Norton and Thevenin Circuits and Op-amps

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Unless otherwise mentioned, all problem solved here are from [1].

(1) Problem 2.38.

Solution:

By killing the current sources, we see that the resistance seen from the two terminal on the left hand side is

$$R_{eq} = 8 \left\| \frac{16}{7} \right. = \frac{16}{9}\Omega$$

By short circuiting the two terminals on the left hand side while keeping all the sources, we can find the short circuit current (apply KCL at the top node, while noting that due to the short circuit, no current will flow through the resistors)

$$I_{sc} = 2 - (-1) = 3 \text{ A}.$$

Thus $V_{Th} = I_{sc}R_{eq} = \frac{16}{3}$ V.

Hence the given circuit can be reduced to either R_{eq} in series with V_{Th} (with plus terminal at top) or with R_{eq} in parallel with I_{sc} (current going upwards).

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(2) Problem 2.39.

Solution:

Consider the circuit on the left. Its terminal equation is

$$v = -2i + 1$$

To find R_{eq1} and V_{Th1} for this circuit, we need to find V_{Th1} and I_{sc1}. From the definitions of these parameters, it follows that

$$V_{Th1} = v|_{i=0} = 1 \text{ V, and}$$
$$I_{sc1} = i|_{v=0} = 0.5 \text{ A}.$$
$$\implies R_{eq1} = \frac{V_{Th1}}{I_{sc1}} = 2\Omega \tag{1}$$

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Similarly, considering the circuit on the right hand side with terminal equation $v = -4i - 7$, we get

$$V_{Th2} = v|_{i=0} = -7 \text{ V, and}$$

$$I_{sc2} = i|_{v=0} = \frac{7}{-4} \text{ A.}$$

$$\implies R_{eq2} = \frac{V_{Th2}}{I_{sc2}} = 4\Omega \tag{2}$$

Hence, we have V_{Th1} in series with R_{eq1} and we have V_{Th2} in series with R_{eq2}. Putting these in series, we get the resultant $R_{eq} = R_{eq1} + R_{eq2} = 6 \ \Omega$, and $V_{Th1} = V_{Th1} + V_{Th2} = -6$ V.

(3) Problem 2.49.

Solution:

From the given terminal law (see Section 2.3 of [1]), network N may be replaced by a resistance of $7\,\Omega$ in series with an independent voltage source of 4 V. In this case, it is easy to see that the equivalent resistance seen from the two terminals on the left with independent sources killed is
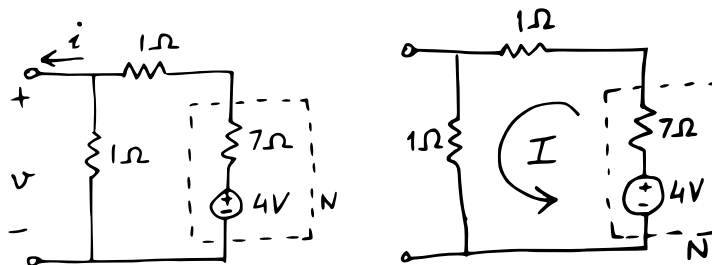
$$R_{eq} = 1\|(1 + 8) = \frac{8}{9} \ \Omega.$$

Figure 1: Network N is represented by its schematic from the given terminal law.

Also, with the sources in the circuit, the circuit consists of just one mesh. Applying KVL around this mesh and computing the drop across the $1 \ \Omega$ resistance on the left gives

$$V_{oc} = \frac{4}{9} \text{ V.}$$
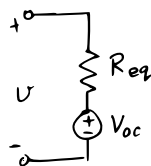
The above two parameters give the Thevenin equivalent circuit.

Figure 2: The Thevenin and Norton equivalent circuits of the given problem.

(4) Problem 2.49.

Solution:

This circuit may be solved by using any method while keeping in mind that we just need R_{eq} with V_{oc} or I_{sc}. Finding R_{eq} is easy by killing all the independent sources (and there are no dependent sources). But finding on of the remaining two parameters can be a bit of work if usual methods (KCL, KVL, superposition) are used. Note that we have many branches with voltage sources in series with resistances and some loops with a current source and a resistance. These little Thevenin and Norton circuits, which are related by $V_{oc} = I_{sc}R_{eq}$, can be transformed in to one another to simplify the given circuit and to arrive at its over all Thevenin and Norton equivalents. This is illustrated in the figure below.
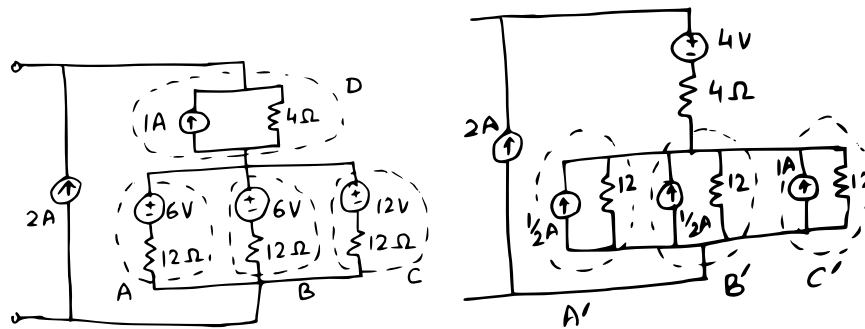

Figure 3:

In the figure above, the circled branches in the lower part of the circuit have been converted in to their Norton equivalents as shown in the figure on the right. The two branches at the top in the figure on the left have been converted in to their Thevenin equivalent as shown in the figure on the right.
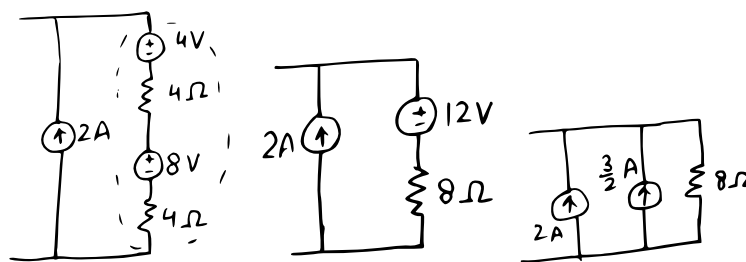

Figure 4:

In the figure above, we continue to simplify the circuit. Note that we are trying to convert sources in to voltage sources when they are in series and in to current sources when they are in parallel.

The resulting equivalent circuits for the given problem are shown below.
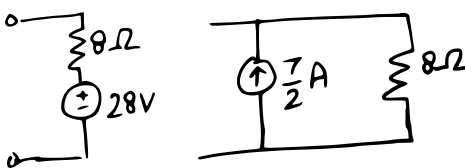

Figure 5:

(5) Problem 2.49.

Solution:

This circuit has a dependent source in it. However, the controlling variable is the current going to the terminals to the left. This is beneficial since V_{oc} is open circuit voltage across the terminals, hence $i = 0$ making the dependent voltage source equal to 0 V. Under these conditions, applying KVL around the lone mesh gives $V_{oc} = 8$ V. And by short circuiting the two terminals on the left hand side and applying KVL again on the resulting mesh, we get $I_{sc} = 14$ A. Hence $R_{eq} = V_{oc}/I_{sc} = \frac{4}{7}$ Ω. Therefore, the Thevenin equivalent circuit of the given circuit is $\frac{4}{7}$ Ω in series with 8 V.

(6) Problem 3.15.

Solution:

In this given circuit, note that the controlling variable in the dependent source in the right hand side part is in the left hand side part. Applying KVL around the loop on the left and solving gives

$$v_1 = 3 \text{ V.}$$

This makes the dependent voltage source on the right hand side 12 V. Using voltage divider and current divided rules, $i = 1$ A.

(7) Problem 3.26.

Solution:

To solve this problem, we assume ideal op-amps and apply KCL on all nodes except at the output nodes of op-amps while using the *virtual open* and *virtual short* principles. The first principle means that there is no current passing through the input terminals of an ideal op-amp and the

second one means that there is no potential difference between the two input terminals of an ideal op-amp.

Applying KCL on the inverting input of the first op-amp and simplifying gives

$$v_1 = -v_{o_1}$$

where v_{o_1} is the potential at the output node of the first op-amp w.r.t. ground (bottom node).

Next, applying KCL on the inverting node of the second op-amp and simplifying, gives

$$v_2 = -2v_{o_1}$$

Combining the above two equations gives

$$v_2 = 2v_1.$$

Hence, $k = 2$.

(8) Problem 3.34.

Solution:

Assume ideal op-amps and apply KCL on all nodes except at the output nodes of op-amps while using the *virtual open* and *virtual short* principles.

Applying KCL on the inverting input of the first op-amp and simplifying gives

$$5v_1 = v_{o_1} \tag{3}$$

where v_{o_1} is the potential at the output node of the first op-amp w.r.t. ground (bottom node).

Next, applying KCL on the inverting node of the second op-amp and simplifying, gives

$$2v_2 = v_{o_2} \tag{4}$$

where v_{o_2} is the potential at the output node of the second op-amp w.r.t. ground (bottom node).

Applying KCL on the inverting node of the third op-amp and simplifying, gives

$$-2v_{o_1} - 4v_{o_2} - 48 = v_3. \tag{5}$$

Elimination v_{o_1} and v_{o_2} from Eq. 5 using Eqs. 3 and 4, we get

$$v_3 = -10v_1 - 8v_2 - 48$$

(9) Problem 3.40.

Solution:

Assume ideal op-amps and apply KCL on all nodes except at the output nodes of op-amps while using the *virtual open* and *virtual short* principles.

Applying KCL on the inverting input of the first op-amp and simplifying gives

$$v_{o_1} = -v_2 \tag{6}$$

where v_{o_1} is the potential at the output node of the first op-amp w.r.t. ground (bottom node).

Next, applying KCL on the inverting node of the second op-amp and simplifying, gives

$$v_{o_2} = 2v_{o_1} \tag{7}$$

where v_{o_2} is the potential at the output node of the second op-amp w.r.t. ground (bottom node).

Applying KCL on the inverting node of the third op-amp and simplifying, gives

$$v_3 = v_{3-} \tag{8}$$

where v_{3-} is the potential at the inverting node of this op-amp.

Finally, applying KCL at the non-inverting node of the third op-amp and simplifying using all the above equations gives

$$v_3 = \frac{2}{3}v_1 - 2v_2$$

References

[1] D. E. Johnson, J. R. Johnson, J. L. Hilburn, and P. D. Scott, *Electric Circuit Analysis.* Upper Saddle River, New Jersey 07458: Prentice Hall, third ed., 1997.