ECSE-210 Fall 2007

RLC Circuits

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Unless otherwise mentioned, all problem solved here are from [1].

(1) Problem 6.18. We are supposed to find $i(t)$ for all $t > 10$s, given $v_c(10-) = 2$V.

Solution:

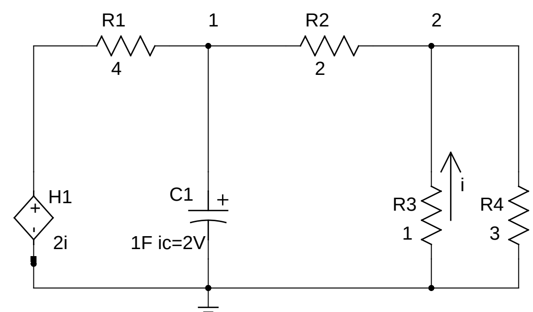

Figure 1: Problem 6.18. All resistances are in Ohms.

Using nodal analysis at nodes 1 and 2, with nodal votages v_1 and v_2 respectively and applying KCL at node 1 we obtain

$$\frac{v_c(t) - 2 \cdot i}{4} + C_1 \frac{dv_c(t)}{dt} + \frac{v_c(t) - v_2(t)}{2} = 0,$$

which upon simplification and using values of the elements from the given circuit along with the fact that $i(t) = \frac{-v_2(t)}{1}$, gives

$$\frac{dv_c(t)}{dt} + \frac{3}{4} v_c(t) = 0. \tag{1}$$

Similarly, applying KCL at node 2, we obtain

$$\frac{v_2(t) - v_c(t)}{2} + \frac{v_2(t)}{1} + \frac{v_2(t)}{3} = 0$$

which upon simplifying gives

$$v_c(t) = \frac{11}{3} v_2(t). \tag{2}$$

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Substituting value of $v_c(t)$ from Eq. 2 into Eq. 1 and simplifying, we obtain

$$\frac{dv_2(t)}{dt} + \frac{3}{4}v_2(t) = 0 \tag{3}$$

Using our method to arrive at the natural solution (the homogenous equation above does not have a particular solution), the solution of this first order linear o.d.e is of the form $v_2(t) = Ke^{-\frac{3}{4}t}$, where $K = V_{2_0}$, the initial conditino of v_2 at time $t = 0$. Now, we know that $V_{c_{10}} = -2\text{V}$, therefore

$$V_{2_0} = \frac{2}{2 + 1\|3} = \ldots = \frac{6}{11}\text{V} \tag{4}$$
$$v_2(t = 10) = \frac{6}{11} = v_2(t = 0)e^{-\frac{3}{4}} \implies K = \frac{6}{11}e^{\frac{30}{4}}$$

Hence

$$v_2(t) = \frac{6}{11}e^{\frac{30}{4}} \cdot e^{-\frac{3}{4}} = \frac{6}{11}e^{-\frac{3}{4}(t-10)}$$
$$i(t) = -\frac{v_2}{1} = -\frac{6}{11}e^{-\frac{3}{4}(t-10)} \tag{5}$$

For the second part, if $v_c(10-) = 0\text{V}$ (using 0 instead of 2 in Eq. 4), $V_{2_0} = 0 \implies K = 0 \implies i(t) = 0\text{A}$.

(2) Problem 6.24.

Solution:

We are supposed to find $v(t); t > 0$ given that the circuit is in dc steady state at $t = 0-$.

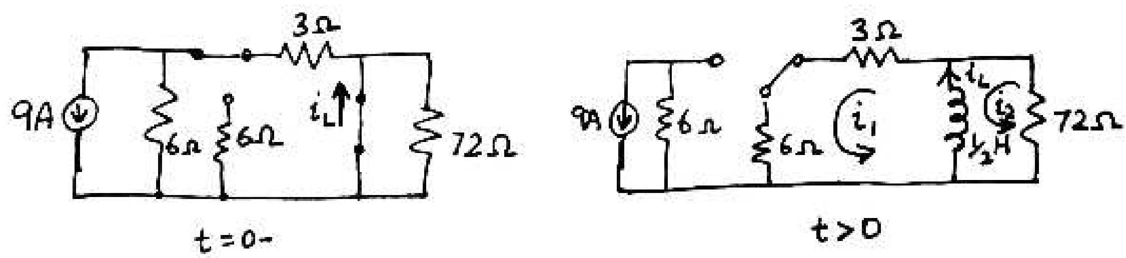

Figure 2: Problem 6.24.

At $t = 0-$, the circuit is in dc steady state and the inductor can be replaced by a short circuit. Also, the 6 Ω resistor is not in the circuit due to the position of the switch. Under these conditions, we have a purely resistive circuit and it is easy to determine (using current divider) that

$$i_L(0-) = 6 \text{ A}. \tag{6}$$

At $t > 0$, the circuit is as shown in the figure above. In this case, the two branches on the left of the original circuit do not play any role for the rest of the circuit so we can ignore them.

Applying KVL on the left hand side mesh of the remaining circuit, we get

$$3i_1 + 6i_i + \frac{1}{2}\frac{di_L}{dt} = 0 \tag{7}$$

Applying KVL around the other mesh, we obtain

$$72i_2 - \frac{1}{2}\frac{di_L}{dt} = 0 \tag{8}$$

Also, note the fact that

$$i_L = i_1 - i_2 \tag{9}$$

Multiplying Eq. 7 by 8 and subtracting Eq. 8 from the result, and using Eq. 9, we obtain

$$\frac{di_L}{dt} + 16i_L = 0 \tag{10}$$

The above o.d.e. is a homogeneous equation. Since its forcing function is zero, the particular solution (or the forced solution) of the o.d.e. is also zero. Hence the complete solution is the natural solution itself. The form of the solution is

$$i_L = Ke^{st}$$

To find the solution, we find the roots of the characteristic equation, which is obtained by replacing the solution in the o.d.e., which gives

$$s + 16 = 0 \implies s = -16 \tag{11}$$

hence,

$$i_L = Ke^{-16t} \tag{12}$$

To find the unknown constant, we use the initial condition given in Eq. 6 in the above equation, which gives us

$$6 = K$$

Hence $i_L = 6e^{-16t}$ A. To find v, we note that

$$\begin{aligned} v &= 6i_1 && (13) \\ &= 6\left(\frac{-1}{18}\frac{di_L}{dt}\right) \ldots \text{from Eq. 7} && (14) \\ &= 32e^{-16t} \text{ V};\ \ t > 0 \text{ in sec.} && (15) \end{aligned}$$

(3) Problem 6.32. We are supposed to find $i(t)$ for all t.

Solution:

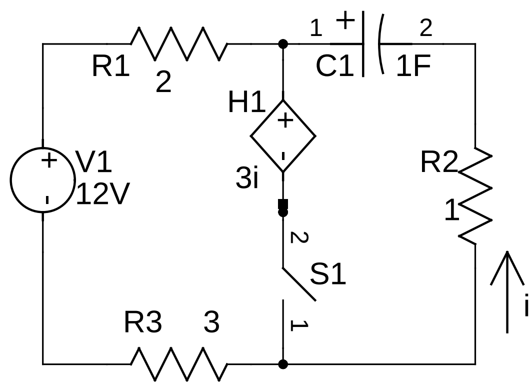

Figure 3: Problem 6.32. All resistances are in Ohms.

At $t < 0$, the circuit is given to be in steady state for long time and thus the capacitor C_1 is fully charged and no current is flowing through it (can be replaced with an open circuit). This implies, $i(t < 0) = 0$, since the loop of C_1 and R_1 is not closed. Consequently, the dependent source $H_1 = 0\text{V}$, hence it can be effectively replaced by a short circuit and hence the voltage across C_1, $v_c(0-) = 0\text{V}$.

At $t = 0$, the switch is opened (effectly removing the dependent source H_1 out of the circuit) and C_1 can be replaced by a $v_c(0+) = v_c(0-) = 0\text{V}$ battery source. In this situation, the initial condition of i is

$$i(0+) = \frac{-12}{2+1+3} = -2\text{A}. \tag{16}$$

For $t > 0$, the circuit is just the outer mesh with C_1 included. In this case, we can apply KVL around the mesh (counter clockwise) to obtain

$$2 \cdot i + 12 + 3 \cdot i + 1 \cdot i + \frac{1}{1} \int_0^t i d\tau = 0.$$

Differentiating the above equation with respect to t and simplifying, we get:

$$\frac{di}{dt} + \frac{1}{6} i = 0, \tag{17}$$

which is a first order linear differential equation. The solution is $i = i_n + i + p$, i.e. sum of the natural and complementary solutions. Since our equation is homogeneous, $i_p = 0$.

The natual solution, $i + n$, has the form $i_n = Ke^{st}$. Substituting this in Eq. 17, we obtain

$$\begin{aligned}
&Kse^{st} + \frac{1}{6} Ke^{st} = 0 \\
&K(s + \frac{1}{6}) = 0 \\
&s + \frac{1}{6} = 0 \qquad\qquad \text{since K cannot be zero} \\
&s = -\frac{1}{6}
\end{aligned} \tag{18}$$

Hence, the solution is of the form

$$i = i_n = Ke^{-\frac{1}{6}t}. \tag{19}$$

To determine K, we use the initial condition from Eq. 16 to evaluation Eq. 19 at $t = 0$, which results in $K = -2$. Hence the solution is

$$i(t) = \begin{cases} 0, & t < 0 \\ -2e^{-\frac{1}{6}t}, & t \geq 0 \end{cases} \tag{20}$$

(4) Problem 6.47.

Solution:

1. The given input is

$$v_g = \begin{cases} 0 & t < 0 \\ 2 & t > 0 \end{cases} \tag{21}$$

Let the voltage at the output of the first opamp be v_0. Then applying KCL at the inverting node of the first opamp, we obtain

$$\frac{0 - v_g}{2} + \frac{0 - v_0}{1} + \frac{1}{2}\frac{d(0 - v_0)}{dt} = 0 \text{ or}$$
$$\frac{dv_0}{dt} + 2v_0 = -2 \qquad (v_g = 1) \tag{22}$$

The natural solution of this o.d.e. is

$$v_{0_n} = Ke^{-2t} \tag{23}$$

and the forced solution assuming to be A and plugging this into the o.d.e. and solving for A gives

$$A = -1 \tag{24}$$

The complete solution is thus

$$v_0 = Ke^{-2t} - 1$$

K is determined by using the initial condition $v_0(0) = 0$ which gives $K = 1$, hence the complete solution is

$$v_0 = e^{-2t} - 1 \tag{25}$$

Applying KCL at the non-inverting node of the second opamp, it is easy to show that

$$v = 4v_0, \tag{26}$$

hence

$$v = 4\left(e^{-2t} - 1\right) \text{ V} \tag{27}$$

2. The input is given by

$$v_g = \begin{cases} 0 & t < 0 \\ 2 & 0 < t < 1 \\ 0 & t > 1 \end{cases} \tag{28}$$

In this case, we already know the solution for $0 < t < 1$ from the first part above. We just need to examine the circuit for $t > 1$. The differential equation is similar to the one before, only that now $v_g = 0$. Thus, the solution is of the form

$$v_0 = K_2 e^{-2(t-1)} \tag{29}$$

where we have accounted for the shift in time by 1 unit (recall linearity: in a linear circuit, if the input is shifted in time, the output is also shifted in time by the same period). To determine the unknown constant, we use the initial condition $v_0(1) = -0.865$ V which is determined from Eq. 25 by plugging $t = 1$ in it. Using this initial condition Eq. 29 gives

$$v_0(1) = -0.865 = K_2,$$

and using Eq. 26, we get

$$v = -3.459\, e^{-2(t-1)},\ t > 1$$

Therefore,

$$v = \begin{cases} 0 \text{ V} & t < 0 \\ 4\left(e^{-2t} - 1\right) \text{ V} & 0 < t < 1 \\ -3.459 e^{-2(t-1)} \text{ V} & t > 1 \end{cases}$$

The solution can be also be obtained in an easier way in terms of unit steps by using the linearity property of the given circuit.

(5) Problem 7.16.

Solution:

First, let's see what is the state of the circuit at $t = 0-$. In this case, since the circuit is in steady state, the inductors may be replaced by short circuits and the resulting circuit is a purely resistive circuit with a dc voltage source. We can then find the current passing through the various branches at this time instant.

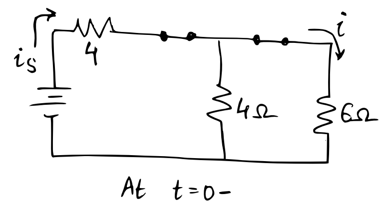

Figure 4: At $t = 0-$.

$$i_s(0-) = \frac{16}{4 + 4\|6} = 2.5 \text{ A}.$$

Also,

$$i(0-) = \frac{2.5(4\|6)}{6} = 1 \text{ A}.$$

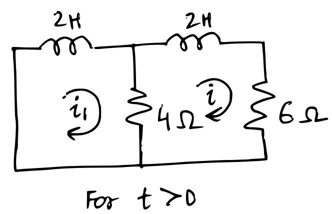

Figure 5: For $t > 0$.

Now, the above two equations give us a couple of initial conditions. Next, we examine the circuit for time $t > 0$. In this case, the circuit is just a two mesh circuit as shown in the figure above.

Applying KVL around mesh # 1 gives

$$2\frac{di_1}{dt} - 4(i_1 - i) = 0 \tag{30}$$

Appyling KVL around mesh # 2 gives

$$2\frac{di}{dt} + 6i + 4(i - i_1) = 0 \tag{31}$$

Differentiating Eq. 31 gives

$$\frac{d^2i}{dt^2} + 5\frac{di}{dt} - 4i + 4i_1 = 0 \tag{32}$$

To get rid of i_1 in the above equation, we use Eq. 31 in the above equation and simplify to obtain the system's ode

$$\frac{d^2i}{dt^2} + 7\frac{di}{dt} + 6i = 0 \tag{33}$$

This is a homogeneous ode hence the forced response is zero. The roots of the corresponding characteristic equations, $s^2 + 7s + 6 = 0$, are $s_{1,2} = -1, -6$.

Thus the natural response is of the form

$$i(t) = K_1 e^{-t} + K_2 e^{-6t}, \ \ t > 0$$

To get the unkown constants, we use the initial conditions. The first one is

$$i(0+) = i(0-) = 1 = i(t)|_{t=0} = K_1 + K_2$$

To get the second initial conditions, we note that the drop across the inductor n the right hand side at $t = 0+$ (note: voltage across an inductor is *not* continuous) at be obtained by examining the circuit at $t = 0+$. In this case, $v_L(0+) = v_{4\Omega} - v_{6\Omega} = 1.5(4) - 1(6) = 0\text{V}$.

Hence,

$$v_L(0+) = 2\frac{di}{dt}|_{t=0} \implies -K_1 - 6K_2 = 0$$

Solving the two equations for the two unkown constants gives $K_1 = 1.2$ and $K_2 = -0.2$. Hence, the response is

$$i(t) = 1.2e^{-t}00.2e^{-6t} \text{ A } \ t > 0$$

and

$$i(t) = 1 \text{ A } \ t < 0$$

(6) Problem 7.19.

Solution:

Let the bottom node be the reference node. The node above the dependent voltage source has a voltage of $2v_1$. Lets name this as node 3. Since the voltage across the capacitor is v_2, the top left node has a voltage of $2v_1 + v_2$. Lets name this node as 2. The remaining node has a voltage of v_1, the voltage across the $\frac{1}{8}$ F capacitor. Lets name this node as node 1.

Applying KCL on node 1

$$\begin{aligned} \frac{1}{8}\frac{dv_1}{dt} &+ \frac{v_1 - (2v_1 + v_2)}{8} \\ &\frac{dv_1}{dt} - v_1 = v_2 \end{aligned} \tag{34}$$

Applying KCL on node 2

$$\frac{2v_1 + v_2}{2} + \frac{(2v_1 + v_2) - v_1}{8} + \frac{1}{8}\frac{d\overbrace{(2v_1 + v_2 - 2v_1)}^{v_2}}{dt} = 0$$

This equation, after using Eq. 34 to eliminate v_2 and simplifying gives

$$\frac{d^2v_1}{dt^2} + 4\frac{dv_1}{dt} + 4v_1 = 0 \tag{35}$$

This is a homogeneous o.d.e.. Its characteristic equation is $s^2 + 4s + 4$, whose roots are found to be $s_{1,s} = -2, -2$. These are real repeated roots, hence the solution is of the form (ref. [1], Section 7.3)

$$v_1 = K_1 e^{-2t} + K_2 t e^{-2t}. \tag{36}$$

Now, we use initial conditions to solve for the unkown constants.

$$v_1(0) = 12 = K_1 + K_2(0) \implies K_1 = 12 \tag{37}$$

We also know that $v_2(0) = 0$ and v_2 is given by Eq. 34. Solving this gives $K_2 = 48$. Hence

$$v_1 = 12e^{-2t} + 48te^{-2t} \tag{38}$$

Also, from the figure, $i = \frac{2v_1}{4}$, hence

$$i = 6e^{-2t} + 24te^{-2t} \text{ A}. \tag{39}$$

(7) Problem 7.32.

Solution:

Obtaining the ode The circuit is an RLC circuit with a single mesh. Applying KVL around that mesh, assuming a clockwise current i, we obtain

$$5i + 1\frac{di}{dt} + 6\int_0^t i d\lambda + v(0) = 26\cos 3t. \tag{40}$$

Differentiating this equation w.r.t., we get

$$\frac{d^2 i}{dt^2} + 5\frac{di}{dt} + 6i = -78\sin 3t. \tag{41}$$

Natural Solution This is obtained by setting the right hand side of the o.d.e. to zero. Assuming the solution to be of the form $i_n = Ke^{st}$ and substituting this in Eq. 41, we obtain the characteristic equation

$$s^2 + 5s + 6 = 0 \tag{42}$$

whose roots are found to be $s_1 = -2$ and $s_2 = -3$. The roots are real and distinct hence the system is overdamped. Also, due to the nature of the roots, we know that the natural solution is of the form

$$i_n = K_1 e^{-2t} + K_2 e^{-3t} \tag{43}$$

Particular Solution Looking at the forcing function, we know that the particular solution is of the form (see [1], Table 7.2, p. 287)

$$\begin{aligned} i_p &= A\sin 3t + B\cos 3t \\ \therefore\ \frac{di_p}{dt} &= 3A\cos 3t - 3B\sin 3t \\ \text{and } \frac{d^2 i_p}{dt^2} &= -9A\sin 3t - 9B\cos 3t \end{aligned} \tag{44}$$

Using these in Eq. 41 and equation the coefficient of sin and cos terms on both sides, we get $A = 1$ and $B = 5$, hence the particular solution is

$$i_p = \sin 3t + 5\cos 3t \tag{45}$$

Complete Solution This is the sum of the natural and particular solutions. From Eqs. 43 and 45

$$i = i_p + i_n = K_1 e^{-2t} + K_2 e^{-3t} + \sin 3t + 5\cos 3t. \tag{46}$$

Finding the unknown constants We use the given initial conditions to find the unknown constants. Since $i(0-) = 2$ A, therefore

$$\begin{aligned} 2 &= i|_{t=0} = K_1 + K_2 + 5 \\ \implies\ & K_1 + K_2 = -3 \end{aligned} \tag{47}$$

Now, since $i(0-) = 2$ A, we can apply KVL around the mesh at $t = 0+$ to find the voltage across the inductor

$$26 = 10 + v_L(0+) + 6 \implies v_L(0+) = 10 \text{ V}.$$

Also, since $v_L = L\frac{di}{dt}$, then

$$v_L(0+) = 10 = \frac{di}{dt}|_{t=0}.$$

Thus, by differentiating the complete solution, Eq. 46, and setting $t = 0$, and simplifying, we get following equation in unknown constants

$$2K_1 + 3K_2 = -7 \tag{48}$$

Solving Eqs. 47 and 48, we obtain $K_1 = -2$ and $K_2 = -1$, hence the complete solution is

$$i = -2e^{-2t} + -1e^{-3t} + \sin 3t + 5\cos 3t \text{ A}; t > 0$$

and where time is in sec.

(8) Problem 7.35.

Solution:

At $t = 0-$, we just have a single mesh RL circuit at dc steady state. At this instance, the inductor acts as a short circuit and hence the current through it is

$$i(0-) = \frac{16}{4+2} = \frac{8}{3} \text{ A.} \tag{49}$$

This is our first initial condition. For $t > 0$, we have an RLC circuit, hence a second order system, therefore we need another initial condition to solve the o.d.e. At $t = 0+$, the capacitor voltage is $v_c(0+) = v_c(0-) = 0$ V, and the current through the mesh is $i(0+)$ A. Hence, applying KVL around the RLC circuit at $t = 0+$, we get

$$\begin{aligned} 16 &= v_L(0+) + \frac{8}{3}4 + 0 \\ \therefore v_L(0+) &= \frac{16}{3} \text{ V.} \end{aligned} \tag{50}$$

Obtaining the ode For $t > 0$, applying KVL around the RLC circuit, we obtain

$$2\frac{di}{dt} + 4i + 8\int_0^t i d\tau + v_c(0) = 16$$

which upon differentiating once w.r.t. once and after simplification, gives

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 4i = 0. \tag{51}$$

Natural Solution This is obtained by assuming the solution of the homogeneous equation is of the form $i = Ke^{st}$. Substituting this into the o.d.e., we obtain the characteristic equation

$$\begin{aligned} &s^2 + 2s + 4 = 0 \\ &\therefore s = -1 + j\sqrt{3},\ -1 - j\sqrt{3} \text{ (roots of the above eq.).} \end{aligned}$$

Hence the natural solution is of the form

$$i_n = e^{-t}(K_1 \sin\sqrt{3}t + K_2 \cos\sqrt{3}t) \tag{52}$$

Particular Solution The forcing function in the o.d.e. is zero hence the particular solution is also zero.

Complete Solution This is given by

$$i = i_n + i_p = e^{-t}(K_1 \sin\sqrt{3}t + K_2 \cos\sqrt{3}t) \tag{53}$$

Finding the unknown constants We use the initial conditions to find the unknown constants. From Eq. 49

$$\frac{8}{3} = i|_{t=0} = K_2 \tag{54}$$

Also, from Eq. 50

$$\frac{16}{3} = 2\left.\frac{di}{dt}\right|_{t=0} = 2\left(\sqrt{3}K_1 - 0 - 0 - K_2\right)$$
$$K_1 = \frac{16}{3\sqrt{3}} \tag{55}$$

Therefore

$$i = e^{-t}\left(\frac{16}{3\sqrt{3}}\sin\sqrt{3}t + \frac{8}{3}\cos\sqrt{3}t\right) \quad t > 0$$

where time is in seconds.

(9) Problem 7.43.

Solution:

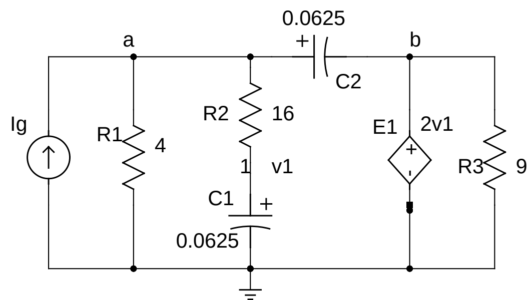

Figure 6: Problem 7.43.

If a step response is desired, that means all initial conditions are zero at $t = 0-$; the stogage elements do not have any energy stored in them.

Differential equation of the circuit Let the nodes be node 1,node a and node b as shown in Figure 6 with node voltages v_1, v_a and $v_b = 2v_1$ respectively.

Applying KCL at node a and implifying, we obtain

$$\frac{dv_a}{dt} + 5v_a - 2\frac{dv_1}{dt} - v_1 = 16i_g \tag{56}$$

Applying KCL at node 1 and simplifying we obtain

$$v_a = v_1 + \frac{dv_1}{dt} \tag{57}$$

Using Eq. 57, Eq. 56 can be simplified to

$$\frac{d^2v_1}{dt^2} + 4\frac{dv_1}{dt} + 4v_1 = 16i_g \tag{58}$$

This is a second order linear ordinary differential equation with the forcing function $f(t) = 16i_g$. The solution of this differential equation is the sum of natural(or complementary) solution and particular(or forced) solution

$$v_1 = v_{1_n} + v_{1_p}. \tag{59}$$

Natural solution Assuming the natual solution to be of the form Ke^{st}, the characteristic equation is

$$s^2 + 4s + 4 = 0 \tag{60}$$

whose roots are

$$s_1 = -2 \text{ and } s_2 = -2. \tag{61}$$

Since the roots are repeated, the natural solution is of the form

$$v_{1_n} = K_1 e^{-2t} + K_2 t e^{-2t}. \tag{62}$$

Particular Solution The forcing function is of the form $f(t) = 1; t \geq 0$, hence the particular is of the form $v_{1_p} = A$. Substituting this into Eq. 58 gives

$$(0) + (0) + 4A = 16 \;\therefore A = 4 \implies v_{1_p} = 4. \tag{63}$$

Hence, from Eq. 59, Eq. 62 and Eq. 63

$$v_1 = K_1 e^{-2t} + K_2 t e^{-2t} + 4. \tag{64}$$

To solve for the constants, we need to use the initial conditions of the system. We know that $v_1(0+) = v_1(0-) = 0$V. Substituting this value of $v_1(0+)$ in Eq. 64 gives $K_1 = -4$. To evaluate K_2 we need another equation. Consider Eq. 57. The right hand side of this equation just contains terms with v_1 which we can evaulate at $t = 0$, but the left hand side contains v_a. To determine the constants, we must know $v_a(0)$.

Due to the initial conditions (recall we have a step input)

$$\begin{gathered} v_1(0) = 0 \implies v_b(0) = 0 \\ v_a(0) = 0 \text{ since } v_{c_2}(0) = 0 \end{gathered} \tag{65}$$

Hence, Eq. 57 becomes

$$\begin{aligned} v_a(0) &= v_1\big|_{t=0} + \left.\frac{dv_1}{dt}\right|_{t=0} \\ &= K_1 + 4 - 2K_1 + K_2 \\ \implies K_2 &= -8 && \text{since } K_1 = -4 \\ \therefore\; v_1 &= -4e^{-2t} - 8te^{-2t} + 4\,\text{V}; t \geq 0. \end{aligned} \tag{66}$$

Hence the general solution to the differential equation of the given sytem is

$$v_1 = \begin{cases} 0\text{ V} & ; t < 0 \\ -4e^{-2t} - 8te^{-2t} + 4\text{ V} & ; t \geq 0 \end{cases}$$

which can also be written as

$$v_1 = (-4e^{-2t} - 8te^{-2t} + 4)u(t)\text{ V}$$

(10) Problem 7.45.

Solution:

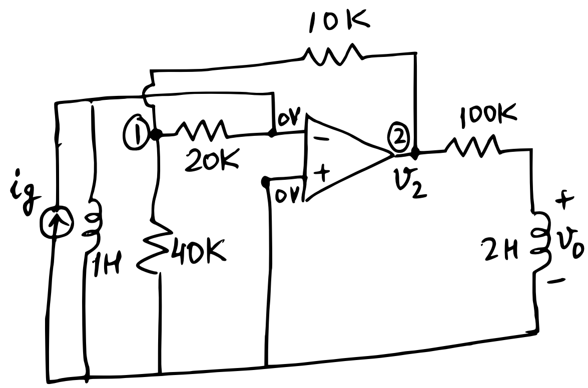

Figure 7: Given circuit

Assume ideal op-amp and use virtual short and virtual open pricipals.

Applying KCL at the inverting input node of the ap-amp, we get

$$\frac{v_1}{40000} + \frac{v_1 - 0}{20000} + \frac{v_1 - v_2}{40000} = 0$$

which, upon simplification, gives

$$v_2 = \frac{7}{4}v_1 \tag{67}$$

Applying KCL at node 1 gives

$$\frac{0 - v_1}{20000} + 0 = i_g$$

which gives

$$v_1 = 20000i_g \tag{68}$$

Using Eq. 68, Eq. 67 becomes

$$v_2 = -350000i_g \tag{69}$$

Next, applying KCL at the output node gives

$$\frac{1}{2}\int_0^t v_o l dl + 0 = \frac{v_2 - 0}{100000}$$

which upon differentiation w.r.t t once simplifying gives

$$50000 v_o = \frac{dv_2}{dt} - \frac{dv_o}{dt}. \tag{70}$$

Using Eq. 69, the above equation becomes

$$50000 v_o = -35000\frac{di_g}{dt} - \frac{dv_o}{dt}. \tag{71}$$

But since $i_g = 1, t > 0$, the derivative of i_g in the above equation is zero. Hence, we are left with

$$\frac{dv_o}{dt} + 50000 v_o = 0$$

The solution to this ode is

$$v_o = Ke^{-50000t} \tag{72}$$

To find the unknown constant, we examine the circuit at $t = 0+$. At this time instant, there is no current flowing through the inductors (due the step input, they had zero initial energry) and the input current is 1 A. Examing the now purely resistive op-amp circuit, it is easy to find that $v_o(0+) = -35000$ V.

Using this initial condition in Eq. 72, we solve for K

$$v_o(0+) = -35000 = K$$

Hence,

$$v_o(t) = -35000e^{-50000t} \text{ V}.$$

(11) Problem 7.48.

Solution:

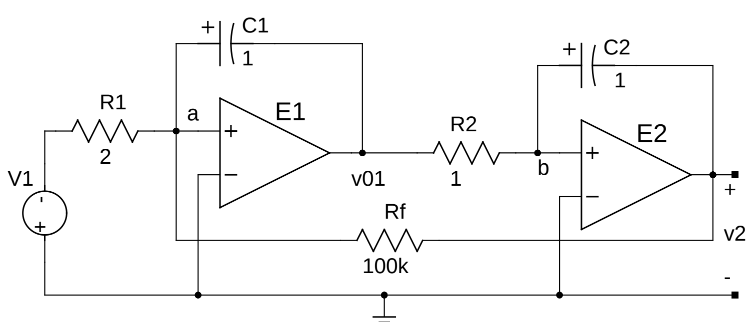

Figure 8: Problem 7.48.

Since only the forced solution is to be found, we do not need the initial conditions (neither are they given). Initial conditions are useful only if the unknown constants are to be found in the general solution of the given 2nd order circuit.

Inspecting the figure we note that the non-inverting terminals of both the op amps are grounded.Using virtual short principle, at node a, $v_a = 0$V and at node b, $v_b = 0$V.

Differential equation of the circuit Applying KCL at node a and simplifying, we obtain

$$-2v_1 - 4\frac{dv_{o_1}}{dt} - v_2 = 0. \tag{73}$$

Applying KCL at node b and simplifying, we obtain

$$v_{o_1} = -\frac{dv_2}{dt} \tag{74}$$

Using Eq. 74, Eq. 73 is simplified to

$$\frac{d^2v_2}{dt^2} - \frac{1}{4}v_2 = 2\cos 2t. \tag{75}$$

Forced response To find the forced solution, or the particular solution, we examine the forcing function which on the left hand side of the differential equation (if the equation is the standard form). In our case, it is $f(t) = 2\cos 2t$, hence the forced solution is of the form $v_{2_f} = A\sin 2t + B\cos 2t$ (see [1], Table 7.2, p. 287). Therefore

$$v_{2_f} = A\sin 2t + B\cos 2t \tag{76}$$
$$\frac{dv_{2_f}}{dt} = 2A\cos 2t - 2B\sin 2t \tag{77}$$
$$\frac{d^2v_{2_f}}{dt^2} = -4A\sin 2t - 4B\cos 2t \tag{78}$$
$$\tag{79}$$

Substituting these values for the variable v_{2_f} and its derivatives into Eq. 75, we obtain

$$-4A\sin 2t - 4B\cos 2t - \frac{1}{4}\left(A\sin 2t + B\cos 2t\right) = 2\cos 2t$$

The unknown constants are found by equating the coefficient of $\cos 2t$ and $\sin 2t$ respectively on both sides of the equation. Thus we get

$$-4B - \frac{B}{4} = 12 \qquad \text{by equating coefficients of } \cos 2t$$
$$-4A - \frac{A}{4} = 12 \qquad \text{by equating coefficients of } \sin 2t$$

Soving these two simultenous linear equations gives $A = 0$ and $B = -\frac{8}{17}$. Substituting these values into Eq. 76 gives us the forced response

$$v_{2_f} = -\frac{8}{17}\cos 2t \text{ V}.$$

References

[1] D. E. Johnson, J. R. Johnson, J. L. Hilburn, and P. D. Scott, *Electric Circuit Analysis*. Upper Saddle River, New Jersey 07458: Prentice Hall, third ed., 1997.