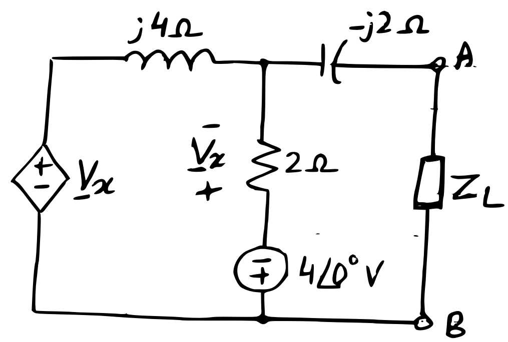
ECSE 210 WINTER 2007

FINAL EXAM SOL.

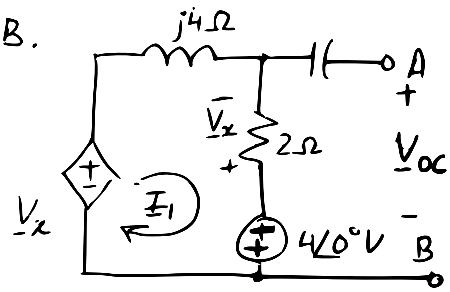
(1) (a) $f = 60$ Hz

$\therefore \omega = 2\pi f$ rad/sec

To find Z_L for max. power transfer, we



need to find impedance seen by the load Z_L. So take Z_L out and find out Thevenin ckt. seen from terminals A & B.



Find $\underline{V}_{oc}$:

Apply KVL around mesh $\underline{I}_1$

$$j4\underline{I}_1 + 2\underline{I}_1 - 4\angle 0 - \underline{V}_x = 0$$

$$j4\underline{I}_1 + 2\underline{I}_1 - 4\angle 0 - \underbrace{2(-\underline{I}_1)}_{\underline{V}_x} = 0$$

$$\underline{I}_1(j4 + 2 + 2) = 4\angle 0 \Rightarrow \underline{I}_1 = \frac{4\angle 0}{4+j4} = \frac{\cancel{4}\angle 0}{\cancel{4}\sqrt{2}\angle 45^\circ}$$

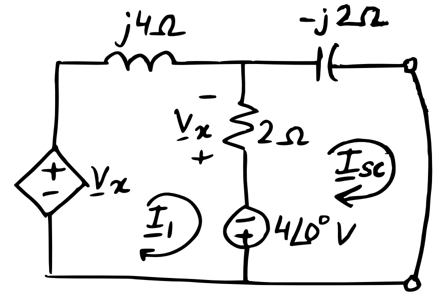
$$\underline{I}_1 = \frac{1}{\sqrt{2}}\angle -45^\circ \text{ A}$$

$$\therefore \underline{V}_{oc} = 2\underline{I}_1 - 4\angle 0 = \sqrt{2}\angle -45^\circ - 4\angle 0 = 1 - j1 - 4$$

$$\underline{V}_{oc} = -3 - j1 = 3.162\angle -161.565^\circ \text{ V} \quad \text{— (1)}$$

Find $\underline{I}_{sc}$

Apply KVL around mesh #1:



$$j4\underline{I}_1 + 2(\underline{I}_1 - \underline{I}_{sc}) - 4\angle 0^\circ - \underline{V}_x = 0$$

$$j4\underline{I}_1 + 2\underline{I}_1 - 2\underline{I}_{sc} - 4\angle 0^\circ - \underbrace{2(\underline{I}_{sc} - \underline{I}_1)}_{\underline{V}_x} = 0$$

$$\underline{I}_1(j4 + 2 + 2) + \underline{I}_{sc}(-2 - 2) = 4\angle 0^\circ$$

$$\underline{I}_1(4 + j4) - 4\underline{I}_{sc} = 4$$

$$\underline{I}_1(1 + j1) - \underline{I}_{sc} = 1 \quad — (2)$$

Apply KVL around mesh I_{sc}:

$$-j2\underline{I}_{sc} + 4\angle 0 + (\underline{I}_{sc} - \underline{I}_1)2 = 0$$

$$-j2\underline{I}_{sc} + 4\angle 0 + 2\underline{I}_{sc} - 2\underline{I}_1 = 0$$

$$-2\underline{I}_1 + \underline{I}_{sc}(2 - j2) = -4$$

$$-\underline{I}_1 + \underline{I}_{sc}(1 - j1) = -2 \quad — (3)$$

From (3):

$$\underline{I}_1 = \underline{I}_{sc}(1 - j1) + 2$$

∴ (2) becomes

$$(\underline{I}_{sc}(1 - j1) + 2)(1 + j1) - \underline{I}_{sc} = 1$$

$$\underline{I}_{sc}\left((1 - j1)(1 + j1) - 1\right) = 1 - 2(1 + j1)$$

$$\underline{I}_{sc} = \frac{-1-j2}{1} = -(1+j2)\ A = 2.236\angle -116.57^\circ \quad \text{--- (4)}$$

Using (2) & (4), $\quad \underline{Z}_{TH} = \dfrac{\underline{V}_{oc}}{\underline{I}_{sc}}$

$$= \frac{3.162\angle -161.57}{2.236\angle -116.57} = \sqrt{2}\angle -45^\circ = 1 - j1\ \Omega$$

$\therefore$ For max. power transfer $\boxed{Z_L = Z_{TH}^{*} = 1 + j1\ \Omega = \sqrt{2}\angle 45^\circ\ \Omega}$

(b) Max. power transfered to the load :

$$P_{L max} = \frac{1}{2} |\underline{I}_L|^2 R_L = \frac{1}{2} \left| \frac{\underline{V}_{oc}}{Z_{TH} + Z_L} \right|^2 R_L$$

$$= \frac{1}{2} \left| \frac{3.162\angle -161.57^\circ}{1+1} \right|^2 \times R_L = \frac{1}{2} \left(\frac{3.162}{2} \right)^2$$

$$\boxed{P_{L max} = 1.25\ W}$$

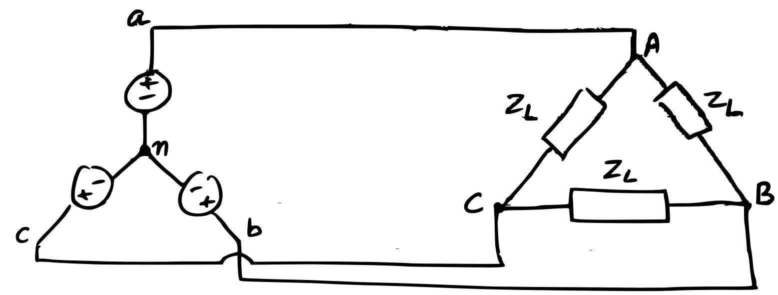
(c): $Z_L = 1 + j1 \quad \leftarrow$ inductive impedance = 

$R_L = 1\ \Omega \qquad jX_L = j1\ \Omega \Rightarrow L\omega = 1 \Rightarrow L = \frac{1}{\omega} = 2.65\,mH$

$\therefore Z_L = R_L + jX_L = R_L + jL\omega$

$$\boxed{\begin{array}{l} R_L = 1\ \Omega \\ L = 2.65\,mH \end{array}}$$

(2)(a)



$V_L = 208$ V rms.

$P_T = 1200$ W $\quad$ pf = 20° lagging $= \cos(-20) = 0.94$

P_L (power consumed per load phase) $= \dfrac{1200}{3} = 400W$

Now, $\quad P_L = V_\Delta I_\Delta \cos\theta$

$\therefore \quad 400 = 208\, I_\Delta \times 0.94$

$I_\Delta = 2.05$ Arms

current in each load phase

$\therefore$ mag. of line current $I_L = \sqrt{3}\, I_\Delta = \boxed{3.55 \text{ Arms}}$

$|Z_L| = \dfrac{V_L}{I_\Delta} = \dfrac{208}{2.05} = 101.46\ \Omega$

Since $\theta = +20°$, $\Rightarrow R_L = 101.46 \cos(+20°) = 95.34\Omega$

$jX_L = j101.46 \sin(+20°) = +j32.61\Omega$

or $\boxed{Z_L = 101.46 \angle 20°\ \Omega}$

② (b) $V_L = 34{,}500$ V rms , $f = 60$ Hz

Balanced 3-ph load power consumption $|\underline{S}_T| = 24\times10^6$ VA at 0.78 pf lag.

So load angle : $\cos^{-1} 0.78 = 38.74°$

$$\therefore \underline{S}_T = 24\times10^6 \underline{/38.74°} = 18.72\times10^6 + j\,15.02\times10^6 \text{ VA}$$
$$= P_T + j\,Q_T$$

Desired new pf = 0.94 lead. $\therefore$ θ_{new} (load angle after pf correction) $= -\cos^{-1} 0.94 = -19.95°$

With pf correction, real power consumption remains the same and new caps consume additional vars.

$$\therefore |\underline{S}_{new}| \cos\theta_{new} = 18.72\times10^6 \text{ W}$$

$$\therefore |\underline{S}_{new}| = \frac{18.72\times10^6}{0.94} = 19.91\times10^6 \text{ VA}$$

$$\therefore \underline{S}_{new} = 19.91\times10^6 \underline{/-19.95°} \text{ VA}$$

$$\therefore jQ_{new} = j\,19.91 \times \sin(-19.95°)\times10^6$$

$$= -j\,6.79\times10^6 \text{ vars}$$

$$\Rightarrow jQ_T + jQ_{cap} = -j\,6.79\times10^6 \Rightarrow jQ_{cap} = -j\,21.81\times10^6 \text{ VARs}$$

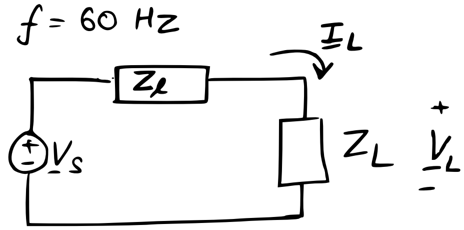
For each cap: $-j\left(\frac{V_L}{\sqrt{3}}\right)^2 C\omega = -j\frac{21.81\times10^6}{3}$

$$C = \frac{21.81\times10^6 \times 3}{3\times 34500^2 \times 2\pi\times 60} = \boxed{48.6\mu F}$$

3

$P_L = 20\,kW$, 0.8 pf lagging. $\Rightarrow \omega = 2\pi f = 377\,r/s$

$\underline{V_L} = 220\angle 0^\circ$ V rms $f = 60$ Hz



$Z_\ell = 0.09 + j0.3\ \Omega$
$= 0.313\angle 73.3^\circ\ \Omega$

$\theta_L = \cos^{-1} 0.8 = 36.87^\circ$

$P_L = V_L I_L \cos\theta_L$

$\therefore\ I_L = \dfrac{P_L}{V_L \cos\theta_L} = \dfrac{20\,000}{220 \times 0.8} = 113.64\,A$ rms

or $\underline{I_L} = 113.64\angle -36.87^\circ$ A rms

So, line impedance drop $\underline{V_\ell} = \underline{I_L}\, Z_\ell = 113.64\angle -36.87^\circ \times 0.313\angle 73.3^\circ$

$\underline{V_\ell} = 35.59\angle 36.43^\circ = 28.64 + j21.14$ V rms

$\therefore\ \underline{V_s} = \underline{V_L} + \underline{V_\ell} = 220 + j0 + 28.64 + j21.14$
$= 248.64 + j21.14 = 249.54\angle 4.86^\circ$ V rms

$\Rightarrow v_s(t) = 249.54 \cos(377t + 4.86^\circ)$ V rms. Ans

(b) pf seen by the source: $\cos(4.86 - -36.87^\circ)$
$= \cos(41.73) = 0.746$ lagging
Ans

4(a) Apply KCL at the inverting node:

$$\frac{V_i - 0}{R_1 + \frac{1}{Cs}} = \frac{0 - V_o}{R_2}$$

$$\Rightarrow H(s) = \frac{V_o}{V_i} = \frac{-R_2}{R_1 + \frac{1}{Cs}} = -\frac{s\frac{R_2}{R_1}}{s + \frac{1}{R_1 C}}$$

$\therefore$ one zero at $s = 0$

one pole at $s = \frac{1}{-R_1 C}$

From the given Bode plot, we know the pole is

at $\omega = 500$ r/s $\therefore 500 = \left|\frac{1}{-R_1 C}\right|$

$$\therefore R_1 = \frac{1}{500 \times 3\times10^{-6}} = \boxed{666.67\Omega}$$

Now, note that as $\omega \to \infty$; $20\log_{10}|H(j\omega)| = 34$dB

or $|H(j\omega)|\Big|_{\omega\to\infty} = 50.119$

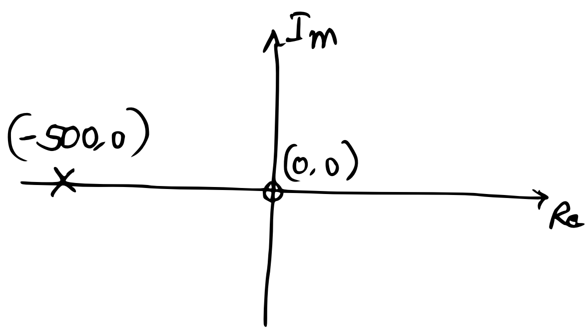
Also $H(j\omega) = \frac{-j\omega R_2/R_1}{j\omega + \frac{1}{R_1 C}}$ $\therefore |H(j\omega)| = \frac{\omega R_2/R_1}{\sqrt{\omega^2 + \frac{1}{R_1^2 C^2}}}$

$$|H(j\omega)| = \frac{R_2}{R_1}\frac{1}{\sqrt{1 + \frac{1}{R_1^2 C_2^2 \omega^2}}} \Rightarrow |H(j\omega)|\Big|_{\omega\to\infty} = \frac{R_2}{R_1}$$

since as $\omega\to\infty$; $\frac{1}{R_1^2 C^2 \omega^2} \to 0$

$\therefore \frac{R_2}{R_1} = 50.119$. With $R_1 = 666.67\Omega$, $\boxed{R_2 = 33.4\,K\Omega}$

(b)

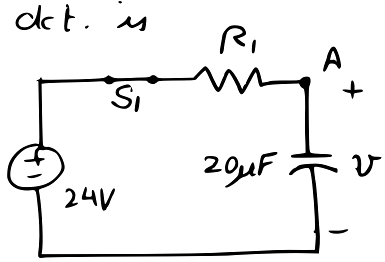


one zero at (0,0)

one pole at $\dfrac{1}{-R_1 C} = (-500, 0)$

(c) Real part of the pole is < 0. $\therefore$ the ckt. is stable.

(5) (a) For $0<t<0.02$ sec, the ckt. is



Apply KCL at A:

$$\frac{v-24}{R_1} = -C\frac{dv}{dt}$$

$$-\frac{v}{R_1}+\frac{24}{R_1} = C\frac{dv}{dt}, \quad \text{or} \quad \frac{dv}{dt}+\frac{1}{R_1C}v = \frac{24}{R_1C}$$

Natural sol. $v_n = K_1 e^{st}$; charac. eq $s+\frac{1}{R_1C}=0$

$$\therefore \; v_n = K_1 e^{-\frac{1}{R_1C}t}$$

Forced sol, $v_f = A$. Putting this into the ode gives

$$0+\frac{A}{R_1C} = \frac{24}{R_1C} \Rightarrow A = 24$$

$$\therefore \; v = v_n + v_f = K_1 e^{-\frac{1}{R_1C}t} + 24$$

$$v(0) = 0 = K_1 + 24 \Rightarrow K_1 = -24$$

$$\therefore \; v = 24\left(1 - e^{-\frac{1}{R_1C}t}\right) \text{ V}$$

Now, $t = 0.02$ sec, $v = 16$ V, $R_1 = ?$

$$16 = 24\left(1 - e^{-\frac{0.02}{R_1C}}\right)$$

$$24 e^{-0.02/R_1C} = 8 \Rightarrow e^{-0.02/R_1C} = \tfrac{1}{3} \Rightarrow \frac{-0.02}{R_1C} = \ln \tfrac{1}{3}$$

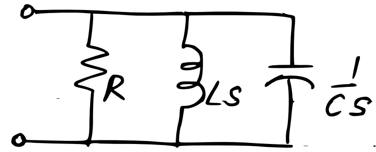
$$\Rightarrow R_1C = \frac{-0.02}{\ln 1/3} = 0.0182 \text{ s}$$

$$\Rightarrow R_1 = \frac{-0.02}{20\times10^{-6}\times\ln 1/3} = \boxed{910.24\,\Omega}$$

(b) We are adding R_2 to the ckt, hence the time constant will get smaller. To maintain 16V for $t > 0.02$ basically means that the ckt. is in steady state wrt the new time constant. In this situation, cap. is open ckted, and using voltage divider:

$$\frac{24}{R_1+R_2} R_2 = 16 \Rightarrow R_2 = R_1 \times 2$$
$$= 1820.48\,\Omega$$

(6) (a) Given the mag. resp. of a parallel RLC ckt.



$$Z(s) = \left(\frac{1}{R} + \frac{1}{Ls} + Cs\right)^{-1}$$

$$= \frac{1}{Cs + \frac{1}{R} + \frac{1}{Ls}}$$

$$Z(j\omega) = \frac{1}{jC\omega + \frac{1}{R} + \frac{1}{jL\omega}} = \frac{1/R - j\left(C\omega - \frac{1}{L\omega}\right)}{1/R^2 + \left(C\omega - \frac{1}{L\omega}\right)^2}$$

(a) At resonance, ω_r, $C\omega_r = \frac{1}{L\omega_r}$

and $Z(j\omega_r) = R$

From the given graph, at resonance, gain $|Z(j\omega_r)| = R$
$= 3998.9\,\Omega$

(b). $Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{2.2491}{2.3322 - 2.172} = 14.04$

(c). ω_0 (resonant freq) $= 2249.1$ Hz (where the peak occurs in the gain plot)

$= (2\pi)\,2249.1$ r/sec
$= 14131.5$ rad/sec

(d). $Z(s) = \left(\frac{1}{R} + \frac{1}{Ls} + Cs\right)^{-1} = \cdots = \frac{s/C}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$

$$Z(j\omega) = \frac{j\,\omega/C}{\frac{1}{LC} - \omega^2 + j\omega\frac{1}{RC}}$$

(e) From (a) $R = 3998.9\,\Omega$
Also, $\omega_0 = \frac{1}{\sqrt{LC}} = 14131.5$

Note that in this case:
Quality factor is also defined as

$$Q = R\sqrt{\frac{C}{L}} = 14.04 \ (\text{from (b)})$$

$$\therefore \sqrt{\frac{C}{L}} = \frac{14.04}{3998.9} \Rightarrow \frac{C}{L} = 12.33 \times 10^{-6} \quad \text{(e1)}$$

Since $\frac{1}{\sqrt{LC}} = 14131.5 \Rightarrow \frac{1}{LC} = 0.1997 \times 10^{9}$ (e2)

Solving equations (e1) and (e2),

$$L = 20.15\,mH$$
$$C = 0.2485\,\mu F$$