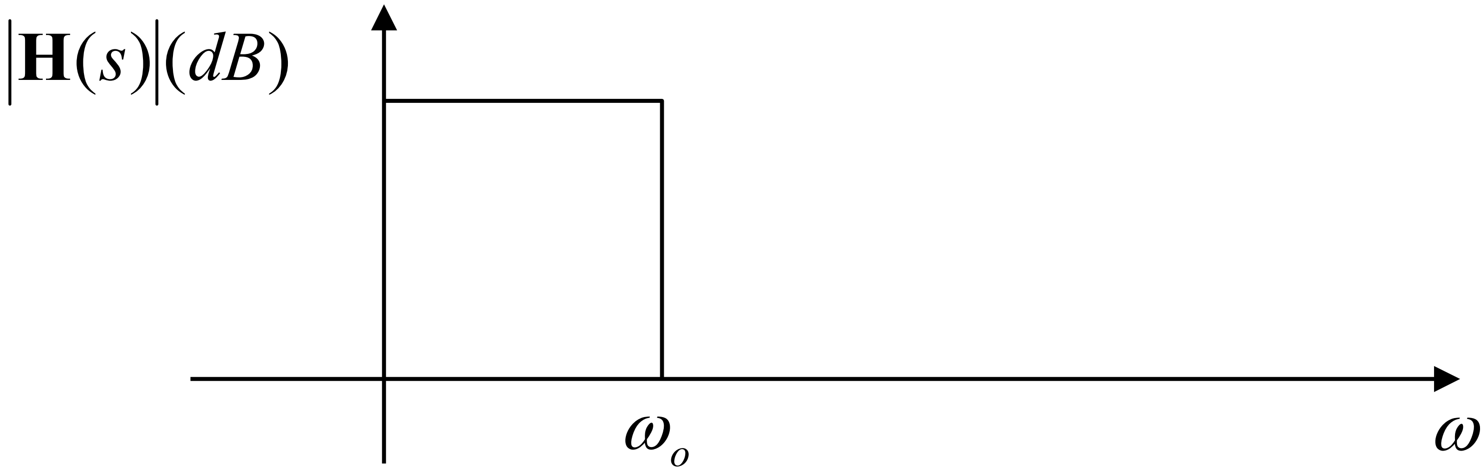
➔ There are four common passive filter types:

1. **Low-pass**: designed to pass low frequencies and reject high frequencies.
2. **High-pass**: designed to pass high frequencies and reject low frequencies.
3. **Band-pass**: designed to pass frequencies within a specific band and reject all others.
4. **Band-reject**: designed to reject frequencies within a specific band are reject all others.

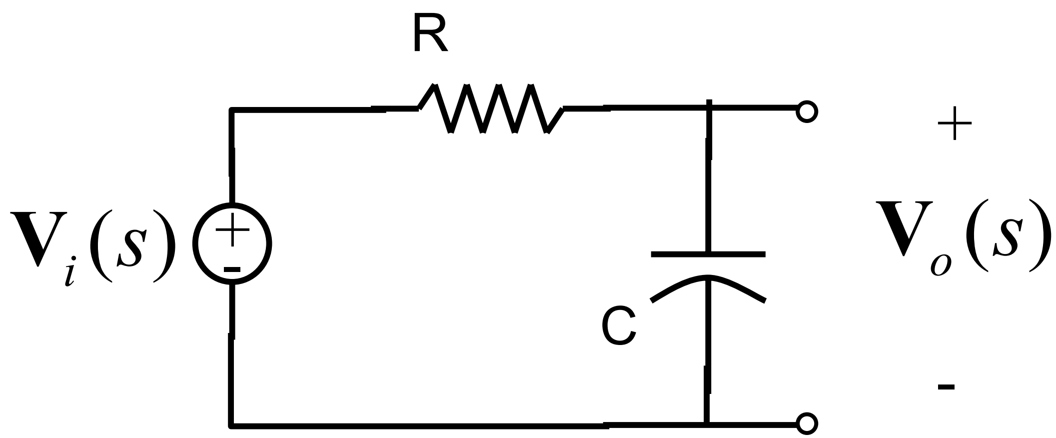
Low-Pass Filter

Ideal response: Pass all frequencies up to a cut-off ω_o, but reject all frequencies above it.





Low Pass Filter Example



$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + sRC} = \frac{\omega_o}{s + \omega_o}$$

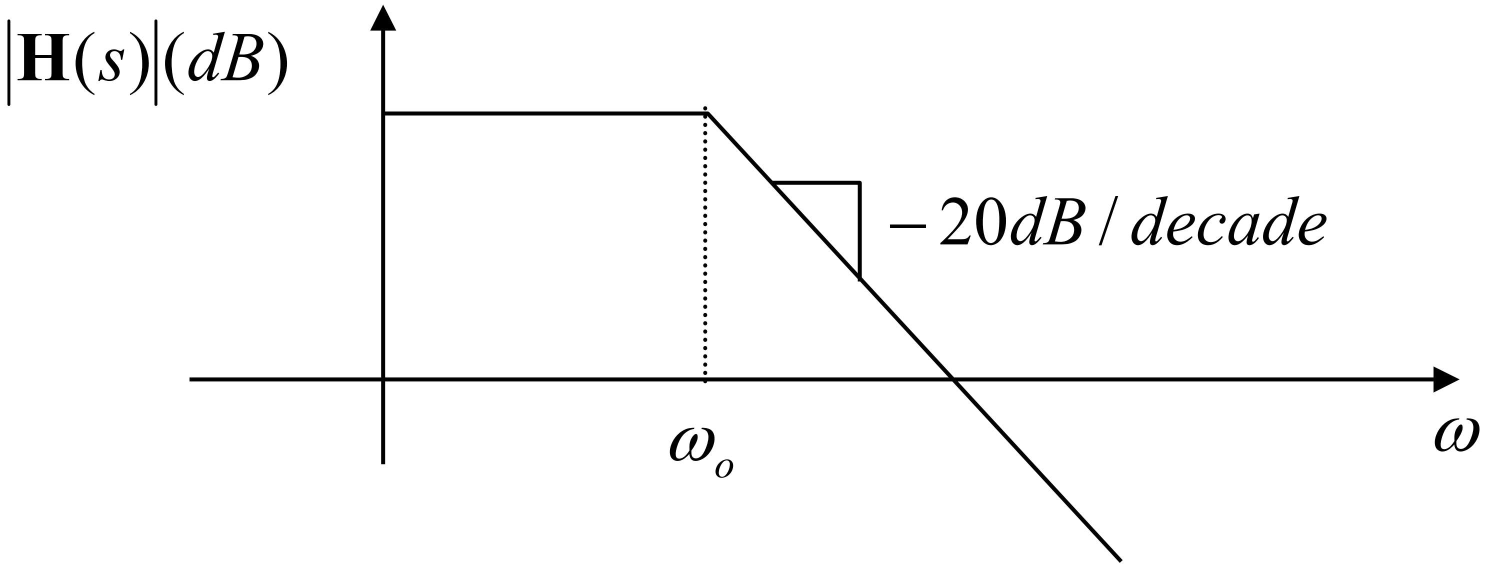
$$\omega_o = \frac{1}{RC}$$

Circuit has a single real pole: $p = -\omega_o$

→ **First order filter.**

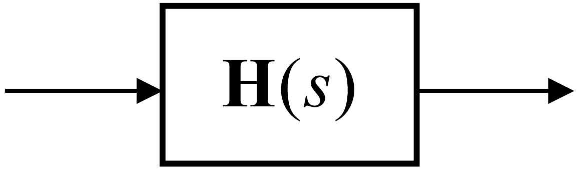
Example Low-Pass Filter

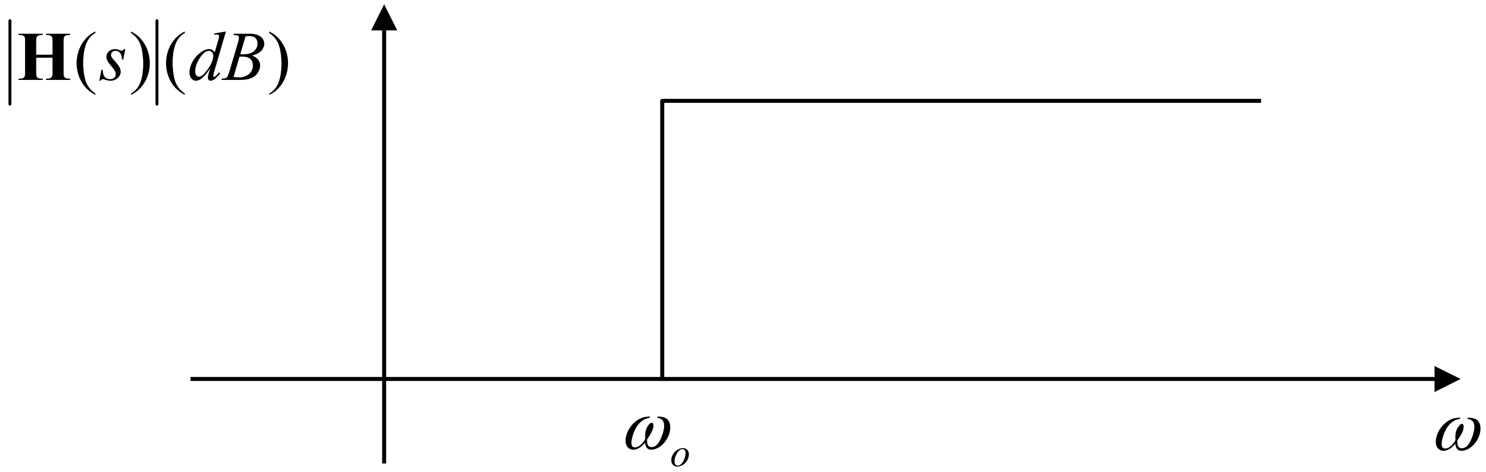
→ First order filter: $\mathbf{H}(s) = \dfrac{\omega_o}{s + \omega_o}$



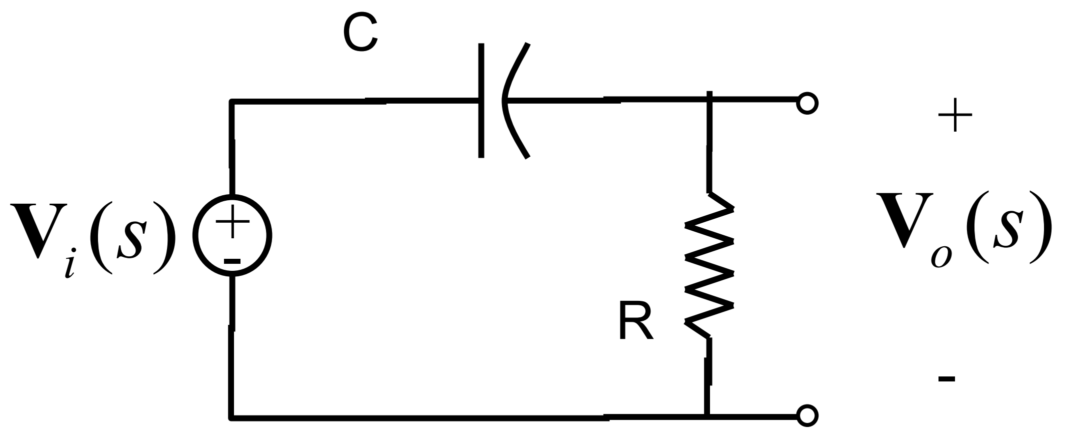
High-Pass Filter

Ideal response: Pass all frequencies above a cut-off ω_o, but reject all frequencies below it.





High-Pass Filter Example



$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/sC} = \frac{sRC}{1 + sRC} = \frac{s}{s + \omega_o} \qquad \omega_o = \frac{1}{RC}$$

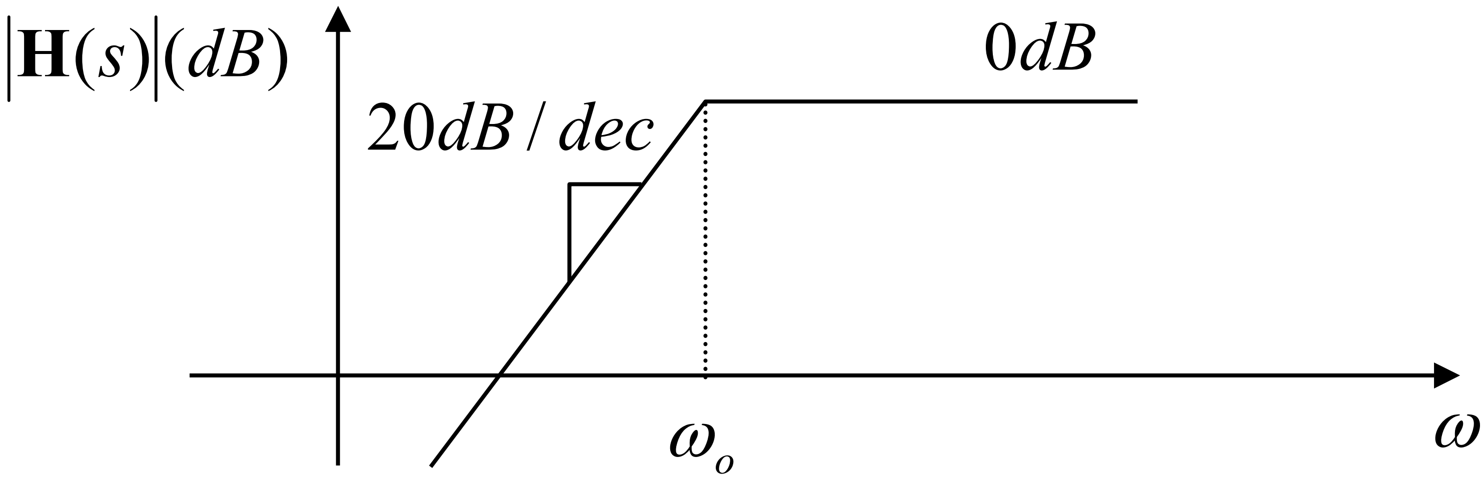
Circuit has a single real pole $p = -\omega_o$
and a single zero at the origin

→ **First order filter.**

Example Low-Pass Filter

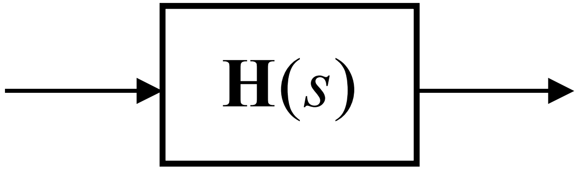
→ First order filter.

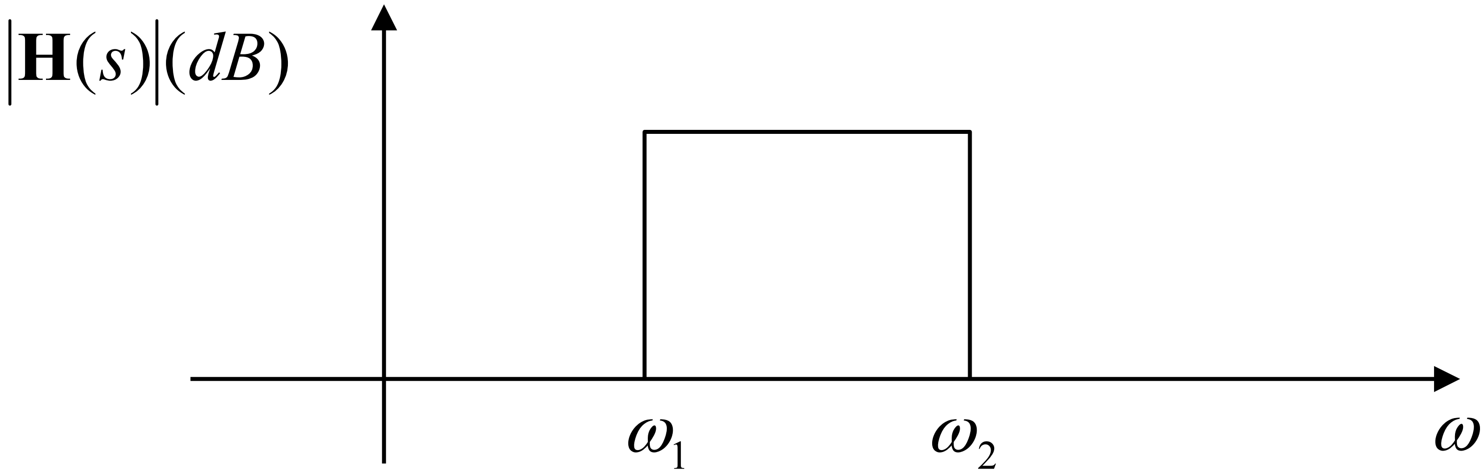
$$\mathbf{H}(s) = \frac{s}{s+\omega_o}$$



Band-Pass Filter

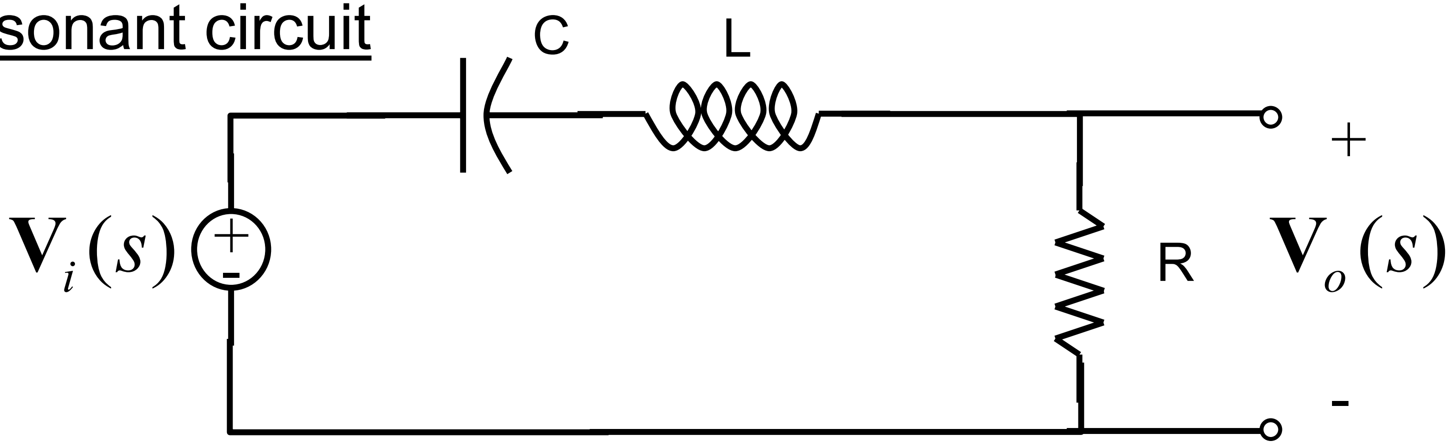
Ideal response: Pass all frequencies within a specific band (frequency range), but reject all other frequencies.





Band-Pass Filter Example

Series Resonant circuit



$$\mathbf{H}(s)=\frac{\mathbf{V}_o}{\mathbf{V}_i}=\frac{R}{R+sL+\frac{1}{sC}}=\frac{sRC}{1+RCs+LCs^2}=\frac{j\omega RC}{1-\omega^2LC+j\omega RC}$$

$$|\mathbf{H}(s)|=\frac{\omega RC}{\sqrt{(1-\omega^2LC)^2+(\omega RC)^2}}$$

Band-Pass Filter Example

$$\left|\mathbf{H}(s)\right| = \frac{\omega RC}{\sqrt{\left(1-\omega^2 LC\right)^2 + \left(\omega RC\right)^2}}$$

$\omega \to 0$ $\Rightarrow$ $\left|\mathbf{H}(s)\right| \to 0$

$\omega \to \infty$ $\Rightarrow$ $\left|\mathbf{H}(s)\right| \to 0$

Response is Maximum at resonant frequency $\omega_o = \dfrac{1}{\sqrt{LC}}$

The pass-band is between the two half-power frequencies.

Band-Pass Filter Example

Half-power frequencies:

$$|\mathbf{H}(s)| = \frac{\omega RC}{\sqrt{\left(1-\omega^2 LC\right)^2 + \left(\omega RC\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{\left(\omega RC\right)^2}{\left(1-\omega^2 LC\right)^2 + \left(\omega RC\right)^2} = \frac{1}{2} \quad \Rightarrow \quad 2\left(\omega RC\right)^2 = \left(1-\omega^2 LC\right)^2 + \left(\omega RC\right)^2$$

$$\Rightarrow \quad \left(1-\omega^2 LC\right)^2 = \left(\omega RC\right)^2 \quad \Rightarrow \quad 1-\omega^2 LC = \pm\omega RC$$

Band-Pass Filter Example

Half-power frequencies:

$$1-\omega^2 LC = \pm \omega RC \quad \Rightarrow \quad \omega^2 \pm \frac{R}{L}\omega - \omega_o^2 = 0$$

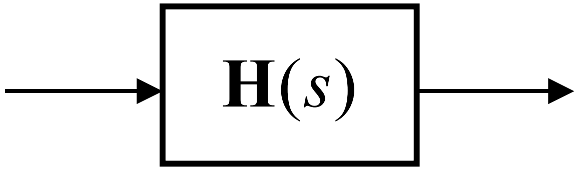
Solving for two positive roots results in half-power or cut-off frequencies:

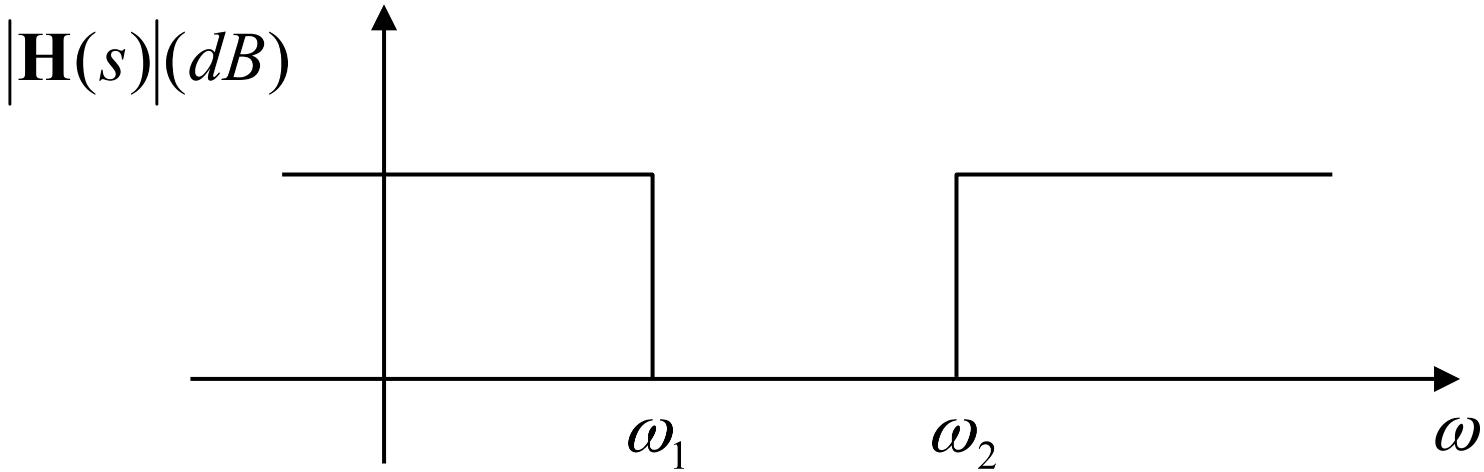
$$\omega_1 = \frac{-\left(R/L\right) + \sqrt{\left(R/L\right)^2 + 4\omega_o^2}}{2} \qquad \omega_2 = \frac{+\left(R/L\right) + \sqrt{\left(R/L\right)^2 + 4\omega_o^2}}{2}$$

$$BW = \omega_2 - \omega_1 = \frac{R}{L}$$

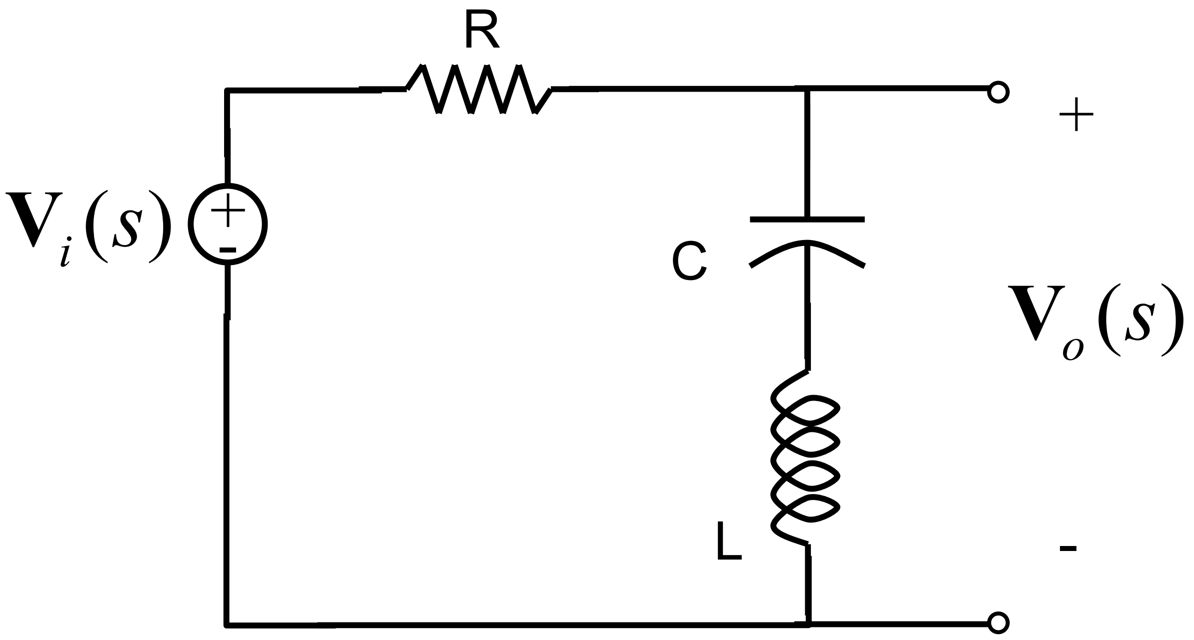
Band-Reject Filter

Ideal response: Rejects all frequencies within a specific band (frequency range), but passes all other frequencies.





Band-Reject Filter Example



$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{sL + 1/sC}{R + sL + 1/sC} = \frac{s^2LC + 1}{1 + RCs + LCs^2} = \frac{1 - \omega^2 LC}{1 - \omega^2 LC + j\omega RC}$$

$$\left|\mathbf{H}(s)\right| = \frac{1 - \omega^2 LC}{\sqrt{\left(1 - \omega^2 LC\right)^2 + \left(\omega RC\right)^2}}$$

Band-Reject Filter Example

$$|\mathbf{H}(s)| = \frac{|1-\omega^2 LC|}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

$\omega \to 0$ ⇒ $|\mathbf{H}(s)| \to 1$

$\omega \to \infty$ ⇒ $|\mathbf{H}(s)| \to 1$

At $\omega_o = \frac{1}{\sqrt{LC}}$ ⇒ $|\mathbf{H}(s)| = 0$

Center frequency = resonant frequency

The rejection band is between the two half- power frequencies.

Band-Pass Filter Example

Half-power frequencies:

$$|\mathbf{H}(s)| = \frac{|1-\omega^2 LC|}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad 2(1-\omega^2 LC)^2 = (1-\omega^2 LC)^2 + (\omega RC)^2$$

$$\Rightarrow \quad 1-\omega^2 LC = \pm\omega RC \quad \Rightarrow \quad \omega^2 \pm \frac{R}{L}\omega - \omega_o^2 = 0$$

Band-Reject Filter Example

Half-power frequencies:

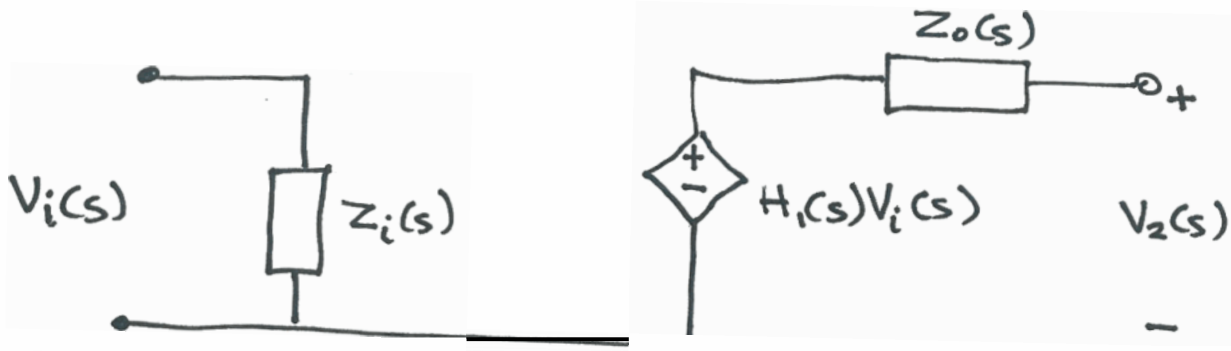
$$\omega^2 \pm \frac{R}{L}\omega - \omega_o^2 = 0$$

Solving for two positive roots results in half-power or cut-off frequencies:

$$\omega_1 = \frac{-\left(R/L\right) + \sqrt{\left(R/L\right)^2 + 4\omega_o^2}}{2} \qquad \omega_2 = \frac{+\left(R/L\right) + \sqrt{\left(R/L\right)^2 + 4\omega_o^2}}{2}$$

$$BW = \omega_2 - \omega_1 = \frac{R}{L}$$

LOADING



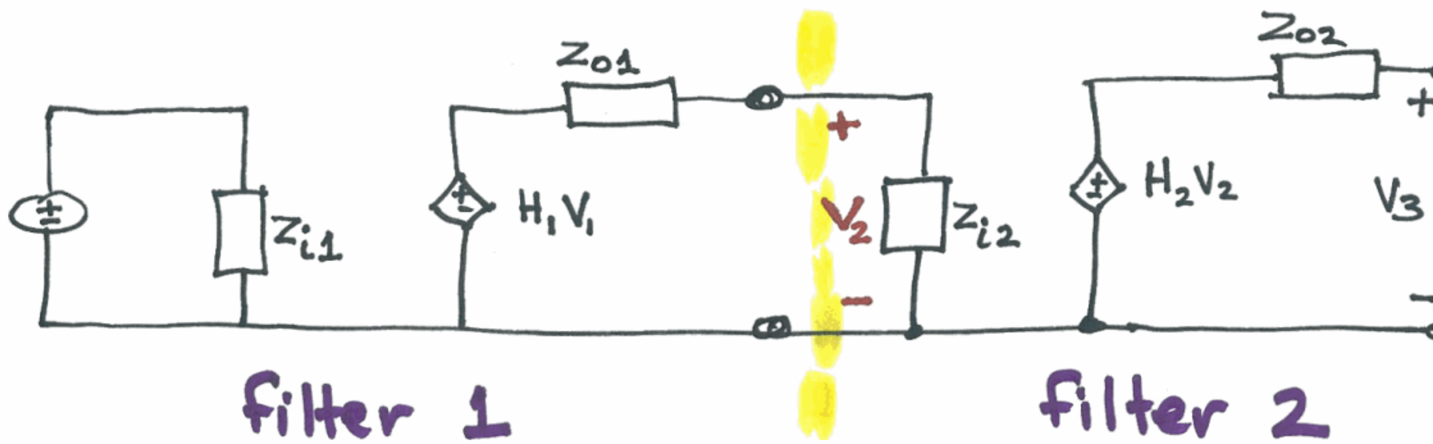
A Model of the first filter stage.

- No current in the output impedance
- $V_2(s) = H_1(s)\,V_i(s)$

⇧

TRANSFER FUNCTION

LOADING FOR A CASCADE



- $$V_3 = H_2 V_2$$
- $$V_2 = \frac{Z_{i2}}{Z_{o1} + Z_{i2}} H_1 V_1$$ **voltage divider**
- Connecting second stage to first stage has changed the output V_2 of the first stage
- The second stage is loading the first stage.
- Now $$V_3 = H_2 \frac{Z_{i2}}{Z_{o1}+Z_{i2}} H_1 V_1$$

  or $$H = \frac{V_3}{V_1} = H_2 \frac{Z_{i2}}{Z_{o1}+Z_{i2}} H_1$$
- $H = H_1 H_2$ if

  ① $Z_{i2} = \infty$ (very large)

  or ② $Z_{o1} = 0$ (very small)

⇒ MINIMAL LOADING