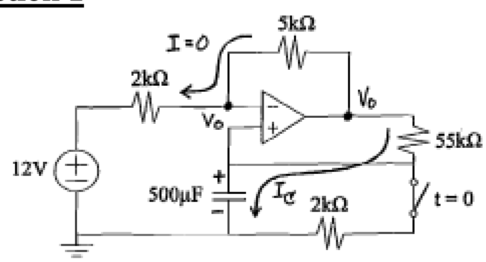
Assignment 6 Solutions
Question 1



(a) Find the net charge stored on the upper plate of the capacitor at time $t = 0^-$.

(b) Find the exponential time constant of the circuit for time $t > 0$.

a) Circuit is in DC steady-state operation at $t = 0^-$;

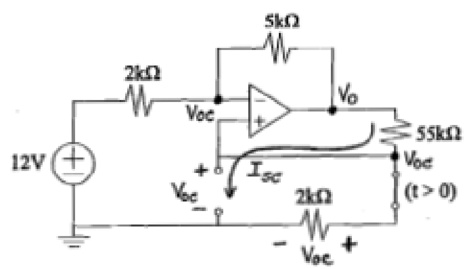
$\Rightarrow \quad I_C = 0 \quad \Rightarrow \quad I_{55k\Omega} = 0$ (Switch open!)

$\Rightarrow \quad V_+ = V_- = V_O = V_C \quad \Rightarrow \quad I_{5k\Omega} = I_{12V} = 0$

$\Rightarrow \quad V_C(0^-) = 12V \; ; \; Q^{UP} = C\,V_C(0^-) = +6mC$

b) $T_C = R_{Th}\,C \; ; \; R_{Th} = V_{OC} / I_{SC}$ ($t > 0$ circuit!)

$\Rightarrow$ Find V_{OC} and I_{SC} (w.r.t. C ; switch closed)



Find V_{OC} :

KCL @ V_- : $(12 - V_{OC}) / 2000 = (V_{OC} - V_O) / 5000$

$\Rightarrow \quad 7\,V_{OC} - 2\,V_O = 60$;

KCL @ V_+ : $(V_O - V_{OC}) / 55000 = V_{OC} / 2000$

$\Rightarrow \quad 2\,V_O = 57\,V_{OC}$;

Therefore: $V_{OC} = -6 / 5$ V

Find I_{SC} :

$I_{SC} = V_O / 55000$; $V_O = -(12 / 2000)(5000) = -30V$

Find R_{Th} : $R_{Th} = V_{OC} / I_{SC} = 2200\Omega$

Find T_C : $T_C = R_{Th}\,C = (2200)(0.0005) = 1.1s$

Question 2

(a) $V_o(t) = V_o(\infty) + [V_o(0) - V_o(\infty)]\, e^{-t/R_{Th}C} \;:\; t > 0;$

$\qquad = 0 + 5\, e^{-500t/R_{Th}} \;:\; t > 0.$

$\Rightarrow R_{Th}$ REQUIRED... (SEEN AT TERMINALS OF "C")

⊛ DRIVE $t>0$ CIRCUIT WITH $-1V$ SOURCE;

$\Rightarrow R_{Th} = R_{IN} = 1/I_o = \left[4\left(\frac{1}{5} + \frac{1}{20}\right)\right]^{-1} = 1\Omega$ //

$\Rightarrow V_o(t) = 5\, e^{-500t}\ V : t > 0$ //

(b) $P_{ABS} = (0.75 I_o)(-V_o)$ WHERE $I_o = -V_o$ (SHOW!)

$\Rightarrow P_{ABS} = 0.75 I_o^2 = 0.75 V_o^2 = 18.75\, e^{-1000t}\ W : t > 0$ //

Question 3

Yes, the op-amp output current I_O must vary with time. The $t > 0$ capacitor charging current varies with time, however, the source current does not. Therefore, KCL at the op-amp output requires I_O must vary with time.

$V_C(t) = V_C(\infty) + [V_C(0\pm) - V_C(\infty)]\, e^{-t/T_C} \;:\; t > 0$

$V_C(0\pm) = 0\,;\; V_C(\infty) = V_O = -(0.005)(1200) = -6V\,;$

$R_{Th} = V_{OC} / I_{SC} = 30k\Omega\,;\; \{V_{OC} = V_O\,;\; I_{SC} = V_O / 30k\Omega\}$

$T_C = R_{Th} C = (30k\Omega)(800nF) = 24ms\,;$

☞ $V_C(t) = -6 + 6\, e^{-t/0.024}\ V \;:\; t > 0.$

$P_{5mA} = 0.005V_1 = (0.005)[(800)(0.005)] = 20mW : t > 0$

Question 4

Switch closed: $T_C = (20k\Omega)(800\mu F) = 16s\,;$

$5T_C \to 10T_C = $ 1min 20s $\to$ 2min 40s.

$V_C(t_0) = 6V \Rightarrow V_S(t_0^+) = (20k\Omega \parallel 5k\Omega)(3mA) = 12V;$

O/C $800\mu F \Rightarrow V_S(t \to \infty) = (5k\Omega)(3mA) = 15V;$

Switch open $\Rightarrow T_C = (20k\Omega + 5k\Omega)(800\mu F) = 20s\,;$

$\Rightarrow V_S(t) = c_1 + c_2 e^{-(t-t_0)/T_C} = 15 - 3\, e^{-0.05(t-t_0)}\ V \;:\; t > t_0.$

$V_C(t \to \infty) = 0 \Rightarrow \min W_C\,;$ Set $(3mA)R = 6V \Rightarrow R = 2k\Omega$

$V_C(t \to \infty) \to \infty \Rightarrow \max W_C\,;$ Set $(3mA)R \to \infty \Rightarrow R \to \infty$

Question 5

$t < 0$ (charging) $T_C = R_{Th} C = (20k\Omega)(5\mu F) = \mathbf{0.1s}$

$\Rightarrow$ Circuit in steady-state at $t = 0^-$, because switch at position "a" for $2s = 20\,T_C$ // ($> 5\text{-}10\ T_C$'s)

$V_C(0^-) \cong (4mA)(20k\Omega) = 80V \quad \Rightarrow V_C(0^+) \cong \mathbf{80V}$

$V_C(t\to\infty) = 120V$; $T_C(t>0) = (25k\Omega)(5\mu F) = \mathbf{0.125s}$

$\Rightarrow V_C(t) \cong \mathbf{120 - 40e^{-8t}\ V : t > 0}$ //

$Q_C(t) = C\,V_C(t)$: $\quad (t = 0^+ \Rightarrow V_{MIN}$; $t\to\infty \Rightarrow V_{MAX})$

$$Q_{MIN} = C\,V_{MIN} \cong (5\mu F)(80V) = \mathbf{400\mu C}$$
$$Q_{MAX} = C\,V_{MAX} = (5\mu F)(120V) = \mathbf{600\mu C} \text{ //}$$

Question 6

(c) Suppose $Q_{MAX}(d) = C(d)\,\underline{V_{MAX}(d)} \cong C_0\underline{V_0}$:

$\Rightarrow Q_{MAX}(a) \cong (3C_0/2)(\mathbf{2}V_0/\mathbf{3}) = \mathbf{C_0V_0}$

$\Rightarrow Q_{MAX}(b) \cong (5C_0/4)\underbrace{(\mathbf{2}V_0/\mathbf{3})}_{?} = \mathbf{\frac{5}{6}C_0V_0}$

But, what is $\underline{V_{MAX}(c)}$? (How can we calculate it?)

For "series" interpretation, $Q(3\epsilon_0) = Q(2\epsilon_0)$?!!

$\Rightarrow C(3\epsilon_0)\,V(3\epsilon_0) = C(2\epsilon_0)\,V(2\epsilon_0)$;

$\Rightarrow V(2\epsilon_0) = 1.5\,V(3\epsilon_0) \quad \Rightarrow \quad V_{MAX}(c) = \mathbf{\frac{5}{6}V_0}$?!!

$\Rightarrow Q_{MAX}(c) \cong (6C_0/5)(\mathbf{5}V_0/\mathbf{6}) = \mathbf{C_0V_0}$

$\Rightarrow Q_{MAX}(\mathbf{a}) \cong Q_{MAX}(\mathbf{c}) \cong Q_{MAX}(\mathbf{d}) > Q_{MAX}(\mathbf{b})$ //

(a) $C(d) = C_0$: $C(a) \cong \mathbf{\frac{3}{2}C_0}$

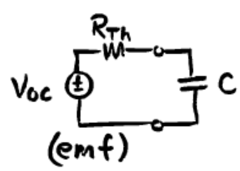
$C(b) \cong C_0/2 + (3C_0/2)/2 = \mathbf{\frac{5}{4}C_0}$

$C(c) \cong [1/(2C_0) + 1/(3C_0)]^{-1} = \mathbf{\frac{6}{5}C_0}$

$\Rightarrow C(\mathbf{d}) < C(\mathbf{c}) < C(\mathbf{b}) < C(\mathbf{a})$ //

(b) Smallest charging circuit $T_C \Rightarrow$ shortest charge time ;

$T_C = [R_{Th}(\text{source})]\,C \Rightarrow$ SMALLEST $C = C(d)$



OR, WHATEVER YOU INDICATED IN YOUR ANSWER TO (a)...

Question 7

Trigger OFF: $V_T = V_C + V_{1\Omega}$; Trigger ON: $I_T = I_C + 3/9$;

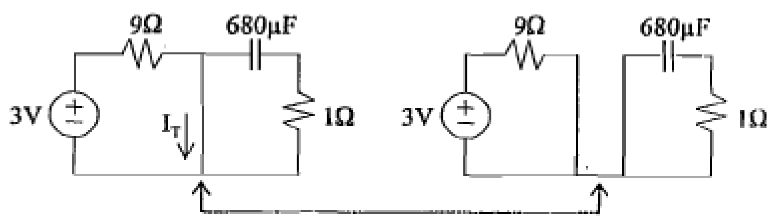
$V_T(0^+) = \overbrace{V_C(0^+)}^{0} + \overbrace{V_{1\Omega}(0^+)} = 0.3\text{V} < 2\text{V} \Rightarrow$ Trigger OFF;

Trigger turns "ON" when $V_C + V_{1\Omega} = 2\text{V}$, i.e. when $V_{9\Omega} = \underline{1\text{V}}$

Therefore, after closing the switch (trigger OFF) the capacitor <u>charges</u> up until I_C drops to $\underline{1/9\text{A}}$; then the trigger turns ON and the capacitor <u>discharges</u> through the bulb (1Ω resistor) until I_C reduces to $0.4 - \underline{0.33} = \underline{0.07}$; then the trigger turns OFF and the capacitor begins to <u>recharge</u>. *Any light when trigger OFF?*

Trigger OFF: max $(P_{Bulb}) = [V_{Bulb}(0^+)]^2 = 90\text{mW} \Rightarrow$ no light.

Trigger ON: max $(P_{Bulb}) = [V_{Bulb}(\text{ON}^+)]^2 = [V_C(\text{ON}^+)]^2$.



KVL & voltage divider $\Rightarrow [V_C(\text{ON}^+)] = 3 - \underline{(9+1)(1/9)} \approx 1.89\text{V}$

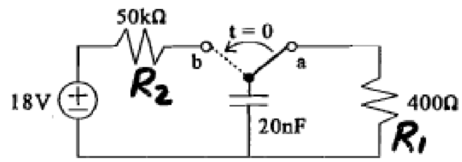
$\Rightarrow$ max $(P_{Bulb}) = [V_C(\text{ON}^+)]^2 \approx \underline{3.57\text{W}}$ ($\Rightarrow$ light output)

Light output <u>continues</u> until V_{Bulb} drops to 0.4V ($I_{Bulb} = 0.4\text{A}$);

$\Rightarrow$ Light output stops <u>before</u> trigger turns OFF.
(because 3V source contributes 1/3A to I_T)

$\Rightarrow$ Bulb flash energy $= 0.5\,C\,[(1.89)^2 - (0.4)^2] \approx \underline{1.16\text{mJ}}$
(energy discharged by capacitor during bulb flash) (?!)

Question 8



NB. $T_c(0^-) = R_1 C = 8\,\mu s$

$\Rightarrow V_c(0^-) = 0$

$$Q_{MAX} = Q(\infty) = C\,V(\infty)$$
$$= 20\times10^{-9}(18) = 360\,nC$$

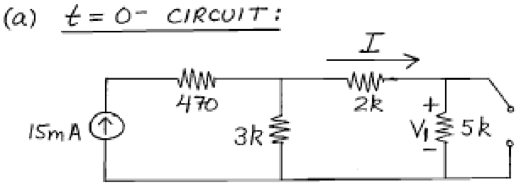
$Q = 0.99\,Q_{MAX}$ AT $t \sim 4.6\,T_c(0^+)$

(SOLVING $0.01 = e^{-t/T_c} \Rightarrow 4.6T_c$)

$T_c(0^+) = R_2 C = 1ms \Rightarrow t = 4.6ms$

Question 9

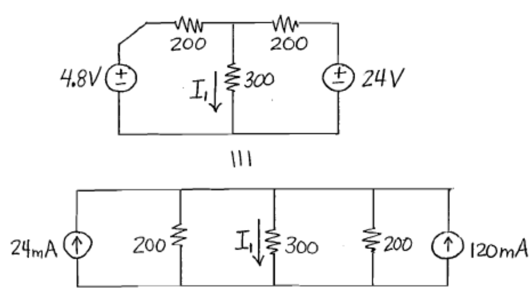
(a) $t = 0^-$ CIRCUIT:



$$I = \frac{3000}{2000 + 5000 + 3000} \times 0.015 = 4.5mA$$

$V_1(0^-) = 5000\,I = 22.5\,V$

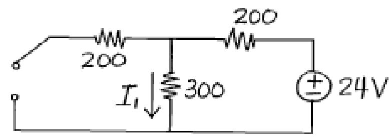
(b) $t = 0^+$ CIRCUIT:



$$I_1(0^+) = \frac{200\|200}{300 + 200\|200}(0.024 + 0.120)$$

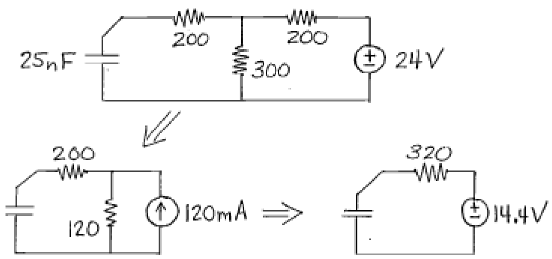
$I_1(0^+) = 36\,mA$

(c) $t \to \infty$ CIRCUIT:



$$I_1(\infty) = \frac{24}{200+300} = 48\,mA$$

(d) $t > 0$ CIRCUIT:



HENCE, $\tau_c(0^+) = R_{Th}\,C$

$$= 320\,(25 \times 10^{-9})$$

$$= 8\,\mu s$$

(e) (BASED ON $t > 0$ CIRCUIT...)

$$V_2(t) = 300\, I_1(t) \quad : \; t > 0$$

$$I_1(t) = I_1(\infty) + [I_1(0^+) - I_1(\infty)]\, e^{-t/\tau_c}$$

$$= 0.048 + (-0.012)\, e^{-125000t}$$

$$\Rightarrow V_2(t) = 14.4 - 3.6\, e^{-125000t}\ V : t > 0$$

Question 10

Will be posted on Monday

Question 11

Will be posted on Monday