Question #1
Dorf, 6th Ed.

Two identical networks connected in parallel will have a total **Y** matrix of

$$\mathbf{Y} = \mathbf{Y}_a + \mathbf{Y}_b$$

Since $\mathbf{Y}_a = \mathbf{Y}_b$, we have

$$\mathbf{Y} = 2\mathbf{Y}_a = \begin{bmatrix} \frac{4}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{4}{3} \end{bmatrix}$$

Finally, when two identical networks are connected in cascade, we have a total **T** matrix of

$$\mathbf{T} = \mathbf{T}_a\mathbf{T}_b = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

Exercise 17.9-1 Determine the total transmission parameters of the cascade connection of three two-port networks shown in Figure E 17.9-1.
Answers: $A = 3$, $B = 21\,\Omega$, $C = 1/6$ S, and $D = 3/2$

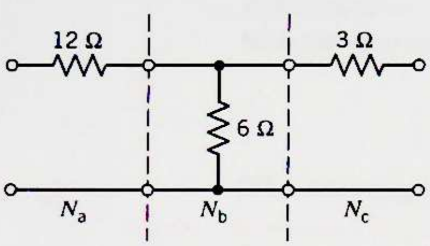


FIGURE E 17.9-1

17.10 Verification Example

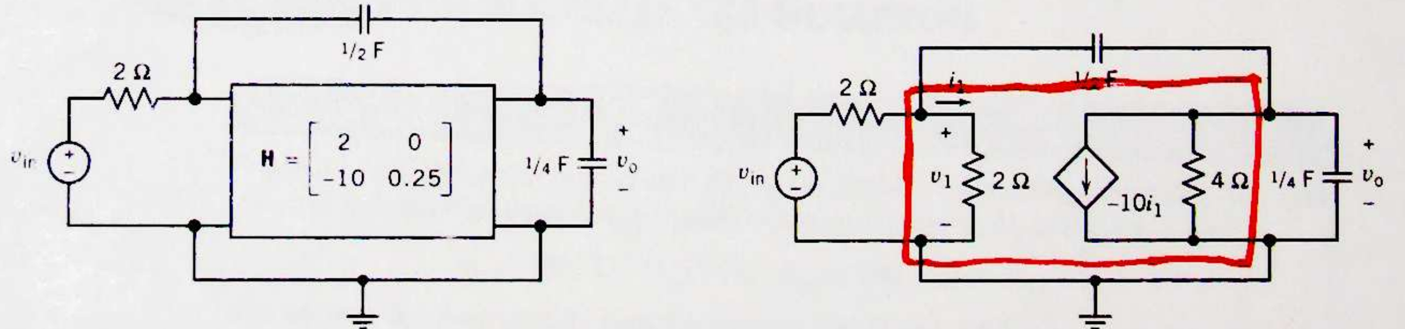
PROBLEM 1

The circuit shown in Figure 17.10-1*a* was designed to have a transfer function given by

$$\frac{V_o(s)}{V_{in}(s)} = \frac{2s - 10}{s^2 + 27s + 2}$$

Does the circuit satisfy this specification?

$V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$



Given

(a) 2 marks

FIGURE 17.10-1
(*a*) A circuit including a two-port network. (*b*) Using the *h*-parameter model to represent the two-port network.

or $V_2 = 4I_2 + 40I_1$

SOLUTION

The h-parameter model from Figure 17.7-1 can be used to redraw the circuit as shown in Figure 17.10-1b. This circuit can be represented by node equations

$$\begin{bmatrix} \left(1+\frac{s}{2}\right) & -\frac{s}{2} \\ \left(-5-\frac{s}{2}\right) & \left(\frac{3s}{4}+\frac{1}{4}\right) \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_o(s) \end{bmatrix} = \begin{bmatrix} \frac{V_{in}(s)}{2} \\ 0 \end{bmatrix}$$

2 marks

(a) $V_1 = 2I_1$

$I_2 = -10I_1 + \frac{V_2}{4}$ } $\frac{1}{2}$ if ok

where $10I_1(s) = 5V_1(s)$ has been used to express the current of the dependent source in terms of the node voltages. Applying Cramer's rule gives

$$\frac{V_o(s)}{V_{in}(s)} = \frac{\frac{1}{2}\left(5+\frac{s}{2}\right)}{\left(1+\frac{s}{2}\right)\left(\frac{3s}{4}+\frac{1}{4}\right)-\frac{s}{2}\left(\frac{s}{2}+5\right)} = \boxed{\frac{2s+20}{s^2-13s+2}}$$

2marks

This is not the required transfer function, so the circuit does not satisfy the specification.

Exercise 17.10-1 Verify that the circuit shown in Figure E 17.10-1 does indeed have the transfer function

$$\cancel{\frac{V_o(s)}{V_{in}(s)} = \frac{2s+10}{s^2+27s+2}}$$

- If no but $\frac{V_o}{V_{in}}$ is wrong, get 0/2.
- 2/2 only if completely correct.

(The circuits in Figures 17.10-1a and E 17.10-1 differ only in the sign of h_{21}.)

$\frac{2}{2}$ if both ok

(b) ~~[illegible]~~ KVL

A) $\frac{V_{in}-V_1}{2} = \frac{V_1}{2} + \frac{(V_1-V_2)s}{2}$

B) $\frac{-V_o s}{4} = \underbrace{\frac{-10V_1}{2} + \frac{V_o}{4}}_{I_2} + (V_2 - V_1)\,s/2$

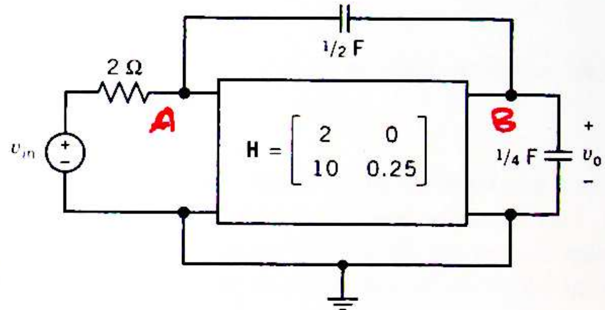


FIGURE E 17.10-1
A modified version of the circuit from Figure 17.10-1.

17.11 DESIGN CHALLENGE SOLUTION

TRANSISTOR AMPLIFIER

Figure 17.11-1 shows the small signal equivalent circuit of a transistor amplifier. The data sheet for the transistor describes the transistor by specifying its h parameters to be

$$h_{ie} = 1250\ \Omega,\quad h_{oe} = 0,\quad h_{fe} = 100,\quad \text{and}\quad h_{re} = 0$$

The value of the resistance R_c must be between 300 Ω and 5000 Ω to ensure that the transistor will be biased correctly. The small signal gain is defined to be

$$A_v = \frac{v_o}{v_{in}}$$

Alternative: Given H(s) at dc (s=0) gives $|H(0)| = 5$
For circuit, C's are open and d.c. analysis gives $|H(0)| = 10$!

Question #2

Alexander, 3rd Ed.

highpass filter (where $\omega_1 = \omega_c$) in Fig. 14.33. However, the result would not be the same as just adding the output of the lowpass filter to the input of the highpass filter, because one circuit loads the other and alters the desired transfer function.

14.7.4 Bandstop Filter

A filter that prevents a band of frequencies between two designated values (ω_1 and ω_2) from passing is variably known as a *bandstop, band-reject*, or *notch* filter. A bandstop filter is formed when the output *RLC* series resonant circuit is taken off the *LC* series combination as shown in Fig. 14.37. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)} \tag{14.56}$$

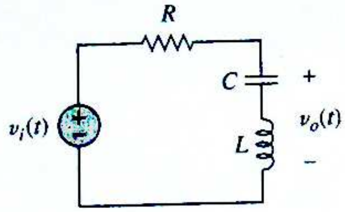


Figure 14.37
A bandstop filter.

Notice that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 1$. Figure 14.38 shows the plot of $|H(\omega)|$. Again, the center frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{14.57}$$

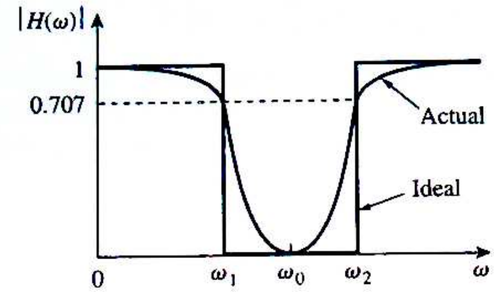


Figure 14.38
Ideal and actual frequency response of a bandstop filter.

while the half-power frequencies, the bandwidth, and the quality factor are calculated using the formulas in Section 14.5 for a series resonant circuit. Here, ω_0 is called the *frequency of rejection*, while the corresponding bandwidth ($B = \omega_2 - \omega_1$) is known as the *bandwidth of rejection*. Thus,

> A **bandstop filter** is designed to stop or eliminate all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

For this circuit $\zeta = 1$ so that there are two real roots (equal)

Need complex poles for resonance.

Notice that adding the transfer functions of the bandpass and the bandstop gives unity at any frequency for the same values of *R*, *L*, and *C*. Of course, this is not true in general but true for the circuits treated here. This is due to the fact that the characteristic of one is the inverse of the other.

In concluding this section, we should note that:

1. From Eqs. (14.50), (14.52), (14.54), and (14.56), the maximum gain of a passive filter is unity. To generate a gain greater than unity, one should use an active filter as the next section shows.
2. There are other ways to get the types of filters treated in this section.
3. The filters treated here are the simple types. Many other filters have sharper and complex frequency responses.

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⟹ 2

Determine what type of filter is shown in Fig. 14.39. Calculate the corner or cutoff frequency. Take $R = 2\ \text{k}\Omega$, $L = 2$ H, and $C = 2\ \mu$F.

Solution:
The transfer function is

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R \parallel 1/sC}{sL + R \parallel 1/sC}, \qquad s = j\omega \tag{14.10.1}$$

(a)

But

$$R \parallel \frac{1}{sC} = \frac{R/sC}{R + 1/sC} = \frac{R}{1 + sRC}$$

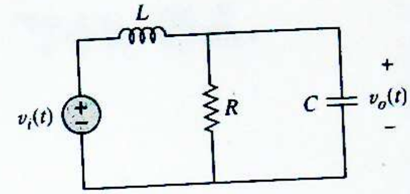

Figure 14.39
For Example 14.10.

Substituting this into Eq. (14.10.1) gives 1

$$\mathbf{H}(s) = \frac{R/(1 + sRC)}{sL + R/(1 + sRC)} = \frac{R}{s^2RLC + sL + R}, \qquad s = j\omega$$

$\frac{1/LC}{s^2 + s/CR + \frac{1}{LC}}$

or

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2RLC + j\omega L + R} \tag{14.10.2}$$

Plot correct 3/3
Plot in error 2/3;
1/3; zer
(b)

1 Since $\mathbf{H}(0) = 1$ and $\mathbf{H}(\infty) = 0$, we conclude from Table 14.5 that the circuit in Fig. 14.39 is a second-order lowpass filter. The magnitude of $\mathbf{H}$ is

$$H = \frac{R}{\sqrt{(R - \omega^2RLC)^2 + \omega^2L^2}} \tag{14.10.3}$$

The corner frequency is the same as the half-power frequency, i.e., where $\mathbf{H}$ is reduced by a factor of $1/\sqrt{2}$. Since the dc value of $H(\omega)$ is 1, at the corner frequency, Eq. (14.10.3) becomes after squaring

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2RLC)^2 + \omega_c^2L^2}$$ 2

or

$$2 = (1 - \omega_c^2LC)^2 + \left(\frac{\omega_cL}{R}\right)^2$$

Substituting the values of R, L, and C, we obtain

$$2 = (1 - \omega_c^2\, 4 \times 10^{-6})^2 + (\omega_c\, 10^{-3})^2$$

Assuming that ω_c is in krad/s,

$$2 = (1 - 4\omega_c^2)^2 + \omega_c^2 \qquad \text{or} \qquad 16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation in ω_c^2, we get $\omega_c^2 = 0.5509$ and -0.1134. Since ω_c is real,

$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$ 1

118.2 hz

Practice Problem 14.10

For the circuit in Fig. 14.40, obtain the transfer function $\mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$. Identify the type of filter the circuit represents and determine the corner frequency. Take $R_1 = 100\ \Omega = R_2$, $L = 2$ mH.

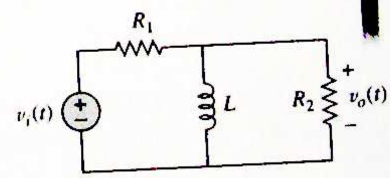

Figure 14.40
For Practice Prob. 14.10.

Answer: $\frac{R_2}{R_1 + R_2}\left(\frac{j\omega}{j\omega + \omega_c}\right)$, highpass filter

$$\omega_c = \frac{R_1R_2}{(R_1 + R_2)L} = 25 \text{ krad/s}.$$

Question #3
Dorf 7th Ed.

EXAMPLE 13.3-1 *Bode Plot*

Find the asymptotic magnitude Bode plot of

$$\mathbf{H}(\omega) = K \frac{j\omega}{1 + j\dfrac{\omega}{p}}$$

Solution

Approximate $\left(1 + j\dfrac{\omega}{p}\right)$ by 1 when $\omega < p$ and by $j\dfrac{\omega}{p}$ when $\omega > p$ to get

$$\mathbf{H}(\omega) \cong \begin{cases} K(j\omega) & \omega < p \\ Kp & \omega > p \end{cases}$$

The logarithmic gain is

$$20 \log_{10}|\mathbf{H}(\omega)| \cong \begin{cases} 20 \log_{10} K + 20 \log_{10}\omega & \omega < p \\ 20 \log_{10}(Kp) & \omega > p \end{cases}$$

The asymptotic magnitude Bode plot is shown in Figure 13.3-5. The $j\omega$ factor in the numerator of $\mathbf{H}(\omega)$ causes the low-frequency asymptote to have a slope of 20 dB/decade. The slope of the asymptotic magnitude Bode plot decreases by 20 dB/decade (from 20 dB/decade to zero) as the frequency increases past $\omega = p$.

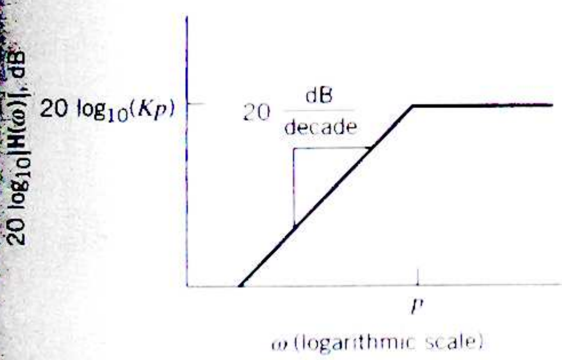

FIGURE 13.3-5
Asymptotic magnitude Bode plot for Example 13.3-1.

EXAMPLE 13.3-2 *Bode Plot of a Circuit*

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 13.3-6*a*. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the node voltage at the output terminal of the op amp, $v_o(t)$. The network function that represents this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} \tag{13.3-4}$$

The corresponding magnitude Bode plot is shown in Figure 13.3-6*b*. Determine the values of the capacitances, C_1 and C_2.

Solution

The network function provides a connection between the circuit and the Bode plot. We can determine the network function from the Bode plot, and we can also analyze the circuit to determine its network function. The values of the capacitances are determined by equating the coefficients of these two network functions.

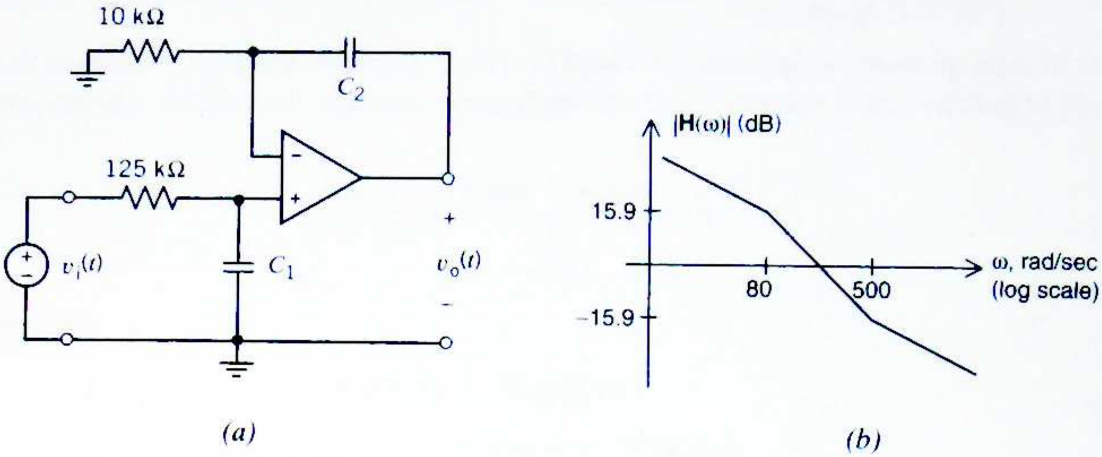

FIGURE 13.3-6 The circuit and Bode plot considered in Example 13.3-2

Step 1: Let's make some observations regarding the Bode plot shown in Figure 13.3-6*b*:

1. There are two corner frequencies, at 80 and 500 rad/s. The corner frequency at 80 rad/s is a pole because the slope of the Bode plot decreases at 80 rad/s. The corner frequency at 500 rad/s is a zero because the slope increases at 500 rad/s. 2
2. The corner frequencies are separated by $\log_{10}\left(\frac{500}{80}\right) = 0.796$ decades. The slope of the Bode plot is $\boxed{\frac{-15.9 - 15.9}{0.796} = -40 \text{ dB/decade}}$ between the corner frequencies. 2
3. At low frequencies—that is, at frequencies smaller than the smallest corner frequency—the slope is -1×20 dB/decade, so the network function includes a factor $(j\omega)^{-1}$ 2

Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega) = k(j\omega)^{-1}\left(\frac{1 + j\dfrac{\omega}{500}}{1 + j\dfrac{\omega}{80}}\right) = k\frac{1 + j\dfrac{\omega}{500}}{j\omega\left(1 + j\dfrac{\omega}{80}\right)} \qquad (13.3\text{-}5)$$

2

where k is a constant that is yet to be determined. $k = 17.84$ 4

Step 2: Next, we analyze the circuit shown in Figure 13.3-6*a* to determine its network function. A network function is the ratio of the output phasor to the input phasor. Phasors exist in the frequency domain. Consequently, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 13.3-7 shows the frequency-domain representation of the circuit from Figure 13.3-6*a*.

To analyze the circuit in Figure 13.3-7, we first write a node equation at the node labeled as node a. (The current entering the noninverting input of the op amp is zero, so there are two currents in this node equation, the currents in the impedances corresponding to 125-kΩ resistor and capacitor C_1.)

$$\frac{\mathbf{V}_i(\omega) - \mathbf{V}_a(\omega)}{125 \times 10^3} = \frac{\mathbf{V}_a(\omega)}{\dfrac{1}{j\omega C_1}}$$

$H(s) = \dfrac{111.5(s+500)}{s(s+80)}$

At $\omega = 80$, $H(j\omega) = 15.9$ db

$\Rightarrow 15.9 = 20\log k + 20\log\left|\dfrac{80j + 500}{80j(80j+80)}\right|$

$0.795 = \log k + \log(0.55946)$

$k = 10^{0.795 - \log(0.55946)} = 11.5$

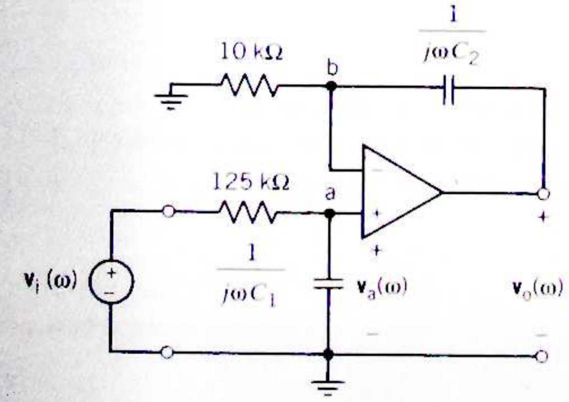

FIGURE 13.3-7
The circuit from Figure 13.3-6*a*, represented in the frequency domain, using impedances and phasors.

where $\mathbf{V}_a(\omega)$ is the node voltage at node a. Doing a little algebra gives

$$\frac{\mathbf{V}_i(\omega)}{125 \times 10^3} = \left(\frac{1}{125 \times 10^3} + j\omega C_1\right)\mathbf{V}_a(\omega)$$

so

$$\mathbf{V}_i(\omega) = (1 + j\omega C_1(125 \times 10^3))\mathbf{V}_a(\omega) \quad \Rightarrow \quad \mathbf{V}_a(\omega) = \frac{\mathbf{V}_i(\omega)}{1 + j\omega C_1(125 \times 10^3)}$$

Next, we write a node equation at the node labeled as node b. (The current entering the inverting input of the op amp is zero, so there are two currents in this node equation, the currents in the impedances corresponding to 10-kΩ resistor and capacitor C_2.)

$$\frac{\mathbf{V}_a(\omega)}{10 \times 10^3} + \frac{\mathbf{V}_a(\omega) - \mathbf{V}_o(\omega)}{\dfrac{1}{j\omega C_2}} = 0$$

Doing some algebra gives

$$\mathbf{V}_a(\omega) + j\omega C_2(10 \times 10^3)(\mathbf{V}_a(\omega) - \mathbf{V}_o(\omega)) = 0$$

$$(1 + j\omega C_2(10 \times 10^3))\mathbf{V}_a(\omega) = j\omega C_2(10 \times 10^3)\mathbf{V}_o(\omega)$$

$$(1 + j\omega C_2(10 \times 10^3))\frac{\mathbf{V}_i(\omega)}{1 + j\omega C_1(125 \times 10^3)} = j\omega C_2(10 \times 10^3)\mathbf{V}_o(\omega)$$

Finally,

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \left(\frac{1}{C_2(10 \times 10^3)}\right)\frac{1 + j\omega C_2(10 \times 10^3)}{(j\omega)(1 + j\omega C_1(125 \times 10^3))} \tag{13.3-6}$$

Step 3: The network functions given in Eqs. 13.3-5 and 13.3-6 must be equal. That is,

$$k\frac{1 + j\dfrac{\omega}{500}}{j\omega\left(1 + j\dfrac{\omega}{80}\right)} = \mathbf{H}(\omega) = \left(\frac{1}{C_2(10 \times 10^3)}\right)\frac{1 + j\omega C_2(10 \times 10^3)}{(j\omega)(1 + j\omega C_1(125 \times 10^3))}$$

Equating coefficients gives

$$\frac{1}{80} = C_1(125 \times 10^3),\ \frac{1}{500} = C_2(10 \times 10^3),\ \text{and } k = \frac{1}{C_2(10 \times 10^3)} = 500$$

because of circuit equivalence.

so

$$C_1 = \frac{1}{80(125 \times 10^3)} = 0.1\ \mu\text{F and } C_2 = \frac{1}{500(10 \times 10^3)} = 0.2\ \mu\text{F}$$

EXAMPLE 13.3-3 *Bode Plot of a Circuit*

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 13.3-8*a*. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the node voltage at the output terminal of the op amp, $v_o(t)$. The network function that represents this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} \tag{13.3-7}$$

The corresponding magnitude Bode plot is also shown in Figure 13.3-8*b*. Determine the values of the capacitances C_1 and C_2.

Solution

The network function provides a connection between the circuit and the Bode plot. We can determine the network function from the Bode plot, and we can also analyze the circuit to determine its network function. The values of the capacitances are determined by equating the coefficients of these two network functions.

Question #4
Dorf 7th Ed.

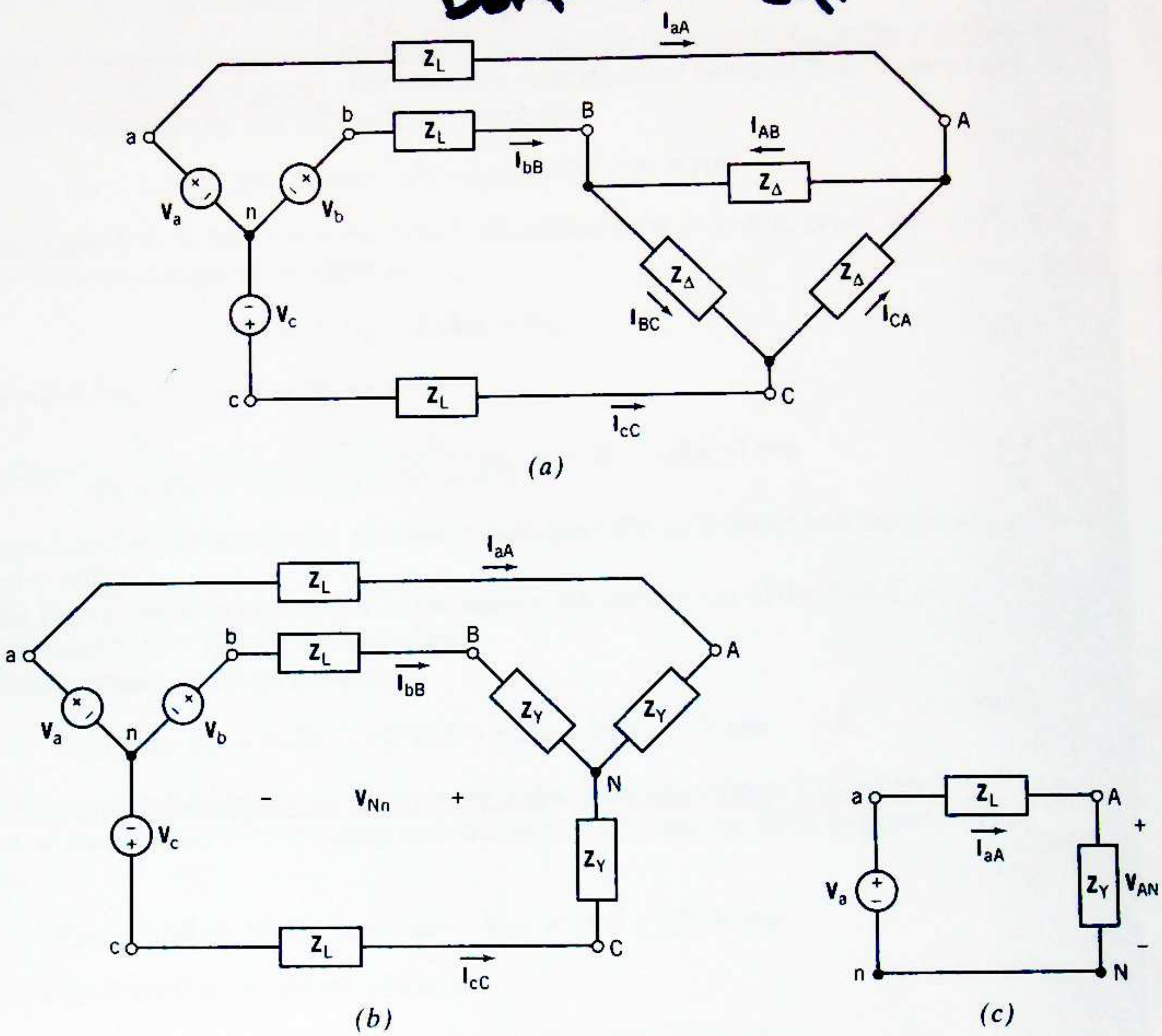

FIGURE 12.6-1 (*a*) A Y-to-Δ circuit, (*b*) the equivalent Y-to-Y circuit, and (*c*) the per-phase equivalent circuit.

The circuit shown in Figure 12.6-1*a* is a balanced Y-to-Δ circuit. Figure 12.6-1*b* shows the equivalent Y-to-Y circuit where

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

This Y-to-Y circuit can be analyzed using the per-phase equivalent circuit shown in Figure 12.6-1*c*.

4

EXAMPLE 12.6-1 *Per-phase Equivalent Circuit*

Figure 12.6-1*a* shows a balanced Y-to-Δ three-phase circuit. The phase voltages of the Y-connected source are $\mathbf{V}_a = 110\,\angle 0^\circ$ V rms, $\mathbf{V}_b = 110\,\angle -120^\circ$ V rms, and $\mathbf{V}_c = 110\,\angle 120^\circ$ V rms. The line impedances are each $\mathbf{Z}_L = 10 + j5\ \Omega$. The impedances of the Δ-connected load are each $\mathbf{Z}_\Delta = 75 + j225\ \Omega$. Determine the phase currents in the Δ-connected load.

Solution

Convert the Δ-connected load to a Y-connected load using the Δ-to-Y transformation summarized in Table 12.4-1. The impedances of the balanced equivalent Y-connected load are

$$\mathbf{Z}_Y = \frac{75 + j225}{3} = 25 + j75\ \Omega$$

The per-phase equivalent circuit for the Y-to-Y circuit is shown in Figure 12.6-1*c*. The line current is given by

0.5 - 1.15j

LINE CURRENT
$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_L + \mathbf{Z}_Y} = \frac{110\,\angle 0^\circ}{(10 + j5) + (25 + j75)} = 1.26\,\angle -66^\circ \text{ A rms} \tag{12.6-1}$$
1

87.32 ∠66.37°

The line current, $\mathbf{I}_{aA}$, calculated using the per-phase equivalent circuit, is also the line current, $\mathbf{I}_{aA}$, in the Y-to-Y circuit, as well as the line current, $\mathbf{I}_{aA}$, in the Y-to-Δ circuit. The other line currents in the balanced Y-to-Y circuit have the same magnitude but differ in phase angle by 120°. These line currents are

$$\mathbf{I}_{bB} = 1.26\,\angle -186^\circ \text{ A rms} \quad \text{and} \quad \mathbf{I}_{cC} = 1.26\,\angle 54^\circ \text{ A rms}$$

To check the value of $\mathbf{I}_{bB}$, apply KVL to the loop in the Y-to-Y circuit that starts at node n, passes through nodes b, B, N, and returns to node n. The resulting KVL equation is

$$\mathbf{V}_b = \mathbf{Z}_L\mathbf{I}_{bB} + \mathbf{Z}_Y\mathbf{I}_{bB} + \mathbf{V}_{Nn}$$

Because the circuit is balanced, $\mathbf{V}_{Nn} = 0$. Solving for $\mathbf{I}_{bB}$ gives

$$\mathbf{I}_{bB} = \frac{\mathbf{V}_b}{\mathbf{Z}_L + \mathbf{Z}_Y} = \frac{110\,\angle -120^\circ}{(10 + j5) + (25 + j75)} = 1.26\,\angle -186^\circ \text{ A rms} \tag{12.6-2}$$

Comparing Eqs. 12.6-1 and 12.6-2 shows that the line currents in the balanced Y-to-Y circuit have the same magnitude but differ in phase angle by 120°.)

The line currents of the Y-to-Δ circuit in Figure 12.6-1*a* are equal to the line currents of the Y-to-Y circuit in Figure 12.6-1*b* because the Y-to-Δ and Y-to-Y circuits are equivalent.

The voltage $\mathbf{V}_{AN}$ in the per-phase equivalent circuit is

$$\mathbf{V}_{AN} = \mathbf{I}_{aA}\mathbf{Z}_Y = (1.26\,\angle -66^\circ)(25 + j75) = 99.6\,\angle 5^\circ \text{ V rms}$$
1 PHASE VOLTAGE Y-Y

The voltage $\mathbf{V}_{AN}$ calculated using the per-phase equivalent circuit is also the phase voltage, $\mathbf{V}_{AN}$, of the Y-to-Y circuit. The other phase voltages of the balanced Y-to-Y circuit have the same magnitude but differ in phase angle by 120°. These phase voltages are

$$\mathbf{V}_{BN} = 99.6\,\angle -115^\circ \text{ V rms} \quad \text{and} \quad \mathbf{V}_{CN} = 99.6\,\angle 125^\circ \text{ V rms}$$

The line-to-line voltages of the Y-to-Y circuit are calculated as

LINE-TO-LINE VOLTAGE

0.58 - .43j

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = 99.5\,\angle 5^\circ - 99.5\,\angle -115^\circ = 172\,\angle 35^\circ \text{ V rms}$$
$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = 99.5\,\angle -115^\circ - 99.5\,\angle 125^\circ = 172\,\angle -85^\circ \text{ V rms}$$
$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN} = 99.5\,\angle 125^\circ - 99.5\,\angle 5^\circ = 172\,\angle 155^\circ \text{ V rms}$$
1

The phase voltages of a Δ-connected load are equal to the line-to-line voltages. The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{172\,\angle 35^\circ}{75 + j225} = 0.727\,\angle -36^\circ \text{ A rms}$$
$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = \frac{172\,\angle -85^\circ}{75 + j225} = 0.727\,\angle -156^\circ \text{ A rms}$$
2
$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = \frac{172\,\angle 155^\circ}{75 + j225} = 0.727\,\angle \cancel{84^\circ} \text{ A rms}$$
83.6 -.5 units

Exercise 12.6-1 Figure 12.6-1*a* shows a balanced Y-to-Δ three-phase circuit. The phase voltages of the Y-connected source are $\mathbf{V}_a = 110\,\angle 0^\circ$ V rms, $\mathbf{V}_b = 110\,\angle -120^\circ$ V rms, and $\mathbf{V}_c = 110\,\angle 120^\circ$ V rms. The line impedances are each $\mathbf{Z}_L = 10 + j25\ \Omega$. The impedances

The total average power delivered to the Δ-connected load in Figure 12.6-1a is

$$P = 3P_{AB} = 3V_{AB}I_{AB}\cos\theta = 3(\sqrt{3}V_P)\frac{I_L}{\sqrt{3}}\cos\theta = 3\ V_P I_L\cos\theta \qquad (12.7\text{-}5)$$

In summary, the total average power delivered to the Δ-connected load in Figure 12.6-1a is equal to the total average power delivered to the balanced Y-connected load in Figure 12.6-1b. That's appropriate because the two circuits are equivalent. Notice that the information required to calculate the power delivered to a balanced load, Y or Δ, is obtained from the per-phase equivalent circuit.

EXAMPLE 12.7-1 *Power Delivered to the Load*

12.6-1a shows a balanced Y-to-Δ three-phase circuit. The phase voltages of the Y-connected source are $\mathbf{V}_a$ = $\underline{/0°}$ V rms, $\mathbf{V}_b = 110\,\underline{/-120°}$ V rms, and $\mathbf{V}_c = 110\,\underline{/120°}$ V rms. The line impedances are each $\mathbf{Z}_L = 10 + j5$ Ω. impedances of the Δ-connected load are each $\mathbf{Z}_\Delta = 75 + j225\ \Omega$. Determine the average power delivered to the

tion

circuit was analyzed in Example 12.6-1. That analysis showed that

$$\mathbf{I}_{aA} = 1.26\,\underline{/-66°}\text{ A rms}$$

$$\mathbf{V}_{AN} = 99.6\,\underline{/5°}\text{ V rms}$$

tal average power delivered to the load is given by Eq. 12.7-3 as

(b) $$P = 3(99.6)(1.26)\cos(5° - (-66°)) = 122.6\text{ W}$$ 1

Exercise 12.7-1 Figure 12.6-1a shows a balanced Y-to-Δ three-phase circuit. The phase voltages of the Y-connected source are $\mathbf{V}_a = 110\,\underline{/0°}$ V rms, $\mathbf{V}_b = 110\,\underline{/-120°}$ V rms, and $\mathbf{V}_c = 110\,\underline{/120°}$ V rms. The line impedances are each $\mathbf{Z}_L = 10 + j25\ \Omega$. The impedances of the Δ-connected load are each $\mathbf{Z}_\Delta = 150 + j\,270\ \Omega$. Determine the average power delivered to the Δ-connected load.

Intermediate Answer: $\mathbf{I}_{aA} = 0.848\,\underline{/-62.5°}$ A rms and $\mathbf{V}_{AN} = 87.3\,\underline{/-1.5°}$ V rms

Answer: $P = 107.9$ W

12.8 Two-Wattmeter Power Measurement

For many load configurations, for example, a three-phase motor, the phase current or voltage is inaccessible. We may wish to measure power with a wattmeter connected to each phase. However, since the phases are not available, we measure the line currents and the line-to-line voltages. A wattmeter provides a reading of $V_L I_L \cos\theta$ where V_L and I_L are the rms magnitudes and θ is the angle between the line voltage, $\mathbf{V}$, and the current, $\mathbf{I}$. We choose to measure V_L and I_L, the line voltage and current, respectively. We will show that two wattmeters are sufficient to read the power delivered to the three-phase load, as shown in Figure 12.8-1. We use cc to denote current coil and vc to denote voltage coil.

Wattmeter 1 reads

$$P_1 = V_{AB}I_A\cos\theta_1 \qquad (12.8\text{-}1)$$

and wattmeter 2 reads

$$P_2 = V_{CB}I_C\cos\theta_2 \qquad (12.8\text{-}2)$$

Question #5
Dorf Seventh Edition

we wish to correct the pf so that $pfc = 0.95$ lagging. Then, we use Eq. 11.6-5 as follows:

$$X_C = \frac{100^2 + 100^2}{100 \tan(\cos^{-1} 0.95) - 100} = -297.9\ \Omega$$

apacitor required is determined from

$$-\frac{1}{\omega C} = X_C$$

fore, since $\omega = 377$ rad/s,

$$C = -\frac{1}{\omega X_C} = \frac{-1}{377(-297.9)} = 8.9\ \mu\text{F}$$

wish to correct the load to $pfc = 1$, we have

$$X_C = \frac{2 \times 10^4}{100 \tan(\cos^{-1} 1) - 100} = -200$$

capacitor required to correct the power factor to 1.0 is determined from

$$C = \frac{-1}{\omega X_C} = \frac{-1}{377(-200)} = 13.3\ \mu\text{F}$$

Since the uncorrected power factor is lagging, we can alternatively use Eq. 11.6-7 to determine C. For example ws that $pfc = 1$. Then $\theta_C = 0°$. Therefore,

$$\omega C = \frac{100}{2 \times 10^4}(\tan\theta - \tan\theta_C) = (5 \times 10^{-3})(\tan(45°) - \tan(0°)) = 5 \times 10^{-3}$$

$$C = \frac{5 \times 10^{-3}}{377} = 13.3\ \mu\text{F}$$

expected, this is the same value of capacitance as was calculated using Eq. 11.6-5.

5

EXAMPLE 11.6-3 *Complex Power*

INTERACTIVE EXAMPLE

he input to the circuit shown in Figure 11.6-6*a* is the voltage of the voltage source,

$$v_s(t) = 7.28 \cos(4t + 77°)\ \text{V}$$

he output is the voltage across the inductor,

$$v_o(t) = 4.254 \cos(4t + 311°)\ \text{V}$$

Determine the following:

a) The average power supplied by the voltage source
b) The average power received by the resistor
c) The average power received by the inductor
d) The power factor of the impedance of the series connection of the resistor and inductor

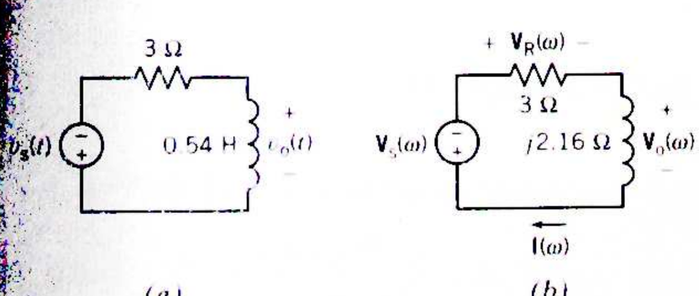

FIGURE 11.6-6
The circuit considered in Example 11.6-3 represented (*a*) in the time domain and (*b*) in the frequency domain.

...tion

...nput voltage is sinusoid. The output voltage is also sinusoid and has the same frequency as the input voltage ...arently the circuit has reached steady state. Consequently, the circuit in Figure 11.6-6*a* can be represented in the ...quency domain, using phasors and impedances. Figure 11.6-6*b* shows the frequency-domain representation of the ...uit from Figure 11.6-6*a*. The impedance of the inductor is $j\omega L = j(4)(0.54) = j2.16\ \Omega$, as shown in Figure 11.6-6*b* ...he phasors corresponding to the input and output sinusoids are

$$\mathbf{V}_S(\omega) = 7.28\,\underline{/77^\circ}\text{ V}$$

$V_R = 5.9 \angle 41^\circ$

$$\mathbf{V}_o(\omega) = 4.254\,\underline{/311^\circ}\text{ V}$$

...e current $\mathbf{I}(\omega)$ in Figure 11.6-6*b* is calculated from $\mathbf{V}_o(\omega)$ and the impedance of the inductor using Ohm's law:

410

$$\mathbf{I}(\omega) = \frac{\mathbf{V}_o(\omega)}{j\,2.16} = \frac{4.254\,\underline{/311^\circ}}{2.16\,\underline{/90^\circ}} = \frac{4.254}{2.16}\,\underline{/311^\circ - 90^\circ} = 1.969\,\underline{/221^\circ}\text{A}$$

1 if correct

...e we know $\mathbf{I}(\omega)$, we are ready to answer the questions asked in this example.

... The average power supplied by the source is calculated from $\mathbf{I}(\omega)$ and $\mathbf{V}_s(\omega)$. The average power of the source is given by

$$\frac{|\mathbf{V}_s(\omega)||\mathbf{I}(\omega)|}{2}\cos\left(\underline{/\mathbf{V}_s(\omega)} - \underline{/\mathbf{I}(\omega)}\right) = \frac{(7.28)(1.969)}{2}\cos(77^\circ - 221^\circ)$$
$$= 7.167\,\cos(-144^\circ) = -5.8\text{ W} \qquad (11.6\text{-}8)$$

Notice that $\mathbf{I}(\omega)$ and $\mathbf{V}_s(\omega)$ adhere to the passive convention. Consequently, Eq. 11.6-8 gives the power received by the voltage source rather than the power supplied by the voltage source. The power supplied is the negative of the power received. Therefore, the power supplied by the voltage source is

$$\boxed{P_s = 5.8\text{ W}}$$

2

(b) The resistor voltage, $\mathbf{V}_R(\omega)$, in Figure 11.6-6*b* is given by

$$\mathbf{V}_R(\omega) = R\,\mathbf{I}(\omega) = 3(1.969\,\underline{/221^\circ}) = 5.907\,\underline{/221^\circ}\text{ V}$$

The average power received by the resistor is calculated from $\mathbf{I}(\omega)$ and $\mathbf{V}_R(\omega)$:

$$P_R = \frac{|\mathbf{V}_R(\omega)|\,|\mathbf{I}(\omega)|}{2}\left(\cos\left(\underline{/\mathbf{V}_R(\omega)} - \underline{/\mathbf{I}(\omega)}\right)\right) = \frac{(5.907)(1.969)}{2}\cos(221^\circ - 221^\circ)$$
$$= 5.8\,\cos(0^\circ) = \boxed{5.8\text{ W}} \qquad (11.6\text{-}9)$$

Notice that $\mathbf{I}(\omega)$ and $\mathbf{V}_R(\omega)$ adhere to the passive convention. Consequently, P_R is the power received by the resistor, as required.

Alternately, the power received by a resistor can be calculated from the current $\mathbf{I}(\omega)$ and the resistance, R. To see how, first notice that the voltage and current of a resistor are related by

$$\mathbf{V}_R(\omega) = R\mathbf{I}(\omega) \Rightarrow |\mathbf{V}_R(\omega)|\,\underline{/\mathbf{V}_R(\omega)} = R(|\mathbf{I}(\omega)|\,\underline{/\mathbf{I}(\omega)}) \Rightarrow \begin{cases} |\mathbf{V}_R(\omega)| = R|\mathbf{I}(\omega)| \\ \underline{/\mathbf{V}_R(\omega)} = \underline{/\mathbf{I}(\omega)} \end{cases}$$

Substituting these expressions for $|\mathbf{V}_R(\omega)|$ and $\underline{/\mathbf{V}_R(\omega)}$ into Eq. 11.6-9 gives

$$P_R = \frac{|R\mathbf{I}(\omega)|\,|\mathbf{I}(\omega)|}{2}\cos\left(\underline{/\mathbf{I}(\omega)} - \underline{/\mathbf{I}(\omega)}\right) = \frac{R|\mathbf{I}(\omega)|^2}{2}$$
$$= \frac{(3)(1.969)^2}{2} = 5.8\text{ W}$$

The average power received by the inductor is calculated from $\mathbf{I}(\omega)$ and $\mathbf{V}_o(\omega)$:

$$P_L = \frac{|\mathbf{V}_o(\omega)||\mathbf{I}(\omega)|}{2}\cos\left(\angle\mathbf{V}_o(\omega) - \angle\mathbf{I}(\omega)\right) = \frac{(4.254)(1.969)}{2}\cos(311^\circ - 221^\circ)$$
$$= 4.188\ \cos(90^\circ) = 0\ \text{W} \qquad (11.6\text{-}10)$$ 1

The phase angle of the inductor voltage is always 90° greater than the phase angle of the inductor current. Consequently, the value of average power received by any inductor is zero.

The power factor of the impedance of the series connection of the resistor and inductor can be calculated from $\mathbf{I}(\omega)$ and the voltage across the impedance. That voltage is $\mathbf{V}_R(\omega) + \mathbf{V}_o(\omega)$, which is calculated by applying Kirchhoff's voltage law to the circuit in Figure 11.6-6b:

$$\mathbf{V}_R(\omega) + \mathbf{V}_o(\omega) + \mathbf{V}_s(\omega) = 0$$
$$\mathbf{V}_R(\omega) + \mathbf{V}_o(\omega) = -\mathbf{V}_s(\omega) = -7.28\angle 77^\circ$$
$$= (1\angle 180^\circ)(7.28\angle 77^\circ)$$
$$= 7.28\angle 257^\circ$$

Now the power factor is calculated as

$$pf = \cos\left(\angle(\mathbf{V}_R(\omega) + \mathbf{V}_o(\omega)) - \angle\mathbf{I}(\omega)\right) = \cos(257^\circ - 221^\circ) = 0.809$$ lagging

The power factor is said to be lagging because $257 - 221 = 36 > 0$. 1

$Z = 3.7\angle 36^\circ$

$pf = 0.81$

$= \cos(36^\circ)$

Average power is conserved. In this example, that means that the average power supplied by the voltage source must be equal to the sum of the average powers received by the resistor and the inductor. This fact provides a check on the accuracy of our calculations.

If the value of $\mathbf{V}_o(\omega)$ had not been given, then $\mathbf{I}(\omega)$ would be calculated by writing and solving a mesh equation. Referring to Figure 11.6-6*b*, the mesh equation is

$$3\mathbf{I}(\omega) + j2.16\,\mathbf{I}(\omega) + 7.28\angle 77^\circ = 0$$

Solving for $\mathbf{I}(\omega)$ gives

$$\mathbf{I}(\omega) = \frac{-7.28\angle 77^\circ}{3 + j2.16} = \frac{(1\angle 180^\circ)(7.28\angle 77^\circ)}{3.697\angle 36^\circ}$$
$$= \frac{(1)(7.28)}{3.697}\angle 180 + 77 - 36 = 1.969\angle 221^\circ\ \text{A}$$

as before.

Exercise 11.6-1 A circuit has a large motor connected to the ac power lines [$\omega = (2\pi)60 = 377$ rad/s]. The model of the motor is a resistor of 100 Ω in series with an inductor of 5 H. Find the power factor of the motor.

Answer: $pf = 0.053$ lagging

Exercise 11.6-2 A circuit has a load impedance $Z = 50 + j80\ \Omega$, as shown in Figure 11.6-5. Determine the power factor of the uncorrected circuit. Determine the impedance $\mathbf{Z}_C$ required to obtain a corrected power factor of 1.0.

Answer: $\mathbf{Z}_C = -j111.25\ \Omega$

The capacitor voltage, $\mathbf{V}_c(\omega)$, in Figure 10.9-5b is given by

$$\begin{aligned}\mathbf{V}_c(\omega) = \mathbf{V}_s(\omega) - \mathbf{V}_o(\omega) &= 7.68\underline{/45^\circ} - 1.59\underline{/125^\circ} \\ &= (5.23 + j5.62) - (-0.91 + 1.30) \\ &= (5.23 + 0.91) + j(5.62 - 1.30) \\ &= 6.14 + j4.32 \\ &= 7.51\underline{/35^\circ}\end{aligned}$$

The impedance of the capacitor is given by

$$-j\frac{1}{2C} = \frac{\mathbf{V}_c(\omega)}{\mathbf{I}(\omega)} = \frac{7.51\underline{/35^\circ}}{1.59\underline{/125^\circ}} = 4.72\underline{/-90^\circ}$$

Solving for C gives

$$C = \frac{-j}{2(4.72\underline{/-90^\circ})} = \frac{1\underline{/-90^\circ}}{2(4.72\underline{/-90^\circ})} = 0.106\ \text{F}$$

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Example 10.9-4

Consider the circuit shown in Figure 10.9-6a. The input to the circuit is the voltage of the voltage source, v the output is the voltage across the 4-Ω resistor, $v_o(t)$. When the input is $v_s(t) = 8.93\cos(2t + 54^\circ)$ V, the cor ing output is $v_o(t) = 3.83\cos(2t + 83^\circ)$ V. Determine the voltage across the 9-Ω resistor, $v_a(t)$, and the val capacitance, C, of the capacitor.

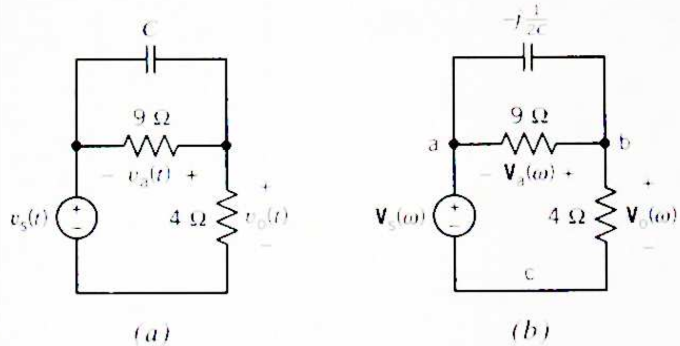

FIGURE 10.9-6 The circuit considered in Example 10.9-4 represented (a) in the ti domain and (b) in the frequency domain.

Solution

The input voltage is a sinusoid. The output voltage is also sinusoid and has the same frequency as the input Apparently the circuit has reached steady state. Consequently, the circuit in Figure 10.9-6a can be represented i quency domain, using phasors and impedances. Figure 10.9-6b shows the frequency-domain representation of tl from Figure 10.9-6a. The voltages $\mathbf{V}_s(\omega)$, $\mathbf{V}_a(\omega)$, and $\mathbf{V}_o(\omega)$ in Figure 10.9-6b are the phasors corresponding $v_a(t)$, and $v_o(t)$ from Figure 10.9-6a. The capacitor and the resistors are represented as impedances in Figure The impedance of the capacitor is $-j\,1/\omega C = -j\,1/2C$ where 2 rad/s is the value of the frequency of $v_s(t)$.

The phasors corresponding to the input and output sinusoids are

$$\mathbf{V}_s(\omega) = 8.93\underline{/54^\circ}\ \text{V}$$

and

$$\mathbf{V}_o(\omega) = 3.83\underline{/83^\circ}\ \text{V}$$

(a) First, we calculate the value of $\mathbf{V}_a(\omega)$. Apply KVL to the mesh in Figure 10.9-6*b* that consists of the two resistors the voltage source to get

$$\begin{aligned}\mathbf{V}_a(\omega) = \mathbf{V}_o(\omega) - \mathbf{V}_s(\omega) &= (3.83\angle 83^\circ) - (8.93\angle 54^\circ)\\ &= (0.47 + j3.80) - (5.25 + j7.22)\\ &= -4.78 - j3.42\\ &= 5.88\angle 216^\circ \quad 1\end{aligned}$$

-144°

The voltage across the 9-Ω resistor, $v_a(t)$, is the sinusoid corresponding to this phasor

$-5.88\cos(2t - 36^\circ)$

$-5.88\cos(2t + 36^\circ)$

$$v_a(t) = 5.88\cos(2t + 216^\circ)\text{ V} \quad 1$$

no units $-.5$

phasor -1

(b) We can determine the value of the capacitance by applying Kirchhoff's current law (KCL) at node b in Figure 10.9-

$$\frac{\mathbf{V}_a(\omega)}{-j\dfrac{1}{2C}} + \frac{\mathbf{V}_a(\omega)}{9} + \frac{\mathbf{V}_o(\omega)}{4} = 0$$

$$(j2C)\mathbf{V}_a(\omega) + \frac{\mathbf{V}_a(\omega)}{9} + \frac{\mathbf{V}_o(\omega)}{4} = 0$$

Solving this equation for $j2C$ gives

$$j2C = \frac{4\mathbf{V}_a(\omega) + 9\mathbf{V}_o(\omega)}{-36\mathbf{V}_a(\omega)}$$

Substituting the values of the phasors $\mathbf{V}_a(\omega)$ and $\mathbf{V}_o(\omega)$ into this equation gives

$$\begin{aligned}j2C &= \frac{4(-4.78 - j3.42) + 9(0.47 + j3.80)}{-36(5.88\angle 216^\circ)}\\ &= \frac{-14.89 + j20.52}{-36(5.88\angle 216^\circ)}\\ &= \frac{25.35\angle 126^\circ}{(36\angle -180^\circ)(5.88\angle 216^\circ)}\\ &= \frac{25.35}{(36)(5.88)}\angle 126^\circ - (-180^\circ + 216^\circ)\\ &= 0.120\angle 90^\circ\\ &= j0.120\end{aligned}$$

Therefore the value of the capacitance is $C = \dfrac{0.12}{2} = 0.06 = 60\text{ mF}.$ 2

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Exercise 10.9-1 Determine the steady-state voltage $v(t)$ for the circuit of Fig E 10.9-1.

Hint: Analyze the circuit in the frequency domain, using impedances and phasors. Use vo age division, twice. Add the results.

Answer: $v(t) = 3.58\cos(5t + 47.2^\circ)$ V

$$\frac{V_o}{V_s} = \frac{4+36Cs}{13+36Cs} = \frac{3.8\angle 83^\circ}{8.9\angle 54} = \frac{4+j72C}{13+j72C}$$