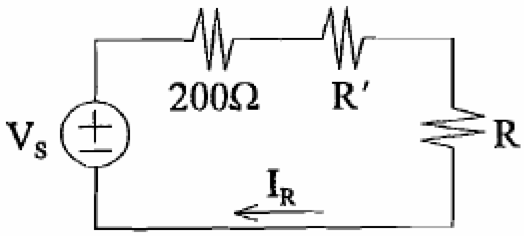
Assignment 5 Solutions

Question 1

(a) Find R_{Th} seen by R, for $R' = 50\Omega$; set $R = R_{Th}$.

$$R_{Th} = R' + (100 + 300) \parallel (V_{OC} / I_{SC})$$
$$= 50 + 400 \parallel 400 = 250\Omega$$

(b)



$$P_R = (I_R)^2 R$$
$$\max(I_R) \Rightarrow \max(P_R)$$
$$R' = 0 \Rightarrow \max(I_R)$$
(R′ non-negative!)

(c) $P_R = (I_R)^2 R$ where $I_R = V_S / (200 + R' + R)$

$\therefore$ $R' = 0 \Rightarrow \max(P_R)$ $\forall$ non-negative R

$\therefore$ Set $R' = 0$ and $R = R_{Th} = 200 + R' = 200\Omega$

Question 2

(a) Max. heating $\Rightarrow$ max. power $\Rightarrow$ set $R = R_{Th}$

(Using "generalized KCL" – see class notes...)

$$-(V_{OC} + 2I')/4 + 8 + I' = 0; \quad I' = -V_{OC}/2$$
$$\Rightarrow V_{OC} = 16V \quad (V_{OC} \text{ circuit ?!})$$

$$-(V_2 + 2I'')/4 + 8 + I'' - V_2/6 = 0;$$
$$I'' = -V_2/2 \Rightarrow V_2 = 12V;$$
$$\Rightarrow I_{SC} = 2A \quad (I_{SC} \text{ circuit ?...})$$

Therefore, set $R = R_{Th} = V_{OC} / I_{SC} = 8\Omega$

(b) $P_R = ½ (V_{OC})^2 / (R + R_{Th}) = ½ (16)^2 / 16 = 8W$

Question 3

MODIFY INPUT CIRCUIT (THEV. EQ.) TO SIMPLIFY:

$V_{OC} = (0.03)250 = 7.5V$; $R_{Th} = 500\Omega$ //

(a) SOLVE $R = R_{Th}$ (SEEN AT TERMINALS OF "R").

$I = 7.5/500 = 15mA$; $V_o = -(0.015)600 = -9V$;

$\Rightarrow V_{OC} = V_o = -9V$; $I_{SC} = V_o/450 = -20mA$;

$\Rightarrow R_{Th} = V_{OC}/I_{SC} = 450\Omega$ // (SET $R = 450\Omega$ //)

(b) P_{30mA} SUPPLIED $= P_{100} + P_{250} + P_{250}$ ABSORBED ;

$\Rightarrow P_{OP\text{-}AMP} = I^2 600 + V_o^2/(900) = 225mW$ //

Question 4

Set $R = R_{Th}$ for circuit connected to R: $(R_{Th} = V_{OC} / I_{SC})$

- ☞ open-circuit R $\Rightarrow I_O = -4I_O \Rightarrow I_O = 0 \Rightarrow V_{OC} = 20V$
- ☞ short-circuit R $\Rightarrow I_{SC} = 5I_O$ <u>and</u> KCL $\Rightarrow V_\diamond = 4V$
- ☞ $I_{SC} = 200mA$ <u>and</u> $R_{Th} = 100\Omega \Rightarrow$ set $R = 100\Omega$.

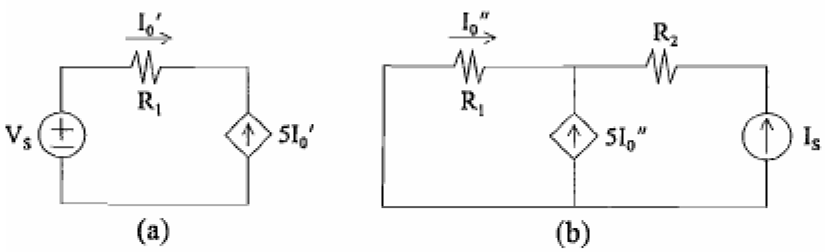
<u>Also:</u> $P_R = ½ P_{20V} = ½ (20)^2 / (100 + 100) = 1W$.

Solve $20I_O = (20 - 400I_O)(4I_O)$ for I_O, to find $P_o = P_\diamond$:

- ☞ $I_O = 37.5mA \rightarrow I_R = 5I_O = 187.5mA$
- ☞ $V_{OC} = I_R (R_{Th} + R) \rightarrow R = V_{OC}/I_R - R_{Th} = 6\,⅔\,\Omega$.

Question 5

Answer: (Two independent sources ⇒ two circuits.)



Also: Briefly outline the steps you would take to solve for the power supplied by the voltage source using the superposition circuit solutions for the current "I_0".

Solve for I_0' using circuit (a).

Solve for I_0'' using circuit (b).

Calculate $P_{Supplied} = V_S (I_0' + I_0'')$.

Question 6

Optimal $R_{BULB} = R_{Th} = V_{OC} / I_{SC} = 12 / 3⅓ = 3.6\,\Omega$;

$\Rightarrow P_{BULB} = 0.5\,(V_{OC})^2 / (R_{BULB} + R_{Th}) = 10W$.

Non-zero $R_{WIRE} \Rightarrow R_{LOAD} = R_{BULB} + R_{WIRE} \neq R_{Th}$;

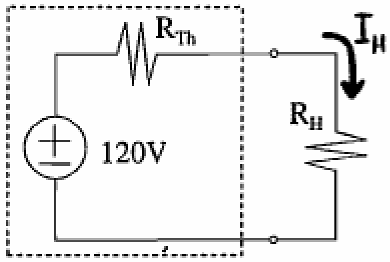
$\Rightarrow$ Unmatched load $\Rightarrow P_{OUT}$ decrease.

Require $R_{EQ} = 3.6\,\Omega$; but, $R_{60W} = (12)^2 / 60 = 2.4\,\Omega$;

$\Rightarrow$ Use three 60W bulbs: one in series with two in parallel.

Note: $2.4\,\Omega + 2.4\,\Omega \,||\, 2.4\,\Omega = 3.6\,\Omega$.

Question 7



(Battery)

$R_{Th} = V_{OC} / I_{SC}$

$= 120V / 15A = 8\Omega$

Set $R_H = R_{Th}$ for P_{MAX}

$\Rightarrow$ Use $8\Omega / 2\Omega m^{-1} = 4m$ //

(b) $P_H = (V_H)^2 / R_H = (120V / 2)^2 / 8\Omega = 450\ W$ //

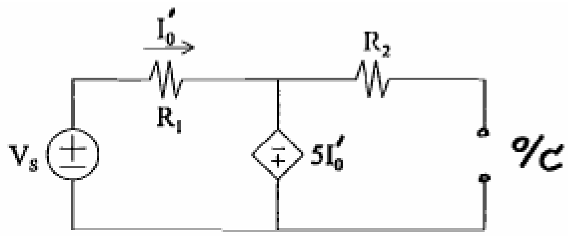
(c) $I_H = V_H / R_H = (120V / 2) / 8\Omega = 7.5A$

$\Rightarrow$ 2.5A limit means I_H must divide 3 ways...

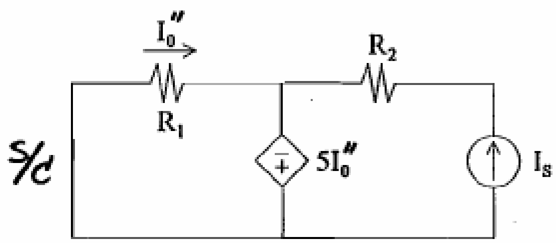
Minimum cost $\Rightarrow$ 3 equal length parallel wires...

$\Rightarrow$ $3 \times 24\Omega$ $\Rightarrow$ $72\Omega / 2\Omega m^{-1} = 36\ m$ //

Question 8



Solve for I_0', current due to V_S alone.



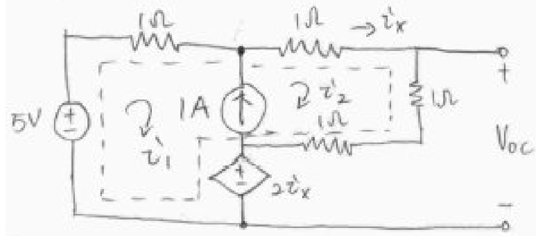
Solve for I_0'', current due to I_S alone.

Total current: $I_0 = I_0' + I_0''$

Question 11

Find the Thevènin Equivalent Circuit, maximum power will be delivered if R = R_{th}.

1) V_{oc}



Supermesh: $5 - i_1 - 3i_2 - 2i_x = 0$

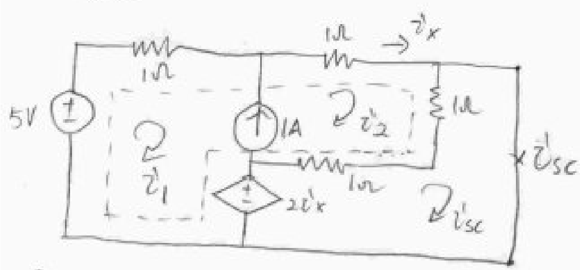
$i_2 - i_1 = 1$

$i_2 = i_x$

$\Rightarrow i_1 = 0$, $i_2 = 1A$

$V_{oc} = 2i_2 + 2i_x = 4V$

2) i_{sc}



Supermesh: $5 - i_1 - 3i_2 - 2i_x + 2i_{sc} = 0$

mesh 3 : $2i_{sc} - 2i_x - 2i_2 = 0$

$i_2 - i_1 = 1$

$\Rightarrow i_1 = 2A$, $i_2 = 3A$, $i_{sc} = 6A$

3) R_{th}

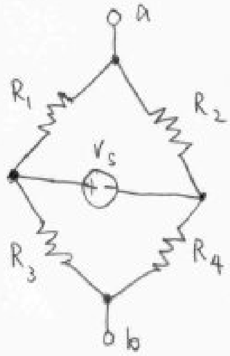
$$R_{th} = \frac{V_{oc}}{i_{sc}} = \frac{4V}{6A} = \frac{2}{3}\Omega$$

Hence, $R = R_{th} = \frac{2}{3}\Omega$ $\quad P = \frac{\left(\frac{V_{oc}}{2}\right)^2}{R} = 6W$

Question 12

First find the Thevenin equivalent of the bridge circuit, simplify the circuit, then find v_o.

1) V_{oc}



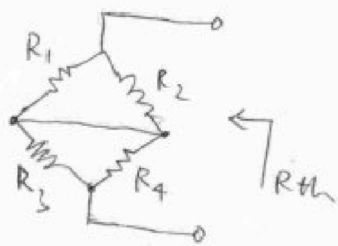
$$V_{oc} = V_{ab} = V_a - V_b$$

By voltage division:

$$V_a = \frac{R_2}{R_1+R_2} V_s \quad , \quad V_b = \frac{R_4}{R_3+R_4} v_s$$

$$V_{oc} = \left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right) v_s$$

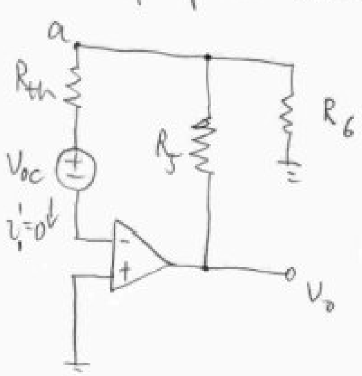
2) R_{th}



$$R_{th} = R_1 // R_2 + R_3 // R_4$$

$$= \frac{R_1 R_2}{R_1+R_2} + \frac{R_3 R_4}{R_3+R_4}$$

3) simplified circuit



$$\left.\begin{array}{l} v_a = v_{oc} + R_{th} i_1 \\ i_1 = 0 \end{array}\right\} \Rightarrow v_a = v_{oc}$$

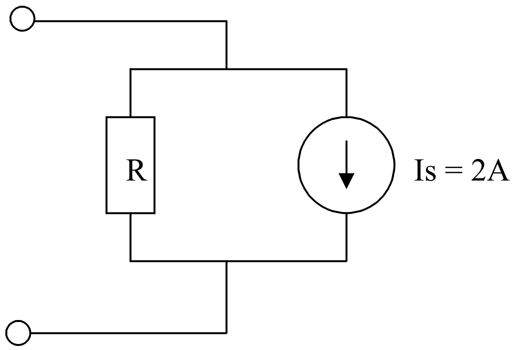
At node a using KCL:

$$i_1 + \frac{v_a - v_o}{R_5} + \frac{v_a}{R_6} = 0$$

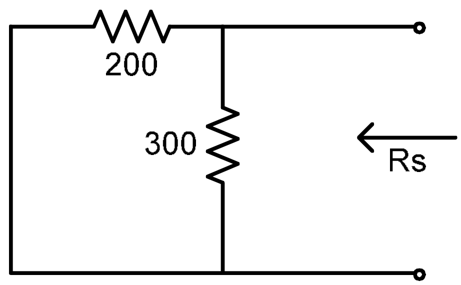
$$\frac{v_{oc} - v_o}{R_5} + \frac{v_{oc}}{R_6} = 0$$

$$v_o = \left(1 + \frac{R_5}{R_6}\right) v_{oc} = \left(1 + \frac{R_5}{R_6}\right)\left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right) v_s \quad //$$

Question 13

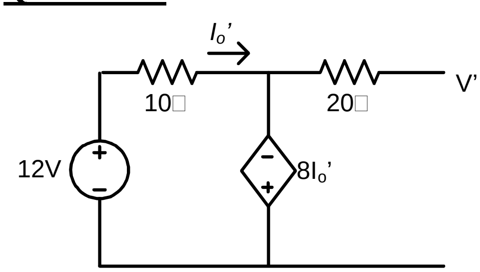


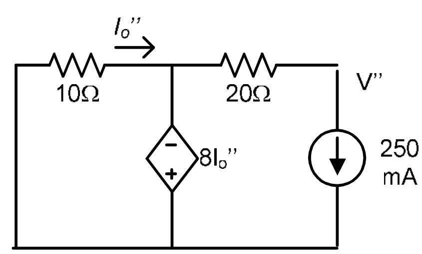
Question 14



Rs = 200||300 = (200*300)/500 = 120
20 || R = 20;
R → ∞

Question 15





10Io’-8Io’-12=0
2Io’=12
Io’=6 ➔ V’=-48V

KVL: -8Io’’+10Io’’= 0 ➔ Io’’=0; V’’= -0.25*20 = -5V
Therefore, P = I(V’+V’’) = 0.25*(-48-5)= -13.25W (Supplied), Watts is Joules/second.